Unit 6 – Solving Oblique Triangles - Classwork

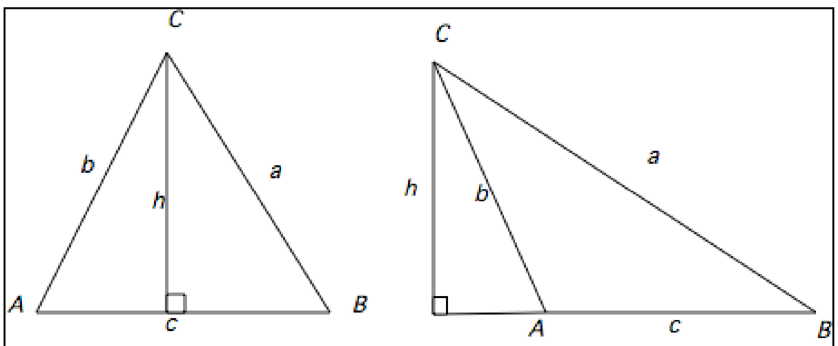
A. The Law of Sines – ASA and AAS

In geometry, we learned to prove congruence of triangles – that is when two triangles are exactly the same. We used several rules to prove congruence: Angle-Side-Angle (ASA), Angle-Angle-Side (AAS), Side-Angle-Side (SAS) and Side-Side-Side (SSS). In trigonometry, we take it a step further. For instance, if we know the values of two angles and a side of a triangle, we can solve that triangle … that is we can find the other angle and the other sides.

We have learned to solve right triangles in Unit 3. In this section we learn how to solve oblique triangles – triangles that **do not** have a right angle.

First, let's start with a generalization for this section. All triangles will have 6 pieces of information – 3 angles and 3 sides. The angles are labeled *A*, *B*, and *C* and the sides are opposite the angles and are labeled *a*, *b*, and *c*.

Note that right angle trigonometry doesn't help us here. There is no right angle, thus no hypotenuse. We need something else. That something is called the Law of Sines.



Acute triangle *ABC* Obtuse triangle *ABC*

In both triangles, we drop a perpendicular h.
In both triangles we can then say:

$\sin A = \dfrac{h}{b}$ and $\sin B = \dfrac{h}{a}$

So $h = b\sin A$ and $h = a\sin B$

It follows that $a\sin B = b\sin A$

and thus $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

The Law of Sines

If *ABC* is a triangle with sides *a, b*, and *c*, then $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

The Law of Sines is really three laws in one: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ and $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$. In each one, there are 4 variables. If you know 3 of them, you can find the 4th.

Example 1) – AAS

$A = 23°$ $a = 14$

$B = 48°$ $\boxed{b = 26.63}$

$\boxed{C = 109°}$ $\boxed{c = 33.88}$

Example 2) - ASA

$A = 72.4°$ $\boxed{a = 3.88 \text{ in}}$

$\boxed{B = 68.9°}$ $b = 3.8 \text{ in}$

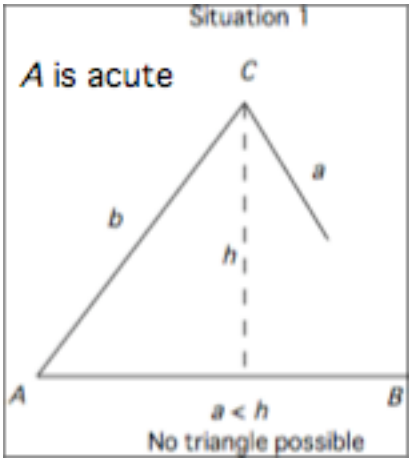
$C = 38.7°$ $\boxed{c = 2.55 \text{ in}}$

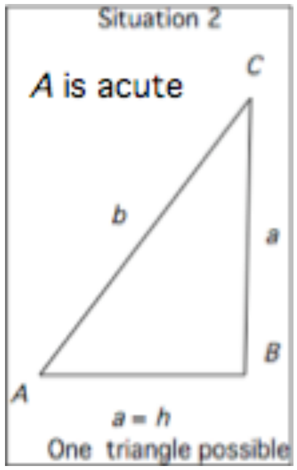
Most work is done on the calculator. Your job is to show what formulas you are using. To check a problem, verify that the largest angle is opposite the largest side and the smallest angle is opposite the smallest side.

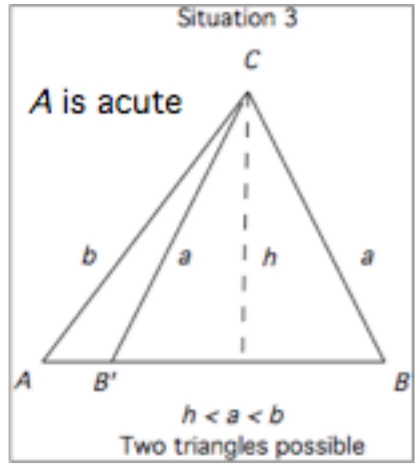
B. The Law of Sines - SSA

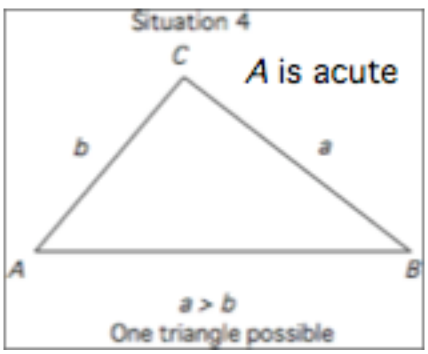
One of the rules for congruence is not Side – Side- Angle (SSA). You may have wondered why not. The problem with SSA is that while 2 sides and an angle may identify a triangle, it is possible that the triangle may not exist with that information. Or it is possible that 2 sides and an angle may identify two possible triangles. Let's examine the SSA case more in depth and find that it breaks up into 6 possible situations.

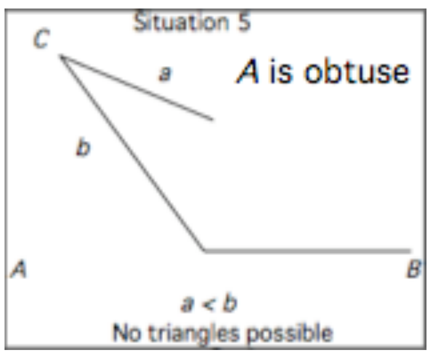
For all 6 situations, we will assume that you are given *a*, *b*, and *A*.

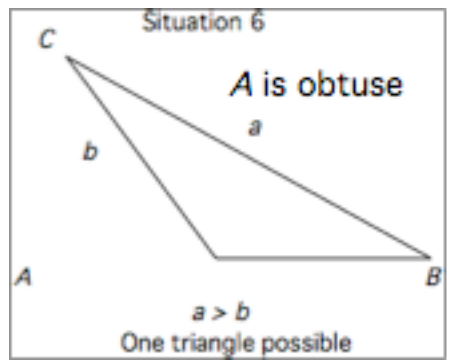












Before we actually attempt to solve a triangle in the SSA case, we must decide which of the 6 situations above the problem form is. In most cases, a simple drawing can help us decide.

Example 3)

$A = 62°$ $\quad$ $a = 5$

$B =$ ________ $\quad$ $b = 15$

$C =$ ________ $\quad$ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{None}}$

Example 4)

$A = 31°$ $\quad$ $a = 22$

$B =$ ________ $\quad$ $b = 9$

$C =$ ________ $\quad$ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{One}}$

Example 5)

$A = 99°$ $a = 9.2$
$B =$ ________ $b = 5.5$
$C =$ ________ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{One}}$

Example 6)

$A = 125°$ $a = 16$
$B =$ ________ $b = 30$
$C =$ ________ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{None}}$

Example 7)

$A = 30°$ $a = 7$
$B =$ ________ $b = 14$
$C =$ ________ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{One}}$

Example 8)

$A = 25°$ $a = 11$
$B =$ ________ $b = 14$
$C =$ ________ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{One}}$

Example 9)

$A = 38°$ $a = 12.9$
$B =$ ________ $b = 21$
$C =$ ________ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{None}}$

Example 10)

$A = 38°$ $a = 13$
$B =$ ________ $b = 21$
$C =$ ________ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{Two}}$

In the last two examples, we find that example 9 is not possible to draw while example 10 is not only possible, but two triangles are possible (it is called the ambiguous case). Yet there is only a 0.1 difference in the value of *a*. It is impossible to tell by eye. So how can you tell?

The answer lies in the Law of Sines. If we set up the Law of Sines with the *A* and *B* family, we get $a\sin B = b\sin A$ and $\sin B = \dfrac{b\sin A}{a}$. Since we know that the largest value of $\sin B$ is 1, we can determine whether a triangle can exist.

If $\sin B = \dfrac{b\sin A}{a} > 1$, the triangle is impossible.	if $\sin B = \dfrac{b\sin A}{a} < 1$, there are two triangles possible.

So let us show that Example 9 has no triangle possible while triangle 10 has two triangles possible.

Example 9) - calculations

$A = 38°$ $\quad a = 12.9$

$B =$ ________ $\quad b = 21$

$C =$ ________ $\quad c =$ __________

$$\sin B = \frac{b\sin A}{a} = 1.00224$$

Example 10) - calculations

$A = 38°$ $\quad a = 13$

$B =$ ________ $\quad b = 21$

$C =$ ________ $\quad c =$ __________

$$\sin B = \frac{b\sin A}{a} = .99453$$

Note that the only time we need to go through this step is when it is unclear from the drawing whether the triangle can be drawn. This occurs when there is a question when the SSA problem is situation 1 or situation 3 (situation is very rare as it yield a perfect right triangle.)

Another way to determine quickly if obtuse triangles are possible is to remember the fact that in any triangle, the largest angle must always be opposite the largest side. Example 6 above clearly is impossible to draw because angle $A = 125°$ must be the largest angle in the triangle. Which means that side *a* must be the largest side. But since *a* is given to be16 and *b* is given to be 30, the triangle is impossible.

<u>IMPORTANT:</u> Also note that this phenomenon only occurs in an SSA situation. ASA and AAS are always defined with one and only one triangle and the Law of Sines solves this easily.

Now that we have decided how to determine whether or not an SSA problem has a solution, we need to actually solve it. Again, let's assume that we are given *a*, *b*, and angle *A* and the SSA problem has only one solution.

Step 1: Find angle *B* by using the fact that $B = \sin^{-1}\left(\dfrac{b\sin A}{a}\right)$

Step 2: Find angle *C* by using the fact that $A + B + C = 180°$ so $C = 180° - A - B$

Step 3: Use the Law of Sines with the *A* family and the *C* family.

$$\frac{a}{\sin A} = \frac{c}{\sin C} \text{ so } c = \frac{a\sin C}{\sin A}$$

Step 4: Quickly verify your answer by checking to see the largest side is opposite the largest angle and the smallest side is opposite the smallest angle.

Let's do several problems that we examined before.

Example 11)

$A = 31°$ $a = 22$

$\boxed{B = 12.16°}$ $b = 9$

$\boxed{C = 136.84°}$ $\boxed{c = 29.22}$

Example 12)

$A = 99°$ $a = 9.2$

$\boxed{B = 36.19°}$ $b = 5.5$

$\boxed{C = 44.81°}$ $\boxed{c = 6.56}$

Example 13)

$\boxed{A = 13.42°}$ $a = 19$

$B = 160°$ $b = 28$

$\boxed{C = 6.58°}$ $\boxed{c = 9.38}$

Example 14)

$\boxed{A = 38°23'}$ $a = 1$ ft 10 in

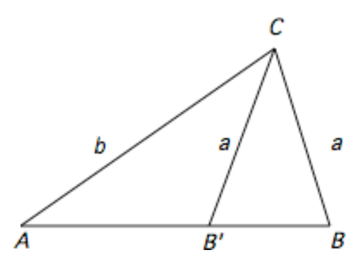
$\boxed{B = 94°24'}$ $\boxed{b = 2 \text{ ft } 11.32 \text{ in}}$

$C = 47°13'$ $c = 2$ ft 2 in

Finally, let's go through the procedure we use when you determine that there are two solutions. Remember, the only way this can happen is if $A < 90°$ and $a < b$. You draw the situation and realize that there are two ways to draw the triangle. When in doubt use the fact that if $\sin B = \dfrac{b \sin A}{a} > 1,$ the triangle is impossible and if $\sin B = \dfrac{b \sin A}{a} < 1,$ there are two triangles possible.

Assuming that there are two triangles possible, here is the procedure.

1. Draw the situation. Show both triangles. ΔABC will be the acute triangle and $\Delta AB'C$ will be the obtuse triangle.
2. Solve ΔABC first using the procedure above. (a reminder)
   a. Find B by using the fact that $B = \sin^{-1}\left(\dfrac{b \sin A}{a}\right)$
   b. Find C by using the fact that $C = 180° - A - B$
   c. Find big c by using the fact that $c = \dfrac{a \sin C}{\sin A}$
3. Solve $\Delta AB'C$ by using the procedure below.
   a. Find angle B' in $\Delta BB'C$ by realizing that $\Delta BB'C$ is isosceles.
   b. Find obtuse angle B' in $\Delta AB'C$ by using the fact that $\angle AB'C = 180° - \angle BB'C$
   c. Find little angle C by using the fact that $C = 180° - A - B'$
   d. Find little c by using the fact that $c = \dfrac{a \sin C}{\sin A}$



This looks like a lot of work but triangle ABC is solved using the basic procedure for one triangle. The fact that triangle $BB'C$ is isosceles makes solving the obtuse triangle easy.

Example 13)

$A = 25°$ $\quad a = 11$

$\boxed{B = 32.54°}$ $\quad b = 14$

$\boxed{C = 122.46°}$ $\quad \boxed{c = 21.96}$

$A = 25°$ $\quad a = 11$

$\boxed{B' = 147.46°}$ $\quad b = 14$

$\boxed{C = 7.54°}$ $\quad \boxed{c = 3.42}$

Example 14)

$A = 38°$ $\quad a = 13$

$\boxed{B = 84°}$ $\quad b = 21$

$\boxed{C = 58°}$ $\quad \boxed{c = 17.91}$

$A = 38°$ $\quad a = 13$

$\boxed{B' = 96°}$ $\quad b = 21$

$\boxed{C = 46°}$ $\quad \boxed{c = 15.19}$

We have now solved triangles in the form of ASA, AAS, and if possible, SSA. That leaves us with the cases of SAS, and SSS. Here are two examples. Explain why the Law of Sines does not help us solve them.

Example 15)

$A = 68°$ $\quad a =$ __________

$B =$ _________ $\quad b = 8$

$C =$ _________ $\quad c = 5$

Example 16)

$A =$ _________ $\quad a = 4$

$B =$ _________ $\quad b = 7$

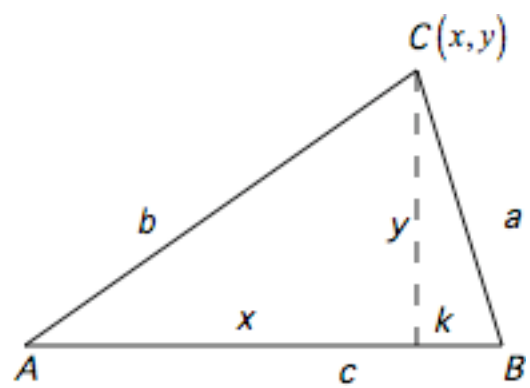
$C =$ _________ $\quad c = 9$

C. The Law of Cosines

Since the Law of Sines is only helpful when we have a complete family (a family or b family, or c family) and a half of a family, we need another law. That law is the Law of Cosines.

In any triangle with sides $a,\ b,$ and c and angles $A,\ B,$ and C.

$$y^2 + k^2 = a^2$$
$$a^2 = (c - x)^2 + y^2$$
$$a^2 = (c - b\cos A)^2 + (b\sin A)^2$$
$$a^2 = c^2 - 2bc\cos A + b^2\cos^2 A + b^2\sin^2 A$$
$$a^2 = c^2 - 2bc\cos A + b^2(\cos^2 A + \sin^2 A)$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Notice that like the Law of Sines, the Law of Cosines is really three laws. In SAS problems, you always use the one for which the angle is given.
So here is the technique to solve problems in the form of SAS. Note that no drawing is really necessary.

Step 1) Use the Law of Cosines for which you are given the angle. For instance, if you are given angle A, you will use $a^2 = b^2 + c^2 - 2bc\cos A$ which allows you to say that $a = \sqrt{b^2 + c^2 - 2bc\cos A}$. Input directly into your calculator.
Step 2) Now that we have a complete family, we now switch to the Law of Sines. Only use the Law of Cosines once in a problem. **IMPORTANT:** to avoid a possible problem, always use the Law of Sines to find the smallest angle remaining. So use the Law of Sines with your complete family and the smallest side remaining.
Step 3) You have two angles. Find the other angle by subtracting the sum of your two known angles from 180°.
Step 4) As before, verify your answers by checking that the largest angle is opposite the largest side and the smallest angle is opposite the smallest side.

Example 17)

$A = 68°$ $\boxed{a = 7.68}$

$\boxed{B = 74.98°}$ $b = 8$

$\boxed{C = 37.02°}$ $c = 5$

Example 18)

$\boxed{A = 4.42°}$ $a = 6$

$B = 14.3°$ $\boxed{b = 19.24}$

$\boxed{C = 161.28°}$ $c = 25$

The technique for solving problems in the form of SSS is similar in that we have to use the Law of Cosines. But since we have 3 sides and no angles, we must solve for an Angle first.

Step 1) You can use any of the 3 Laws of Cosines, However to avoid a potential problem, use the Law of Cosines to find the largest angle. If a is the largest side, use $a^2 = b^2 + c^2 - 2bc\cos A$ which allows you to say that $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
Calculate the value of the right side of this equation on your calculator and take the inverse cosine of that answer to find A.
Step 2) Now that we have a complete family, we now switch to the Law of Sines. Only use the Law of Cosines once in a problem. So use the Law of Sines with your complete family and any other half family.
Step 3) You have two angles. Find the other angle by subtracting the sum of your two known angles from 180°.
Step 4) As before, verify your answers by checking that the largest angle is opposite the largest side and the smallest angle is opposite the smallest side.

Example 17)

$\boxed{A = 35.43°}$ $a = 7$

$\boxed{B = 48.19°}$ $b = 9$

$\boxed{C = 96.38°}$ $c = 12$

Example 18)

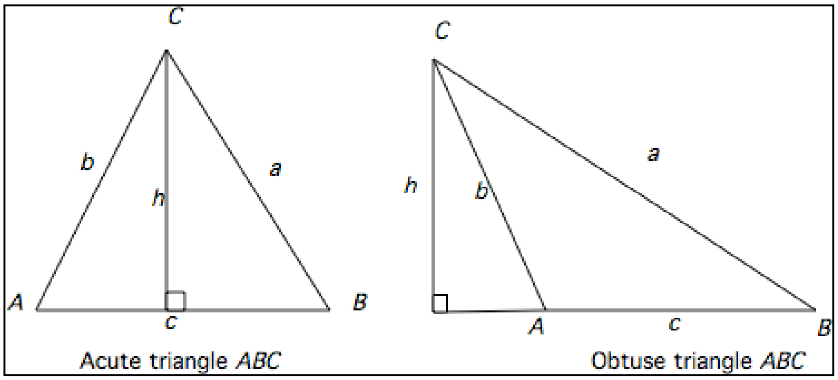
$\boxed{A = 82.82°}$ $a = 6000$

$\boxed{B = 41.41°}$ $b = 4000$

$\boxed{C = 55.77°}$ $c = 5000$

D. Area of Oblique Triangles

There are two formulas using trigonometry that will allow us to find the area of oblique triangles based on given information. Obviously, if the triangle is a right triangle, we only need both legs: Area $= \frac{1}{2}(\text{base})(\text{height})$.



$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$

$$\text{Area} = \frac{1}{2}cb\sin A$$

$$\text{So we say Area} = \frac{1}{2}bc\sin A$$

$$\text{Similarly, Area} = \frac{1}{2}ac\sin B$$

$$\text{Similarly, Area} = \frac{1}{2}ab\sin C$$

This formula works when you have two sides and the included angle (SAS). But frequently you have three sides of a triangle and wish to determine the area. In that case, we have another formula that will determine the area of that triangle. It is called Heron's (pronounced Hero's) formula.

Heron's Formula

If ΔABC has sides $a, b,$ and $c,$ the area of the triangle is given by

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$

Find the areas of the following triangles:

Example 19)

$A = 68°$ $a =$ ________
$B =$ ________ $b = 8$
$C =$ ________ $c = 5$

Area = $\boxed{18.54}$

Example 20)

$A =$ ________ $a = 6$
$B = 14.3°$ $b =$ ________
$C =$ ________ $c = 25$

Area = $\boxed{18.52}$

Example 21)

$A =$ ________ $a = 7$
$B =$ ________ $b = 9$
$C =$ ________ $c = 12$

Area = $\boxed{31.30}$

Example 22)

$A =$ ________ $a = 6000$
$B =$ ________ $b = 4000$
$C =$ ________ $c = 5000$

Area = $\boxed{9{,}921{,}567.42}$

Example 23)

$A = 31°$ $a = 22$

$\boxed{B = 12.16°}$ $b = 9$

$\boxed{C = 136.84°}$ $c =$ __________

Area = $\boxed{67.62}$

Example 24)

$A = 25°$ $a = 11$

$\boxed{B = 32.54°}$ $b = 14$

$\boxed{C = 122.46°}$ $c =$ __________

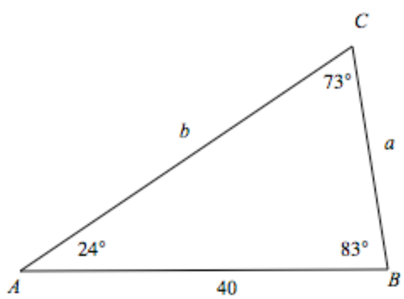
Area = $\boxed{64.97}$

E. Applications of Solving Oblique Triangles

Finally, Now that we can solve oblique triangles and find their area, we turn to applications of these topics. When solving applications, always following these procedures.

1) Draw a picture describing the situation.
2) Label the picture with variables and be sure you use these variables in your solution.
3) Identify whether the resulting triangle is an ASA, AAS, or SSA situation requiring the Law of Sines or an SAS or SSS requiring the Law of Cosines. Sometimes you will have to use these laws first and then use right angle trigonometry.
4) If an area is required decide whether the SAS formula $\left(\text{Area} = \frac{1}{2}bc\sin A\right)$ is needed or Heron's formula where SSS is required.
5) Always remember to answer the question(s) asked.
6) Always remember to supply proper units to the answer(s). Circle the answer(s).

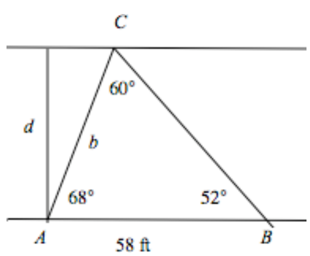
Example 25) A tree leans $7°$ to the vertical. At a point 40 feet from the tree (on the side closest to the lean), the angle of elevation to the top of the tree is $24°$. Find the height of the tree.



$$\boxed{a = \frac{40\sin 24°}{\sin 73°} = 17.01 \text{ ft}}$$

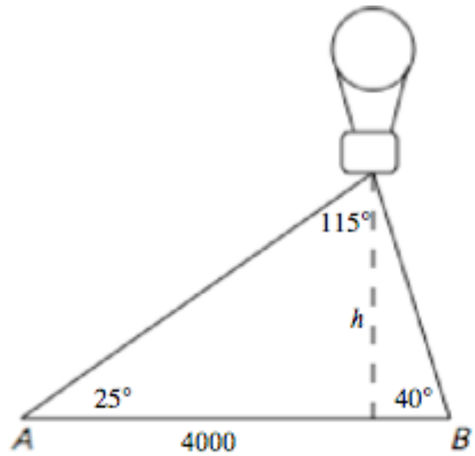
Example 26) Two markers A and B are on the same side of a river are 58 feet apart. A third marker is located across the river at point C. A surveyor determines that $\angle CAB = 68°$ and $\angle ABC = 52°$.

a) What is the distance between points A and C? b) What is the distance across the river?



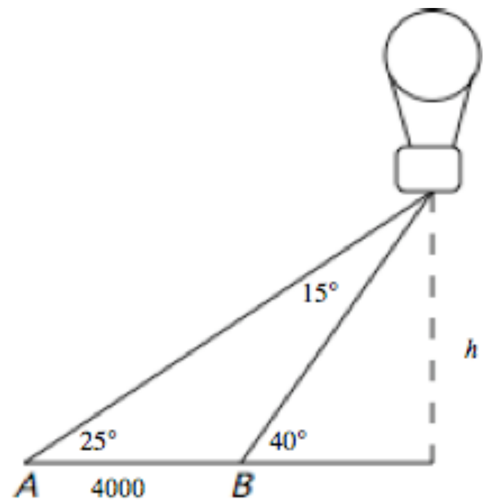
$$\boxed{\begin{aligned} b &= \frac{58\sin 52°}{\sin 60°} = 52.78 \text{ ft} \\ d &= 52.78\cos 22° = 48.93 \text{ ft} \end{aligned}}$$

Example 27) A hot air balloon is hovering over Valley Forge. Person A views the balloon at an angle of elevation of 25° while person B views the balloon at an angle of elevation of 40°. If A and B are 4000 feet apart, find the height of the balloon.



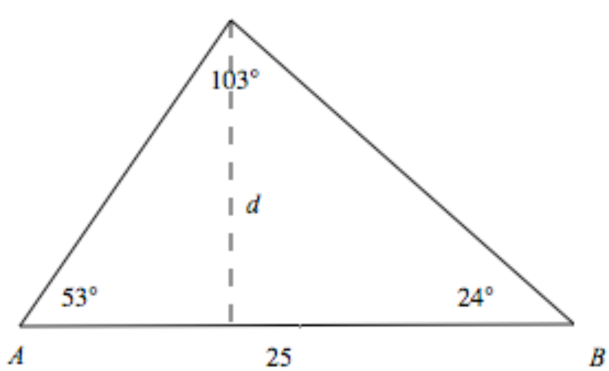
$$\boxed{\begin{aligned} a &= \frac{4000\sin 25°}{\sin 115°} = 1865.23 \text{ ft} \\ h &= 1865.23\sin 40° = 1{,}198.25 \text{ ft} \end{aligned}}$$

Example 28) Do the problem above with the same angles and distances but assume that the people are now on the same side of the balloon as seen in the accompanying picture.



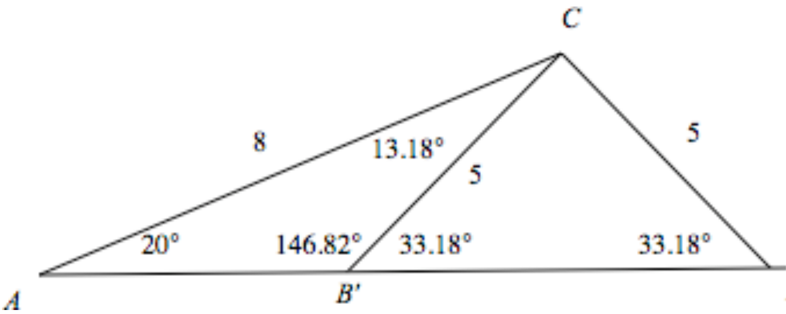
$$\boxed{\begin{aligned} a &= \frac{4000\sin 25°}{\sin 15°} = 6531.49 \text{ ft} \\ h &= 6531.49\sin 40° = 4{,}198.36 \text{ ft} \end{aligned}}$$

Example 29) Two tornado spotters are on a road running east-west and are 25 miles apart. The west man spots a tornado at bearing $N37°E$ and the east man spots the same tornado on a bearing of $N56°W$. How far is the tornado from each man and how far is the tornado from the road?



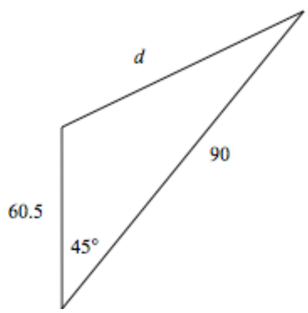
$$\boxed{\begin{aligned} a &= \frac{25\sin 53°}{\sin 103°} = 20.49 \text{ miles} \\ b &= \frac{25\sin 24°}{\sin 103°} = 10.44 \text{ miles} \\ d &= 20.49\sin 53° = 16.36 \text{ miles} \end{aligned}}$$

Example 30) A small ship travels from a lighthouse whose light only shines east on a bearing of $N70°E$ a distance of 8 miles. It then makes a turn to the right and travels for 5 miles and finds itself directly in the beam of the lighthouse's light. How far from the lighthouse is the ship?



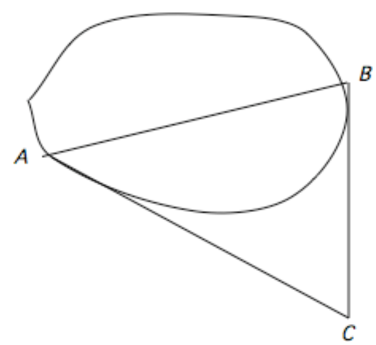
$$c = \frac{5\sin 126.82°}{\sin 20°} = 11.70 \text{ miles or}$$
$$c = \frac{5\sin 13.18°}{\sin 20°} = 3.33 \text{ miles}$$

Example 31) On a regulation baseball field, the four bases form a square whose sides are 90 feet long. The center of the pitching mound is 60.5 feet from home plate. How far is the mound from first base?



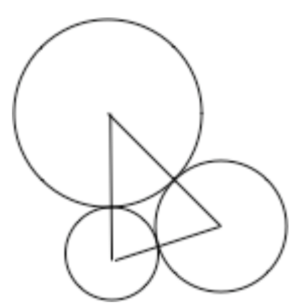
$$d = \sqrt{60.5^2 + 90^2 - 2(60.5)(90)\cos 45°}$$
$$d = 63.72 \text{ ft}$$

Example 32) To determine the distance across a lake AB, a surveyor goes to point C where he can measure the distance from C to A and from C to B as well as the angle ACB. If AB is 865 feet and BC is 188 feet and $\angle ACB = 42°18'$, find the distance AB across the lake.



$$d = \sqrt{865^2 + 188^2 - 2(865)(188)\cos 42°18'}$$
$$d = 736.89 \text{ ft}$$

Example 33) Circular tracts of land with diameters 900 meters, 700 meters and 600 meters are tangent to each other externally. There are houses directly in the center of each circle. What are the angles of the triangle connecting the houses and what is the area of that triangle?



$\boxed{A = 69.28°}$ $\quad a = 800$

$\boxed{B = 61.26°}$ $\quad b = 750$

$\boxed{C = 49.46°}$ $\quad c = 650$

$$A = \cos^{-1}\left(\frac{750^2 + 650^2 - 800^2}{2(750)(650)}\right) \qquad C = \sin^{-1}\left(\frac{650\sin 69.28°}{800}\right)$$

$$\text{Area} = \sqrt{1100(300)(350)(450)} = 68{,}738.64 \text{ meters}^2$$

Unit 6 – Solving Oblique Triangles - Homework

1. Solve each of the following triangles. 3 decimal places and decimal degrees unless DMS given.

a. $A = 62°$, $B = 78°$, $\boxed{C = 40°}$; $a = \boxed{a = 9.93}$, $b = 11$, $c = \boxed{c = 7.23}$

b. $\boxed{A = 68.90°}$, $B = 72.4°$, $C = 38.7°$; $a = 19.4$, $\boxed{b = 19.82}$, $\boxed{c = 13.00}$

c. $A = 19.2°$, $\boxed{B = 152.10°}$, $C = 8.7°$; $\boxed{a = 26.96}$, $\boxed{b = 38.36}$, $c = 12.4$

d. $A = 122.5°$, $B = 38.2°$, $\boxed{C = 19.30°}$; $\boxed{a = 5.87 \text{ ft}}$, $\boxed{b = 4.30 \text{ ft}}$, $c = 2.3$ ft

e. $A = 19°15'$, $\boxed{B = 103°57'}$, $C = 56°48'$; $a = 1.01$, $\boxed{b = 2.97}$, $\boxed{c = 2.56}$

f. $\boxed{A = 24°32'}$, $B = 82°49'$, $C = 72°39'$; $\boxed{a = 1.61 \text{ mm}}$, $b = 3.84$ mm, $\boxed{c = 3.69 \text{ mm}}$

2. Determine how many triangles are possible in the following SSA situations. Do not actually solve the triangles.

a. $A = 152°$, $B =$ ________, $C =$ ________; $a = 42$, $b = 55$, $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{None}}$

b. $A = 31°$, $B =$ ________, $C =$ ________; $a = 6$, $b = 4.5$, $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{One}}$

c. $A = 151°$ $a = 125$
$B =$ ________ $b = 79$
$C =$ ________ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{One}}$

d. $A = 68°$ $a = 12$
$B =$ ________ $b = 55$
$C =$ ________ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{None}}$

e. $A = 27°$ $a = 12$
$B =$ ________ $b = 17$
$C =$ ________ $c =$ __________

Drawing:

Triangles Possible: $\boxed{\text{Two}}$

f. $A = 58°$ $a = 25$
$B =$ ________ $b = 30$
$C =$ ________ $c =$ __________

Drawing:

Triangles possible: $\boxed{\text{None}}$

g. $A =$ ________ $a = 40$
$B = 122°$ $b = 38$
$C =$ ________ $c =$ __________

h. $A =$ ________ $a =$ __________
$B = 142°$ $b = 19$
$C =$ ________ $c = 6$

Don't get upset that you aren't given A, a, and b in exercises g - j. You can still draw the pictures.

Drawing:

Triangles Possible: $\boxed{\text{None}}$

Drawing:

Triangles possible: $\boxed{\text{One}}$

i. $A =$ ________ $a =$ __________
$B =$ ________ $b = 6$
$C = 36°12'$ $c = 16$

Drawing:

Triangles Possible: $\boxed{\text{One}}$

j. $A =$ ________ $a =$ __________
$B =$ ________ $b = 50$
$C = 26°49'$ $c = 38$

Drawing:

Triangles possible: $\boxed{\text{Two}}$

3. These are some of the same problems in section 2 above –those that have 1 or 2 solutions. Solve them. If they have two solutions, you will need to make another chart.

a. $A = 31°$ $a = 6$
$\boxed{B = 22.72°}$ $b = 4.5$
$\boxed{C = 126.28°}$ $\boxed{c = 9.39}$

b. $A = 151°$ $a = 125$
$\boxed{B = 17.84°}$ $b = 79$
$\boxed{C = 11.16°}$ $\boxed{c = 49.89}$

c. $\boxed{A = 26.79°}$ $a = 13.91$
$B = 142°$ $b = 19$
$\boxed{C = 11.21°}$ $c = 6$

d. $\boxed{A = 131°00'}$ $\boxed{a = 20.44}$
$\boxed{B = 12°48'}$ $b = 6$
$C = 36°12'$ $c = 16$

e.

$A = 27°$ $a = 12$
$\boxed{B = 40.03°}$ $b = 17$
$\boxed{C = 112.97°}$ $\boxed{c = 24.34}$

$A = 27°$ $a = 12$
$\boxed{B' = 139.97°}$ $b = 17$
$\boxed{C = 13.03°}$ $\boxed{c = 5.96}$

f.

$\boxed{A = 12°30'}$ $\boxed{a = 3.31 \text{ ft}}$
$\boxed{B = 66°16'}$ $b = 14$ feet
$C = 101°14'$ $c = 5$ yards

g.

$\boxed{A = 116°46'}$ $\boxed{a = 75.20}$
$\boxed{B = 36°25'}$ $b = 50$
$C = 26°49'$ $c = 38$

$\boxed{A = 9°36'}$ $\boxed{a = 14.05}$
$\boxed{B' = 143°35'}$ $b = 50$
$C = 26°49'$ $c = 38$

h.

$A = 58°$ $a = 25$
$B =$ ________ $b = 30$
$C =$ ________ $c =$ __________

$\boxed{\text{Not Possible}}$

4. Solve the following triangles using the Law of Cosines (and then the law of Sines).

a.

$\boxed{A = 91.36°}$ $a = 22$
$B = 42°$ $\boxed{b = 14.73}$
$\boxed{C = 46.64°}$ $c = 16$

b.

$\boxed{A = 41.18°}$ $a = 99$
$\boxed{B = 15.82°}$ $b = 41$
$C = 123°$ $\boxed{c = 126.11}$

c.

$\boxed{A = 29.90°}$ $a = 28$
$\boxed{B = 48.40°}$ $b = 42$
$\boxed{C = 101.70°}$ $c = 55$

d.

$\boxed{A = 141.24°}$ $a = 1$ yard
$\boxed{B = 24.67°}$ $b = 2$ feet
$\boxed{C = 14.09°}$ $c = 14$ inches

e.
$\boxed{A = 58.75°}$ $a = 4500$ mm
$\boxed{B = 71.79°}$ $b = 5000$ mm
$\boxed{C = 49.46°}$ $c = 4000$ mm

f.
$A =$ ________ $a = 7$
$B =$ ________ $b = 5$
$C =$ ________ $c = 20$

$\boxed{\text{Impossible}}$

5. Find the areas of the following triangles.

a.
$A =$ ________ $a = 22$
$B = 42°$ $b =$ ________
$C =$ ________ $c = 16$

Area = $\boxed{117.77}$

b.
$A =$ ________ $a = 99$
$B =$ ________ $b = 41$
$C = 123°$ $c =$ ________

Area = $\boxed{1702.08}$

c.
$A =$ ________ $a = 28$
$B =$ ________ $b = 42$
$C =$ ________ $c = 55$

Area = $\boxed{575.58}$

d.
$A =$ ________ $a = 1$ yard
$B =$ ________ $b = 2$ feet
$C =$ ________ $c = 14$ inches

Area = $\boxed{105.18 \text{ in}^2}$

e.
$A =$ ________ $a = 4500$ mm
$B =$ ________ $b = 5000$ mm
$C =$ ________ $c = 4000$ mm

Area = $\boxed{8,549,259.84}$

f.
$A =$ ________ $a = 7$
$B =$ ________ $b = 5$
$C =$ ________ $c = 20$

Area = $\boxed{\text{Impossible}}$

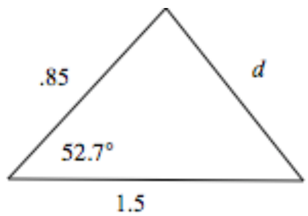
g.
$A = 151°$ $a = 125$
$\boxed{B = 17.84°}$ $b = 79$
$\boxed{C = 11.16°}$ $c =$ ________

Area = $\boxed{955.65}$

h.
$A = 58°$ $a = 25$
$B =$ ________ $b = 30$
$C =$ ________ $c =$ ________

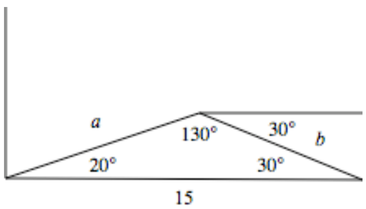
Area = $\boxed{\text{Impossible}}$

6. Two roads intersect at an angle of 52.7° with a field in between. A girl is walking on one of the roads 1.5 miles from their intersection. Her house lies 0.85 miles from the intersection along the other road. If she cuts across the field to her house, how much walking mileage will she save?



$$\boxed{\begin{array}{l} d^2 = .85^2 + 1.5^2 - 2(.85)(1.5)\cos 52.7° \\ d = 1.19 \quad \text{Distance saved } = .85 + 1.5 - 1.19 = 1.16 \text{ miles} \end{array}}$$

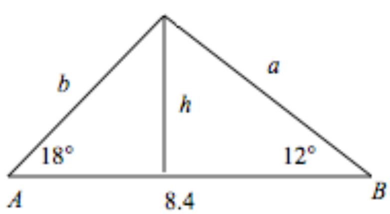
7. Kurt wants to sail his boat from a marina to an island 15 miles east of the marina. Along the course, there are several small islands they must avoid. He sails first on a heading of 70° and then on a heading of 120° (remember that headings angle measures rotated clockwise from the north). What is the total distance he travels before reaching the island?

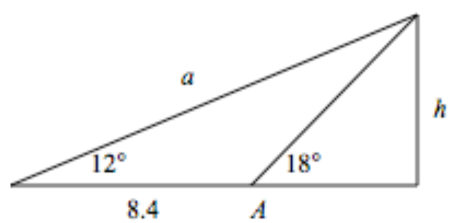


$A = 30°$	$a =$ __
$B = 20°$	$b =$ __
$C = 130°$	$c = 15$

$$\boxed{\begin{array}{l} a = \dfrac{15\sin 30°}{\sin 130°} = 9.79 \\ b = \dfrac{15\sin 20°}{\sin 130°} = 6.70 \quad \text{Total distance: } 16.49 \text{ miles} \end{array}}$$

8. A balloon is sighted from two points on level ground. From point *A*, the angle of elevation is 18° and from point *B* the angle of elevation is 12°. *A* and *B* are 8.4 miles apart. Find the height of the balloon if a) *A* and *B* are on opposite sides of the balloon and b) *A* and *B* are on the same sides of the balloon.

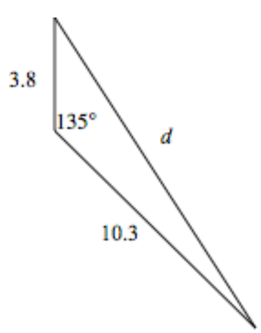




$$\boxed{\begin{array}{l} a = \dfrac{8.4\sin 18°}{\sin 150°} = 5.19 \\ h = 5.19\sin 12° = 1.08 \text{ miles} \end{array}}$$

$$\boxed{\begin{array}{l} a = \dfrac{8.4\sin 12°}{\sin 6°} = 16.71 \\ h = 16.71\sin 12° = 3.47 \text{ miles} \end{array}}$$

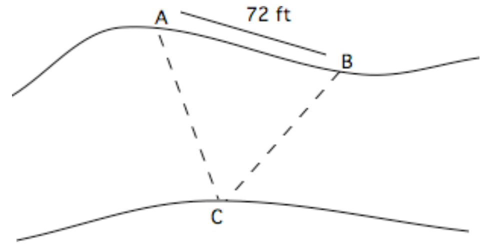
9. A golfer takes two putts to get the ball into the hole. The first putt rolls the ball 10.3 feet in the northwest direction and the second putt sends the ball due north 3.8 feet into the hole. How far was the ball originally from the hole?



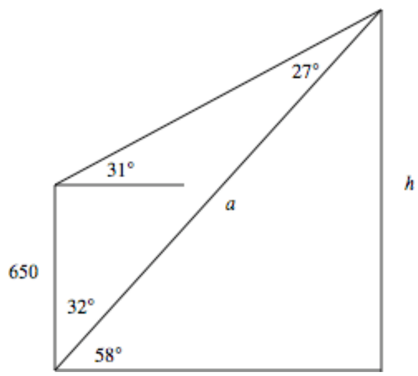
$$\boxed{d^2 = 3.8^2 + 10.3^2 - 2(3.8)(10.3)\cos 135° = 13.26 \text{ ft}}$$

10. Two markers A and B are on the same side of a canyon rim and are 72 feet apart. A third marker is located across the rim at point C. A surveyor determines that $\angle BAC = 70°12'$ and $\angle ABC = 51°38'$. Find the distance between A and C.

$$\boxed{AC = \frac{72\sin 51°38'}{\sin 58°10'} = 66.45 \text{ ft}}$$

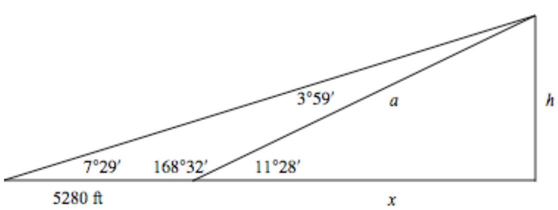


11. A blimp is sighted simultaneously by two observers: A at the top of a 650-foot tower and B at the base of the tower. Find the distance of the blimp from observer A if the angle of elevation as viewed by A is $31°$ and the angle of elevation as viewed by B is $58°$. Find the height of the blimp also.



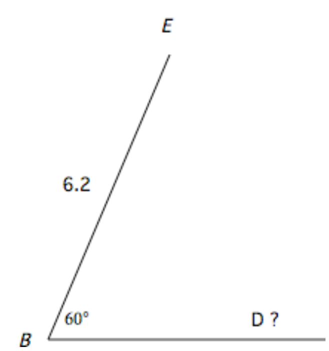
$$\boxed{\begin{array}{l} a = \dfrac{650\sin 121°}{\sin 27°} = 1227.25 \\ h = 1227.25\sin 58 = 1{,}040.76 \text{ ft} \end{array}}$$

12. As I complete the Boston Marathon, I view the top of the Prudential Center building at an angle of elevation of $7°29'$. I run one mile closer and now view the angle of elevation as $11°28'$. How tall is the Prudential building and how far away from it am I now?



$$\boxed{\begin{array}{l} a = \dfrac{5280\sin 7°29'}{\sin 3°59'} = 9899.13 \\ h = 9899.13\sin 11°28'. = 1{,}967.92 \text{ ft} \\ x = 9899.13\sin 11°28' = 9701.54 \text{ ft } = 1.84 \text{ miles} \end{array}}$$

13. A triangular piece of land in a park is to be made into a flower-bed. Stakes have been driven into the ground at the vertices of the triangle which we will call B, E, and D. The gardener can only locate the two stakes at B and E. BE measures 6.2 meters, and the gardener recalls that the angle at B is $60°$ and the side opposite the $60°$ angle is to be 5.5 meters in length. Based on this information how far from B should the gardener search for the missing stake?



$B = 60°$	$b = 5.5$		$B = 60°$	$b = 5.5$
$E = 42.51°$	$\boxed{e = 4.29 \text{ ft}}$	or	$E = 17.49°$	$\boxed{e = 1.91 \text{ ft}}$
$D = 77.49°$	$d = 6.2$		$D = 102.51°$	$d = 6.2$