Patterns and Numerical Relationships

Unit 1

Unit Overview

In this unit you will investigate patterns, rational and irrational numbers, and the properties of exponents. You will apply your knowledge of fractions and decimals to practical situations.

Academic Vocabulary

Add these words to your vocabulary notebook.

- hypothesis
- power
- reciprocal
- scientific notation

Essential Questions

- How are fractions, percents and decimals related?
- Why is it important to understand the procedures for working with different kinds of numbers?

EMBEDDED ASSESSMENTS

This unit has two Embedded Assessments, following Activities 1.2 and 1.5. These Embedded Assessments allow you to demonstrate your understanding of patterns, exponents, and scientific notation. You will also display your ability to work with equivalent forms of numbers, fractions, decimals, and percents—and distinguish between rational and irrational numbers.

Getting Ready

Write your answers on notebook paper. Show your work.

1. Give a fraction, a decimal and a percent that represents the shaded portion of the figure shown.

2. Draw a visual representation of each of the following. Use a circle for at least one of your figures.

a. $\frac{5}{6}$

b. 30%

c. 0.75

3. Arrange each of the following groups of numbers in increasing order.

a. 1.345, 1.6789, 1.12

b. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$

4. Which of the following is larger? Justify your answer.

$\frac{2}{3} + \frac{4}{5}$ or $\frac{2}{5} + \frac{5}{6}$

5. How are mixed numbers and improper fractions related?

6. Which of the following is the product of 2.4 and 3.1?

a. 0.55

b. 5.5

c. 7.44

d. 74.4

7. Evaluate each of the following.

a. $\frac{3}{4} + \frac{2}{3}$

b. $\frac{3}{4} - \frac{2}{3}$

c. $\frac{3}{4} \cdot \frac{2}{3}$

d. $\frac{3}{4} \div \frac{2}{3}$

e. $4.6 + 0.23$

f. $4.6 - 0.23$

g. $4.6 \cdot 0.23$

h. $4.6 \div 0.23$

8. Explain why 49 is a perfect square and 50 is not a perfect square. Give three other numbers that are perfect squares and three that are not.

Investigating Patterns
Laws and Orders

ACTIVITY
1.1

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Visualization, Think/Pair/Share

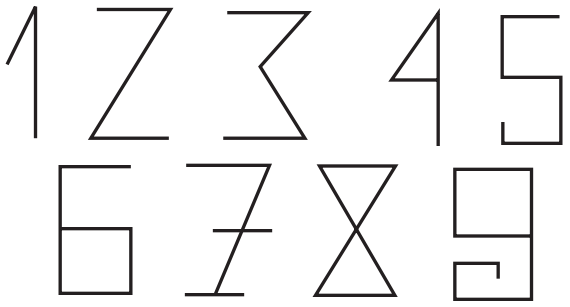
My Notes

For much of the late 19th and early 20th centuries, the fictional character Sherlock Holmes was known for his great detective work. In this activity, you will be asked to perform many tasks similar to those Sherlock used to solve his cases.

In order to solve mysteries, Holmes used a deductive process that led him to a logical conclusion. First, he would *observe* a situation and gather as many facts as possible. Next, he would *analyze* each fact to determine its relevance to the situation. Then he would *search* for even more clues by considering the smallest of details. Finally, he would use his *imagination* to link all of the clues together in the most logical manner.

The Case of the Arabic Symbols

At first glance, the following picture would appear to be a representation of the numbers one through nine. However, the way they are drawn gives a clue to how the symbols for each number were originally created.



1. Observe, analyze, and search for clues in the diagram to come up with a guess about why the numbers were first written this way. Don't forget to use your imagination!

My Notes

SUGGESTED LEARNING STRATEGIES: Discussion Group, Visualization, Look for a Pattern

2. Discuss your observations with your group.

a. List some observations that you had in common with your group members.

b. List some observations that your group members had that you didn't think of.

c. As a group, write an official conclusion based on your shared observations.

3. Based on your group's conclusion, explain how this pattern could also be used to describe zero with the symbol **0**.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Quickwrite

My Notes

The Case of the Multiple Viewpoints

The next case at hand involves investigating the pattern shown below. In order to reconstruct the pattern and solve the mystery, several witnesses have been asked to describe the pattern.

Figure 1 Figure 2 Figure 3

4. The description provided by the first witness is given in terms of percents. Provide an example of what this description might be.

5. The second witness used fractions to describe the pattern. Provide an example of what this description might be.

6. Witnesses #3 and #4 have provided different accounts of the pattern. One of them describes the pattern as increasing, and the other describes it as decreasing. Explain how both of their descriptions could be considered correct.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Quickwrite

7. Analyze the descriptions of all four witnesses and draw a representation of what Figure 4 would look like if the pattern continued.

8. Explain why it is necessary to gather as much evidence about the pattern from as many different viewpoints as possible.

The Case of the Revolving Figure

Case #3 involves the pattern shown below:

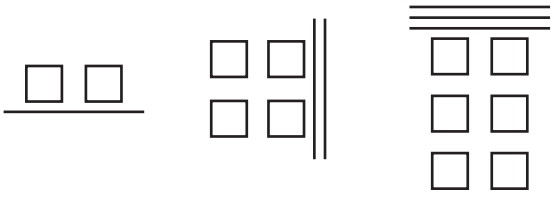


Figure 1 Figure 2 Figure 3

9. Observe and analyze the pattern, then describe it in as much detail as possible.

SUGGESTED LEARNING STRATEGIES: Discussion Group, Self/Peer Revision, Look for a Pattern, Create Representations

My Notes

10. Share your description with your group members and list any details you may not have considered before.

11. Use the evidence gathered in Questions 9 and 10 to draw representations of the fourth and fifth figures in the pattern.

12. Answer the following based on your observations of the pattern.

a. Describe the pattern for the number of line segments in each figure.

b. How many line segments would appear in Figure 16?

c. How many line segments would appear in Figure 49?

d. Explain how you could determine the number of line segments in *any* figure in the pattern.

WRITING MATH

One way to describe number patterns is to list several terms in order, followed by ellipses (...) to indicate that the pattern continues. For example, writing

1, 3, 5, 7, ...

implies that the pattern of adding two to each digit continues indefinitely.

My Notes

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Look for a Pattern, Work Backward

13. Organize the evidence you gathered about line segments, and continue to explore the pattern in the table below.

Figure	Number of Line Segments	Number of Squares	Sum of Line Segments and Squares
1			
2			
3			
4			
5			
16			
49			

14. Explain how you could determine the number of squares and the sum of line segments and squares in any figure in the pattern.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Work Backward, Discussion Group, Graphic Organizer

My Notes

TRY THESE **A**

Describe the pattern, and list the next three terms in the following sequences:

a. 0, 8, 16, 24, …

b. 2, −4, −10, −16, …

c. 27, 9, 3, 1, …

15. Complete the table below by investigating each sequence.

Sequence	Increasing or Decreasing?	Next Term in the Sequence	Description of Pattern
0, 5, 10, 15…			
−8, −4, −2, −1, …			
1.5, 2.75, 4, …			
$\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \ldots$			
$2, \frac{5}{4}, \frac{1}{2}, \ldots$			

SUGGESTED LEARNING STRATEGIES: Self/Peer Revision, Think/Pair/Share

My Notes

MATH TERMS
The **absolute value** of a number is its distance (number of units) from 0 on a number line. Absolute value is always nonnegative.

16. Consider this sequence that uses absolute values of numerical expressions.

$|5 - 2|, |5 - 3|, |5 - 4|, |5 - 5|, |5 - 6|, |5 - 7|$

a. Is this sequence increasing or decreasing? Explain.

b. Express the next term in the sequence two ways.

17. Write the information requested in the bulleted list for each set of numerical expressions.

a. $|-16| \times |5|, |-18 - 2|, |3 - 13|, |38| + |-2|, \left|-\frac{10}{2}\right|$

- Evaluate each absolute-value expression.
- Order the numbers so that they form a sequence.
- Describe the pattern in the sequence.

b. $-2|-4|, \frac{|-24|}{6}, |-2| \times |-5|, 40|2 \div 5|, |7| - |-9|$

- Evaluate each absolute-value expression.
- Order the numbers so that they form a sequence.
- Describe the pattern in the sequence.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Discussion Group, Self/Peer Revision

My Notes

The Case of the Missing Term

In the previous cases, the given patterns contained at least three terms. In this case, however, only the first two terms have been provided.

12, 6, …

18. Explain how it would be possible to use any of the four mathematical operations to generate the first two numbers in the pattern. Then write the first four terms of the series based on your explanation.

a. Addition

b. Subtraction

c. Multiplication

d. Division

19. Observe and analyze your results from Question 18. Compare and contrast the patterns that were generated.

My Notes

SUGGESTED LEARNING STRATEGIES: Quickwrite, Group Presentation

TRY THESE **B**

Describe two ways to generate the next term in each of the following patterns.

a. 9, 27, …

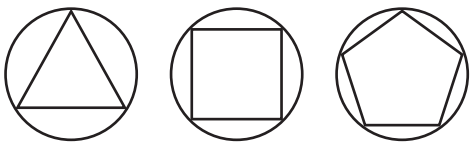
b. 12, 48, …

20. Explain why it is usually necessary to be given at least three terms in order to establish a pattern.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Describe the following patterns and determine the next two terms:

a. 

b. 1, 3, 2, 6, 4, 9, …

c. 0.2, 0.04, 0.008, …

d. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \ldots$

e. $|-3 + -2|, |-3 + 0|, |-3 + 2|, |-3 + 4|, \ldots$

2. Come up with two different ways to represent the fourth term in each of the following patterns.

a. 

b. $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \ldots$

3. The numbers below are known as Fibonacci numbers. Can you discover the pattern?

1, 1, 2, 3, 5, 8, …

4. MATHEMATICAL REFLECTION Describe the different ways that patterns can be represented as well as the process you use to discover whether a pattern exists.

Properties of Exponents
That's a Lot of Cats!

SUGGESTED LEARNING STRATEGIES: Discussion Group, Work Backward, Guess and Check

My Notes

As I was going to St. Ives,
I met a man with seven wives.
Every wife had seven sacks,
And every sack had seven cats.
Every cat had seven kittens
Kittens, cats, sacks, wives,
How many were going to St. Ives?

In addition to being an 18th century translation of what the *Guinness Book of World Records* claims is the oldest mathematical riddle in history, the above poem has also been the subject of great debate over the years.

CONNECT TO HISTORY

Problem 79 on the Rhind Mathematical Papyrus (circa 1650 BCE) contains the algorithm that is said to be the basis for the mathematics in this riddle.

1. Depending on the interpretation of the question, the riddle is said to have multiple answers. Determine whether or not each of the following could be considered a reasonable answer to the riddle. Justify your reasoning.

a. 28

b. 29

c. 2801

d. 1

SUGGESTED LEARNING STRATEGIES: Guess and Check, Look for a Pattern, Simplify the Problem.

My Notes

MATH TERMS
An **exponent** is a number that indicates how many times a **base** is used as a factor. For example, if you have 5 factors of x, $(x \cdot x \cdot x \cdot x \cdot x)$, they can be written as x^5. In this example, x is the base, and 5 is the exponent.

Regardless of the many interpretations of the poem, the riddle can be used to explain how **exponents** work.

2. Complete the following table.

	Number Written in…			
	Expanded Form	**Exponential Form**	**Standard Form**	**Base**
Wives		7^1		7
Sacks	$7 \cdot 7$	7^2		
Cats	$7 \cdot 7 \cdot 7$		343	
Kittens				

3. If each kitten had seven stripes, we could multiply $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ to determine the number of stripes.

a. How would this be expressed in exponential form?

b. What is the value in standard form?

MATH TERMS
The **associative property of multiplication** states that for all real numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

4. You could also use the **associative property of multiplication** to group the product in various ways such as $(7) \cdot (7 \cdot 7 \cdot 7 \cdot 7)$.

a. Determine three additional ways to regroup the product.

b. Rewrite each of your responses in part a using exponents.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Look for a Pattern

My Notes

5. Describe any patterns you notice about the exponents in Question 4.

6. Consider the product $(x \cdot x) \cdot (x \cdot x \cdot x)$

a. Rewrite the product using exponents.

b. Simplify the expression, and write the answer in exponential form.

c. Describe the similarities and differences between this product and the products you examined in Question 5.

d. Based on your observations, write a rule for multiplying terms with exponents that have the same base.

7. If you wanted to divide the number of kittens by the number of cats, we could express this as $\frac{2401}{343} = 7$.

a. Rewrite this equation by expressing the numerator, denominator, and the answer in exponential form.

b. Describe how you could simplify the fraction.

My Notes

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Vocabulary Organizer, Discussion Group

8. Consider the expression $\frac{x^4}{x^3}$.

a. Rewrite the expression with the numerator and denominator in expanded form.

b. Describe how you could simplify the expression.

c. Write a general rule for dividing terms with exponents that have the same base.

ACADEMIC VOCABULARY

A **power** consists of two parts, a base and an exponent. For example, in the power 5^3, 5 is the base and 3 is the exponent.

9. Consider the expression $\frac{x^3}{x^5}$.

a. Rewrite the numerator and denominator in expanded form and simplify.

b. Now apply the general rule you found in item 8c.

c. Compare your results for parts a and b, What do you notice?

d. Write an expression for 4^{-3} without exponents. Show your work.

10. Although there is only one narrator in *As I was going to St. Ives*, the number 1 can be written as a **power** of 7. To see how this works, you can examine several ways to express the number 1.

a. Explain why each of the following is equal to 1.

$\frac{7}{7}$, $\frac{49}{49}$, $\frac{343}{343}$, and $\frac{2401}{2401}$

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revisions

My Notes

b. Rewrite each fraction by expressing each numerator and denominator in exponential form.

c. Explain how it is possible for the fractions in part b to simplify to 7^0.

11. Write a general rule that describes the outcome of raising any base to the power 0.

12. Use your rule to explain how the product $10{,}324^0 \cdot 8{,}576^0$ can be done using mental math.

Assume each stripe on each kitten contains seven spots. The situation is becoming more complicated, and the need for using exponents has grown…uh…exponentially.

13. Determine the total number of spots on the kittens. Explain how you determined your solution.

14. In expanded form, the product could be written as

$(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)$.

a. Explain why this could also be written as $(7^3)^2$.

b. Express this product in its simplest exponential form.

c. Based on this example, what operation is being performed on the exponents?

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Vocabulary Organizer, Self/Peer Revision, Group Presentation

My Notes

15. In the table below, summarize the rules for exponents you discovered in this activity.

Situation	Verbal Description	Numeric Example
Multiplying powers with the same base		
Dividing powers with the same base		
Raising a term to an exponent of zero		
Raising a power to another exponent		

CONNECT TO AP

In advanced math courses, you will use the same rules to simplify and rewrite expressions in which the exponents are fractions, decimals, or negative numbers. For example, you discovered that $x^3 \cdot x^4 = x^7$. A similar problem in an advanced math course might be $x^{\frac{1}{5}} \cdot x^{\frac{2}{5}} = x^{\frac{3}{5}}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Simplify in exponential form.

1. $3^5 \cdot 3^4$
2. $5^6 \cdot 3^2$
3. $x^{16} \cdot x^1$
4. $\frac{4^3}{4^2}$
5. $\frac{10^8}{10^6}$
6. $\frac{x^{11}}{x^4}$
7. 5^0
8. $\frac{2^4}{2^7}$
9. Write your answer to item 8 as an expression without exponents.
10. **MATHEMATICAL REFLECTION** Explain why the bases of powers must be the same in order for the rules of exponents to work.

Patterns and Exponents

CONTAGIOUS MATHEMATICS

Embedded Assessment 1

Use after Activity 1.2.

While checking her email, Luisa stumbles across a cryptic message from someone named 5up3r H4xx0r. In the message, 5up3r 4axx0r claims to have developed a computer virus and is set to release it on the Internet. Once the virus has infected two computers, the potential exists for it to spread exponentially because each infected computer has a chance to pass it along to the next computer it connects with.

The only way for the virus to be stopped, says the hacker, is if Luisa correctly answers each of the following questions.

1. The pattern of the spread of the virus will be 1, 2, 4, 8, … Identify the next three numbers in this pattern.

2. Express the first seven numbers in the pattern as a power of 2.

3. Describe the following patterns:

a. The pattern of the number of computers infected

b. The pattern of the exponents of two

c. The pattern of the difference between each term

d. The pattern of the last digit in each term

4. Describe how the 18^{th} term in the pattern could be determined.

5. Express each of the factors for the following products in exponential form.

a. 32 • 128

b. 4 • 256

c. 16 • 64

6. Simplify each of the products in Item 5. Leave your answers in exponential form.

Patterns and Exponents

CONTAGIOUS MATHEMATICS

7. Simplify each of the following expressions and determine what the last digit would be.

a. $(2^{15})^4$

b. $\frac{2^{12}}{2^3}$

8. Write a reply to 5up3r H4xx0r about your success in foiling the virus plan. Include in your reply a description of the problems that were difficult for you and the ones you were able to complete easily.

	Exemplary	Proficient	Emerging
Math Knowledge #1, 2, 6, 7	• Correctly identifies the next three numbers in the given pattern (1) • Correctly expresses each of the first seven terms as a power of 2 (2) • Finds correct products and quotients in exponential form and simplifies them (6, 7) • Determines the last digit of the result of the standard form (7)	Provides justification for items in questions 1, 2, 6, and 7 but only three are correct	Provides at least three justifications for questions 1, 2, 6, and 7 but they may be incorrect or incomplete
Problem Solving #4	Determines the 18^{th} term (4)	Uses a correct method to find the 18^{th} term but makes a computational error	Uses an incorrect method or is unable to determine the 18^{th} term
Representation #5a, b, c	Expresses both factors of the product in exponential form (5)	Expresses most of the factors in exponential form	Is unable to express the factors in exponential form.
Communication #3a–d, 4, 8	• Correctly describes the pattern of the number of computers infected (3a), the pattern of the exponents of two (3b), the pattern of the difference between each term (3c), and the pattern of the last digit in each term (3d) • Correctly describes how the 18^{th} pattern in the term could be determined (4) • Provides a detailed explanation about the problems solved in the EA that includes information about areas of both strength and difficulty (8)	Gives explanations for items in questions 3, 4, and 8 but only two are complete and correct	Gives at least two of the three required explanations for questions 3, 4, and 8 but they are incomplete and incorrect

Operations with Decimals
Driving for Points

SUGGESTED LEARNING STRATEGIES: Mark the Text, Think Outloud, Group Discussion

My Notes

Tony, Leilani, Chrissy, and Dale are starting a NASCAR fantasy league this season. Each team will consist of five of the top twenty drivers on the circuit. At the beginning of the season, the drivers are assigned point values based on their performances in previous years. The top twenty drivers and their point values are shown in the table below.

Driver	Point Value	Driver	Point Value
Sparky Pluggs	23.7	Victor E. Lane	18.6
Bump Ertle	23.1	Turner Wedge	18.3
Sara Wheeler	22.6	Shayla Crews	17.5
Kevin Fender	22.4	Ian Jun	16.8
Bonnie Checker	22.2	Trey Oval	16.7
Carl Burator	21.5	Camber LePointe	15.4
Chaz Errol	21.1	Stockton Carr	15.1
Ray De Ator	20.8	Tank Topov	14.9
Cam Shaffer	20.2	Forrest Gere	14.1
Monty Carlo	19.4	Roland Springs	13.8

1. The rules of the league state that the sum of the point values of the drivers on a team must not exceed 100 points. Take turns choosing teams in your groups, and complete the table below.

Team 1		Team 2		Team 3		Team 4	
Driver	Point Value	Driver	Point Value	Driver	Point Value	Driver	Point Value
Sum:		Sum:		Sum:		Sum:	

My Notes

SUGGESTED LEARNING STRATEGIES: Guess and Check, Group Discussion, Work Backwards

2. In the first four rounds of the draft, Leilani has chosen Sara Wheeler, Carl Burator, Turner Wedge, and Camber LePointe.

a. How many points does Leilani have available to spend on her fifth driver?

b. Explain the process you used to determine your answer for part a.

c. Assuming she's still available, can Leilani choose Bonnie Checker in the fifth round? Explain your reasoning.

As the season progresses, the players may trade drivers as long as the total point value for each team does not exceed 100 points.

3. Tony's team consists of Sparky Pluggs, Bonnie Checker, Trey Oval, Tank Topov, and Forrest Gere. He's having second thoughts about choosing Tank Topov, however, and is considering trying to make a trade for another driver.

a. Calculate the current point value of Tony's team.

b. How many points below the limit is Tony's current team?

c. If Tony decides to trade Topov, how many points will he have to spend on another driver?

d. Who is the highest ranked driver that Tony can trade for? Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Questioning the Text, Group Discussion, Identify a Subtask

My Notes

After the first five races of the season, the point values of the drivers are adjusted according to how they have performed thus far. The new point values are shown below.

Driver	Point Value	Driver	Point Value
Sparky Pluggs	24.1	Kevin Fender	18.5
Carl Burator	23.9	Turner Wedge	18.2
Bonnie Checker	22.4	Trey Oval	17.9
Victor E. Lane	22.1	Forrest Gere	16.6
Sara Wheeler	21.6	Shayla Crews	16.1
Bump Ertle	21.3	Camber LePointe	15.7
Monty Carlo	20.9	Roland Springs	15.5
Tank Topov	20.1	Ray De Ator	14.8
Cam Shaffer	19.5	Ian Jun	14.3
Chaz Errol	19.2	Stockton Carr	13.2

4. Recalculate the values of the teams you chose in Item 1 based on the updated point values.

Team 1		Team 2		Team 3		Team 4	
Driver	Point Value	Driver	Point Value	Driver	Point Value	Driver	Point Value
Sum:		Sum:		Sum:		Sum:	

5. Which driver had the greatest change in point values? By how much?

6. Which team in your group had the greatest change in total value? By how much?

My Notes

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/ Retell Quickwrite, Look for a Pattern, Group Presentation

In most fantasy leagues, adjusted values only take effect when a driver is released or traded from a team. This is good news for Tony's team.

Tony's Team		
Driver	**Initial Value**	**Current Value**
Sparky Pluggs	23.7	24.1
Bonnie Checker	22.2	22.4
Trey Oval	16.7	17.9
Tank Topov	14.9	20.1
Forrest Gere	14.1	16.6
Sum:		

7. If Tony had gone through with trading Tank Topov, explain why he would not be able to add him back to his team now. Justify your answer by providing specific values in the table above.

8. When adding and subtracting decimals by hand, explain the process you use to ensure that your calculations are done correctly.

SUGGESTED LEARNING STRATEGIES: Mark the Text, Identify a Subtask, Discussion Group

My Notes

Team points are awarded each week based on where the drivers finish in the race. A driver's point value is multiplied by the number of cars he beat in the race. NASCAR races consist of 43 cars, so this formula would be used.

(Fantasy Point Value) • (43 − Finishing Position)

For example, the points for a driver who finished 10th in the race and had a fantasy point value of 18.4 would be calculated as follows:

(18.4) • (43 − 10) = (18.4) • (33) = 607.2 points

9. The results of the sixth race are shown in the table below.

Driver	Point Value	Finish		Driver	Point Value	Finish	
Sparky Pluggs	24.1	1		Kevin Fender	18.5	5	
Carl Burator	23.9	12		Turner Wedge	18.2	30	
Bonnie Checker	22.4	6		Trey Oval	17.9	22	
Victor E. Lane	22.1	8		Forrest Gere	16.6	9	
Sara Wheeler	21.6	16		Shayla Crews	16.1	28	
Bump Ertle	21.3	21		Camber LePointe	15.7	30	
Monty Carlo	20.9	4		Roland Springs	15.5	19	
Tank Topov	20.1	3		Ray De Ator	14.8	11	
Cam Shaffer	19.5	23		Ian Jun	14.3	32	
Chaz Errol	19.2	34		Stockton Carr	13.2	41	

a. Calculate the number of points earned by each driver on your team.

b. Calculate the total number of points earned by your team.

c. Rank the total values of the teams in your group from greatest to least.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Work Backwards, Vocabulary Organizer

My Notes

10. In the following week, Victor E. Lane finished eighth. When calculating the fantasy points, Dale multiplied the numbers as follows:

$$(2.21) \cdot (35) = 77.35 \text{ points}$$

a. Identify the error Dale made in his calculation.

b. Calculate the correct value of fantasy points earned by Victor E. Lane.

c. What do you notice about Dale's initial calculation and the one you made in part b?

d. Explain why it would not have been necessary for Dale to recalculate the values.

ACADEMIC VOCABULARY

A **hypothesis** is an assumption or guess that is based on the observation of an event or series of events. The word is derived from the Greek *hypotethenai*, which means "to suppose."

11. Dale is fascinated by the observations made in Item 10 and wonders if it would work for any multiplication with decimals. He uses the following sample problems to test his **hypothesis**.

a. $25 \cdot 25$
b. $2.5 \cdot 25$
c. $2.5 \cdot 2.5$
d. $0.25 \cdot 2.5$
e. $0.25 \cdot 0.25$

a. Determine the value for each of Dale's examples.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Guess and Check, Think/Pair/Share

My Notes

b. Do you think his hypothesis was correct? Justify your answer.

c. How could you determine the final position of the decimal point before multiplying the numbers? Explain your reasoning.

d. Based on your answer to part c, which of the sample problems would have the same product as 25 • 0.25? Explain your reasoning.

TRY THESE **A**

a. 3.45 • 28.7

b. 88 • 2.6

c. 0.4 • 8.1

d. 40 • 0.81

e. Explain how you could have completed Item d without calculating the values after having completed Item c.

At the end of the season, bonus points are awarded by multiplying the starting point value and the total earnings of each driver. Chrissy's top driver was Ray De Ator, who earned $8,000,000 and had a starting point value of 20.8.

12. Chrissy's calculator only displays eight digits. Explain why displaying Ray De Ator's bonus points on her calculator could be a problem.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Look for a Pattern

My Notes

ACADEMIC VOCABULARY

Scientific notation is a way of writing a number in terms of a decimal greater than or equal to 1 and less than 10, multiplied by a power of 10. Using symbols, a number is written in scientific notation when it is expressed in the form $a \cdot 10^n$, where $1 \leq a < 10$ and n is an integer.

Scientific notation is used to simplify numbers that are very large or very small in order to make calculations easier. Chrissy decides that this is how she wants to calculate the bonus points, so she uses the following values:

$$(8 \cdot 10^6) \cdot (2.08 \cdot 10^1)$$

To understand how Chrissy determined these values, it is necessary to know how scientific notation works. Converting a number into scientific notation involves multiplying numbers by a factor of 10.

13. Consider the following products:

$$80{,}000 = 8 \cdot 10{,}000 = 8 \cdot 10^4$$
$$8{,}000 = 8 \cdot 1{,}000 = 8 \cdot 10^3$$
$$800 = 8 \cdot 100 = 8 \cdot 10^2$$

a. Continue the pattern.

$$80{,}000 = 8 \cdot 10{,}000 = 8 \cdot 10^4$$
$$8{,}000 = 8 \cdot 1{,}000 = 8 \cdot 10^3$$
$$800 = 8 \cdot 100 = 8 \cdot 10^2$$
$$80 = 8 \cdot 10 = \_\_\_\_\_$$
$$8 = 8 \cdot 1 = \_\_\_\_\_$$
$$0.8 = 8 \cdot 0.1 = \_\_\_\_\_$$
$$0.08 = 8 \cdot 0.01 = \_\_\_\_\_$$

TECHNOLOGY TIP

Calculators use a form of scientific notation to express very large and very small numbers. For example:

8,000,000,000,000 would be expressed as 8E12. The 12 indicates that 8 is multiplied by 10 to the 12th power.

0.000000000008 would be expressed as 8E − 12. The −12 indicates that 8 is multiplied by 10 to the −12th power,

b. Explain how Chrissy could use this pattern to express Ray De Ator's winnings in scientific notation.

c. Why was Chrissy correct in writing 20.8 as $2.08 \cdot 10^1$?

14. Write these numbers in scientific notation.

a. 56, 300

b. 110,000

c. 6,000

d. 0.0072

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Discussion Group, Quickwrite, Vocabulary Organizer

My Notes

Another way to determine proper scientific notation is to count the number of places the decimal would have to be moved in order to be located to the right of the non-zero first digit in the number.

15. Consider this table.

	Numbers greater than 1	Numbers between 0 and 1
A	$89{,}200 = 8.92 \cdot 10^4$	$0.000892 = 8.92 \cdot 10^{-4}$
B	$362 = 3.62 \cdot 10^2$	$0.0362 = 3.62 \cdot 10^{-2}$
C	$52.3 = 5.23 \cdot 10^1$	$0.523 = 5.23 \cdot 10^{-1}$

a. Compare and contrast the scientific notation for the numbers in Row A.

b. Compare and contrast the scientific notation for the numbers in Row B.

c. Compare and contrast the scientific notation for the numbers in Row C.

d. Make a conjecture about the relationship between a number in scientific notation and the same number in standard form.

TRY THESE **B**

Express the following in scientific notation using any method.

a. 12,300 **b.** 0.45 **c.** 23,500,000,000

d. How many times as great as $4.25 \cdot 10^8$ is $4.25 \cdot 10^{10}$?

Now that Chrissy has converted her numbers to scientific notation, she needs to find the product to determine Ray De Ator's bonus points.

$$(8 \cdot 10^6) \cdot (2.08 \cdot 10^1)$$

Since multiplication is **commutative**, she knows that she can rearrange the product in the following manner.

$$(8 \cdot 2.08) \cdot (10^6 \cdot 10^1)$$

MATH TERMS

The **commutative property of multiplication** states that for any real numbers a and b, $ab = ba$.

SUGGESTED LEARNING STRATEGIES: Self/Peer Revision, Group Presentaion

My Notes

16. Chrissy performs the multiplication and expresses the answer in scientific notation as $16.64 \cdot 10^6$.

a. Explain why Chrissy's answer is incorrect.

b. Describe the process she should use to correct her answer.

c. Write the correct amount of bonus points earned by Ray De Ator in scientific notation and in expanded form.

TRY THESE **C**

Express each product in scientific notation and in standard form.

a. $(16 \cdot 3.9) \cdot (10^4 \cdot 10^3)$

b. $(1.4 \cdot 32.7) \cdot (10^9 \cdot 10^{13})$

c. $(7.3 \cdot 9) \cdot (10^5 \cdot 10^2)$

d. $(25 \cdot 2.2) \cdot (10^2 \cdot 10^{-6})$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Over the last four weeks, Kevin has saved \$18.75, \$22.30, \$15.00, and \$26.50. If his goal is to save \$100, how much more does he need to save?

2. Kevin's sister, Olivia, has saved an average of \$19.37 over the same four-week period.

a. How much has Olivia saved?

b. What is the difference in the amount of money Kevin and Olivia have saved?

3. The #1 pick in the NFL draft has just signed a contract that will pay him a total of \$75 million over six years.

a. Express the total amount of the contract in scientific notation.

b. Express his yearly salary in scientific notation.

4. The thickness of a strand of hair can be about 0.0005 millimeter. What is this measurement in scientific notation?

5. MATHEMATICAL REFLECTION Explain how you can determine the number of decimal places in your final answer when adding or subtracting numbers with decimals. Explain how you can determine the number of decimal places in your final answer when multiplying numbers with decimals.

Operations with Fractions
And the Beat Goes On

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Group Discussion, Create Representations

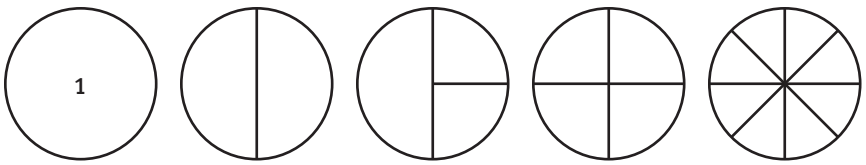
My Notes

Before we get into any math, let's talk about music.

1. Take a few moments to think about the following questions and discuss them with your group.

What is your favorite song?
Why do you like it?
Why do you prefer one type of music over another?

Believe it or not, the answers you came up with may have more to do with mathematics than you may realize. Consider the following diagrams:



2. The first circle has no divisions and is represented by the number 1.

a. Write the fractional equivalents in the portions of the other circles.

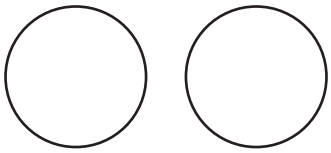
b. For each of the circles, justify that the sum of each combination of fractions is 1.

c. Which circle was not like the others? Explain what made it different and the process you used to determine your answer.

SUGGESTED LEARNING STRATEGIES: Create Representations, Questioning the Text, Think/Pair/Share

My Notes

3. Determine two additional ways to divide the circle using combinations of the same fractions from Item 1. Label each portion with the appropriate fractions, and justify that the sum of each combination of fractions is 1.



So what does all this have to do with music? As shown in the chart below, certain musical notes correspond to different fractional portions of a measure. In order to write sheet music, a composer must have a working knowledge of fractions.

CONNECT TO MUSIC

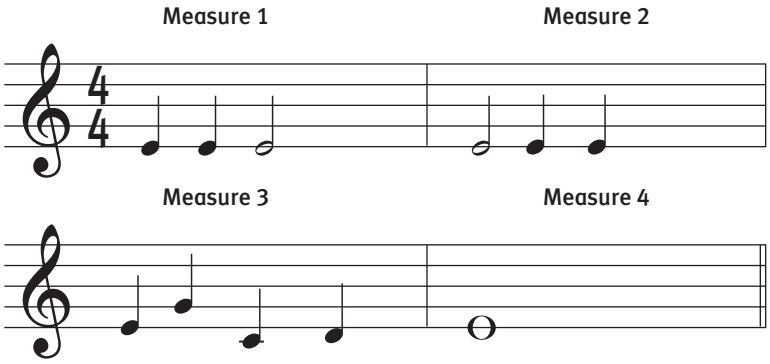
In music, **common time** is also referred to as $\frac{4}{4}$ time. This means that each measure contains four beats, and a quarter note is equal to one beat. Common time is also the most frequently used beat in rock, pop, funk, country, and R & B music.

Note	Relative Length in Common Time	Beats in Common Time	Fraction of Measure
𝅝	Whole note	4 beats	1
𝅗𝅥	Half note	2 beats	$\frac{1}{2}$
♩	Quarter note	1 beat	$\frac{1}{4}$
♪	Eighth note	$\frac{1}{2}$ beat	$\frac{1}{8}$
𝅘𝅥𝅯	Sixteenth note	$\frac{1}{4}$ beat	$\frac{1}{16}$

4. Based on the note chart, how many of each note would it take to fill one measure in common time? Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Self/Peer Revision, Discussion Group

My Notes



5. For each measure shown above, express each note as a fraction of its respective measure.

a. Measure 1:

b. Measure 2:

c. Measure 3:

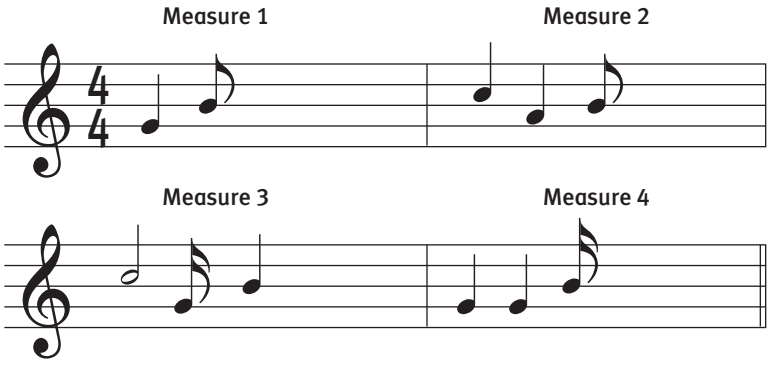
d. Measure 4:

6. Show that the sum of the fractions in each measure is equal to 1.

7. Based on your observations in Item 4, explain why you think the note in the fourth measure is called a "whole note."

My Notes

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Think/Pair/Share, Quickwrite



8. The measures shown above do not contain the required amount of beats. Use the table below to explore various ways to complete each measure.

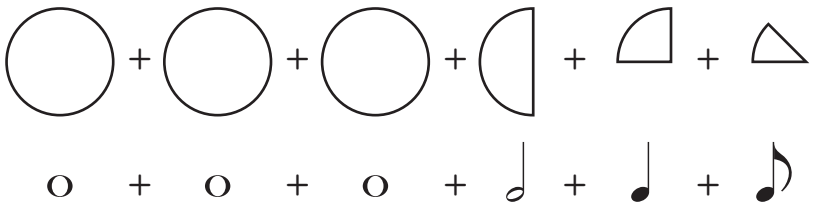
Measure	Fraction of Measure Shown	Fraction of Measure Remaining	Notes to Complete Measure (Example 1)	Notes to Complete Measure (Example 2)
1				
2				
3				
4				

9. Explain the processes you used to determine the fraction of the measures shown and the fraction of the measures remaining.

SUGGESTED LEARNING STRATEGIES: Create Representations, Predict and Confirm, Group Presentation

My Notes

10. What can you conclude about each of the following expressions? Explain your reasoning.



$1 + 1 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

11. Write the sum of the expressions in Item 10 as

a. A mixed number.

b. An improper fraction.

c. A decimal.

d. A percent.

TRY THESE **A**

a. $\frac{1}{3} + \frac{1}{6}$

b. $\frac{2}{5} + \frac{3}{8}$

c. $\frac{5}{12} - \frac{5}{6}$

d. $4\frac{1}{4} + 2\frac{1}{2}$

e. $6\frac{3}{5} - 2\frac{2}{3}$

f. Create graphical representations of Items a and d. Use the My Notes space.

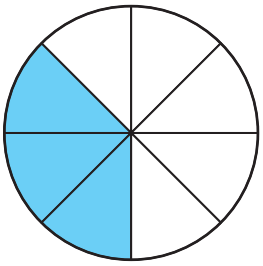
My Notes

SUGGESTED LEARNING STRATEGIES: Questioning the Text, Predict and Confirm, Create Representations

When multiplying fractions such as $\frac{3}{4} \cdot \frac{1}{2}$, it is helpful to think of the multiplication symbol • as the word "of." So you read, "Find $\frac{3}{4}$ of $\frac{1}{2}$."

12. Without completing the problem, do you think the answer will be greater than or less than $\frac{1}{2}$? Explain your reasoning.

13. Consider the picture below.



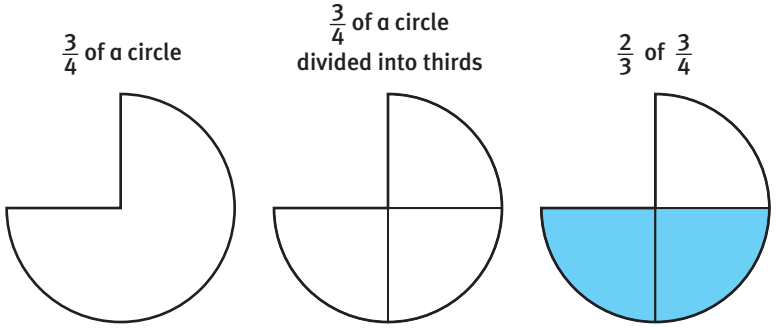
a. Explain how this picture could be used to represent $\frac{3}{4}$ of $\frac{1}{2}$.

b. Express the shaded portion of the circle as a single fraction.

SUGGESTED LEARNING STRATEGIES: Visualization, Simplify Think/Pair/Share

My Notes

The problem $\frac{2}{3} \cdot \frac{3}{4}$ could be modeled in the same way. This problem is asking you to find $\frac{2}{3}$ of $\frac{3}{4}$.



Depending on how the problem is completed, several "different" answers could be given. Using the diagrams above, the answer could be expressed as $\frac{2}{4}$ or $\frac{1}{2}$.

By simply multiplying the numerators and denominators, the answer could be shown as $\frac{2}{3} \cdot \frac{3}{4} = \frac{2 \cdot 3}{3 \cdot 4} = \frac{6}{12}$.

14. Explain why $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{6}{12}$ are the same number.

TRY THESE **B**

a. $\frac{4}{5} \cdot \frac{1}{3}$

b. $\frac{6}{7} \cdot \frac{3}{10}$

c. $\frac{8}{3} \cdot \frac{13}{15}$

d. $\frac{4}{12} \cdot \frac{6}{8}$

e. Create a graphical representation of Item a.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Mark the Text, Self/Peer Revision

My Notes

MATH TIP

You can gain a greater understanding of a concept by learning about "why" something works as opposed to memorizing "how" something works.

ACADEMIC VOCABULARY

The **reciprocal** of a number is its multiplicative inverse.

A common strategy used to divide fractions uses the directions "invert and multiply." The problem, however, is that the phrase can often raise more questions than answers. For example:

What does invert mean?
What do I invert?
Why do I invert?
Why do I multiply?

These are all valid questions that deserve equally valid answers.

The first two questions are easy to answer. To invert something is to turn it upside down. In mathematics, this is referred to as finding the **reciprocal** of a number. In other words, inverting the fraction $\frac{2}{3}$ would give us the reciprocal, or $\frac{3}{2}$. When dividing fractions such as in the problem $\frac{2}{5} \div \frac{2}{3}$, the second fraction is inverted to produce $\frac{2}{5} \cdot \frac{3}{2}$.

15. Determine the reciprocal of the following:

a. $\frac{4}{3}$

b. $\frac{5}{18}$

c. 15

d. $5\frac{3}{5}$

As for the last two questions concerning *why* this works, it's important to understand the concept of inverse operations.

16. Consider the problem $10 \div 2$.

a. Without using the words "divided by," explain what this problem is asking you to do.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Discussion Group, Activating Prior Knowledge, Group Presentation

My Notes

b. What number could be multiplied by 10 to reach the same solution?

c. What do you notice about the number 2 and your answer to part b?

d. Use your answer to part b to rewrite the problem using multiplication.

e. Based on your observation, explain what is meant by describing multiplication and division as inverse operations of one another.

Background music in movies and television shows is not chosen at random. In fact, the music is often chosen so that it directly relates to the emotions being portrayed on the screen.

17. The following table lists the average beats per minute of music meant to express various emotions.

Emotion	Beats per Minute
Joy and Triumph	120
Mystery and Suspense	115
Comfort and Peace	100
Loneliness and Regret	120

a. Describe a scene from a movie or television show that would parallel each of the four categories in the table.

SUGGESTED LEARNING STRATEGIES: Quickwrite

My Notes

b. Explain how you could use multiplication or division to determine the number of beats in a three-minute song.

TRY THESE **C**

a. $\frac{4}{5} \div \frac{3}{7}$

b. $\frac{6}{15} \div \frac{2}{9}$

c. $\frac{14}{3} \div 4$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Express the solutions to the following problems in two different forms.

   a. $\frac{3}{5} + \frac{4}{6}$ **b.** $\frac{5}{8} - \frac{1}{3}$

   c. $\frac{2}{5} \cdot \frac{7}{12}$ **d.** $\frac{4}{3} \div \frac{4}{9}$

2. Describe the process you would use to find the solution to the problem below. Express your answer numerically and with a graphical representation.

   $$3\frac{1}{4} - 1\frac{1}{3}$$

3. Without performing any calculations, determine which of the following problems will produce the greatest answer. Explain your reasoning.

   $\frac{3}{5} \cdot \frac{4}{7}$ or $\frac{3}{5} \div \frac{4}{7}$

4. Tony has 36 pages left in the book he is reading. He plans to read $\frac{1}{4}$ of the pages tonight before going to bed. Explain how you can determine the number of pages Tony will read by using either multiplication or division.

5. MATHEMATICAL REFLECTION Explain the similarities and differences between adding, subtracting, multiplying, and dividing fractions.

Rational and Irrational Numbers
Know When to Fold 'Em

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Graphic Organizer, Discussion Group

My Notes

A popular urban myth is that it is impossible to fold a piece of paper in half more than seven times.

1. Remove a sheet of paper from your notebook, and try it for yourself. You may fold the paper in any direction you wish as long as you fold the paper in half each time. When you have finished experimenting, share your results with your group members.

While the validity of the myth itself is not relevant to this activity, folding the paper in half repeatedly presents some interesting theories about numbers.

2. Complete the following table based on the first six folds you made in your paper.

Folds	Number of Regions on Paper	Each Region's Fraction of the Original Paper	Sketch of Unfolded Paper Showing Folds
0			
1			
2			
3			
4			
5			
6			

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Predict and Confirm, Use Manipulatives, Graphic Organizer

3. Consider the dimensions of the resulting quadrilateral each time the paper is folded.

a. What is happening to the dimensions of the paper?

b. In theory, what size is the resulting quadrilateral approaching as you keep folding?

c. Is it possible for a resulting quadrilateral to actually reach this size? Explain your answer.

4. Fold your paper into thirds as shown below.

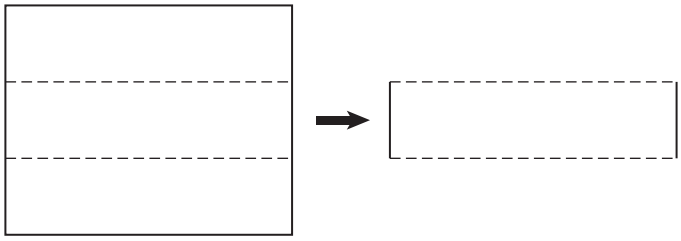


Figure 1 Figure 2

a. Beginning with your paper folded into a third as shown in Figure 2 above. Then fold the paper in half repeatedly and complete the table below.

	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7
Each Region's Fraction of Original Paper						

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Self/Peer Revision, Discussion Group

My Notes

b. Are any of these fractions the same as those from the table in Item 2? Explain why this might be true.

5. Use another piece of paper to show how you could find other fractional values between 0 and 1. Discuss your results with your group.

6. Fill in the missing values in the following table.

Fraction in simplest terms	Decimal Form	Percent	Graphical Representation
$\frac{1}{10}$			
	0.2		
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}\%$	
		40%	
		60%	
$\frac{2}{3}$	$0.\overline{6}$	$66.\overline{6}\%$	
	0.75		
		80%	
$\frac{9}{10}$			

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Mark the Text, Identify a Subtask, Predict and Confirm

My Notes

MATH TERMS

A **rational number** is any number that can be written as the ratio of two integers where the divisor is not zero. All of the fractions and percents in this unit so far have been *rational* numbers.

An **irrational number** cannot be expressed as the ratio of two integers. Nor can it be expressed as a terminating or repeating decimal. *Pi* is an irrational number.

Many early mathematicians believed that all numbers were **rational;** that is, they could be written as a quotient of two integers. However, as early as the 7th century BC, mathematicians from India were aware of numbers that could not be expressed as the quotient of two integers. Eventually, it became accepted that the square roots of most real numbers could not be expressed rationally. Not only that, but **irrational** numbers cannot be expressed as terminating or repeating decimals.

It is possible, however, to determine reasonable estimates for these numbers. To do so, it's helpful to become familiar with the relative size of some common rational numbers.

7. Consider the number $\sqrt{43}$.

a. Between which two perfect squares is the integer 43?

b. Between which two integers is $\sqrt{43}$? Explain your reasoning.

READING MATH

The number $\sqrt{43}$ is read "the square root of 43."

c. What rational number is exactly halfway between the integers you determined in part b?

MATH TERMS

The rational numbers together with the irrational numbers form the set of **real numbers**.

d. Square the rational number you found in part c. Is this number greater than or less than 43?

e. Use the information discovered in parts a to d to estimate $\sqrt{43}$ to the nearest hundredth. Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Quickwrite, Group Presentation

My Notes

f. Use a calculator to determine $\sqrt{43}$, and compare it to your estimate.

TRY THESE **A**

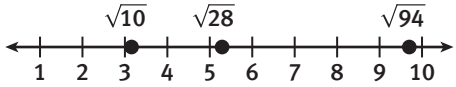
Estimate the following square roots to the hundredths place without using a calculator.

a. $\sqrt{10}$

b. $\sqrt{28}$

c. $\sqrt{94}$

d. Give the approximate location of each irrational number on the number line below.



CONNECT TO AP

In the branch of mathematics known as calculus, you will learn how to estimate the square root of a number using a linear equation.

8. Using a calculator, $\sqrt{3}$ is shown to be 1.7320508….

a. Explain how you know this is between the estimates of $1\frac{7}{10}$ and $1\frac{4}{5}$.

b. Explain the connection between these estimates and the actual value with rational and irrational numbers.

When you cube a number, you raise the number to the third power. For example, 4 cubed $= 4^3 = 4 \cdot 4 \cdot 4 = 64$.

The cube root of a number n is the number that when used as a factor three times gives a product of n. The symbol for cube root is $\sqrt[3]{\ }$. The expression $\sqrt[3]{64} = 4$ because $4 \cdot 4 \cdot 4 = 64$.

9. Is $\sqrt[3]{64}$ rational or irrational? Explain.

A cube root that cannot be simplified to a rational number is irrational.

10. Is $\sqrt[3]{12}$ rational or irrational? Explain.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Determine a rational and an irrational number between each of the following pairs of numbers.

1. $12\frac{1}{10}$ and $12\frac{3}{5}$

2. $4\frac{2}{3}$ and $4\frac{8}{9}$

3. $\frac{1}{2}$ and 1

4. $100\frac{2}{3}$ and $100\frac{5}{6}$

Tell if each of the following sums is rational or irrational. Explain your reasoning.

5. $\frac{3}{4} + 0.8 + 12\%$

6. $2\frac{1}{5} + 0.75 + \sqrt{27}$

7. $24\% + \sqrt{16} + 0.3$

8. $\frac{33}{8} + \pi + \frac{\pi}{3}$

9. $0.\overline{77} + 4 + 5\frac{1}{3}$

Determine a reasonable estimate to the hundredths place for each of the following. Then give their approximate location on a number line.

10. $\sqrt{18}$ **11.** $\sqrt{130}$

12. $\sqrt{2}$ **13.** $\sqrt{86}$

14. $\sqrt{81}$ **15.** $\sqrt{144}$

16. $\sqrt[3]{8}$ **17.** $\sqrt[3]{125}$

18. MATHEMATICAL REFLECTION Explain why it is always possible to find another number (rational or irrational) between any two numbers.

Rational Numbers

A RECIPE FOR SUCCESS

Embedded Assessment 2

Use after Activity 1.5.

On May 15, 2005, a group of volunteers met in Las Vegas to create what would become the world's largest cake. In order for the cake to qualify for the *Guinness Book of World Records*, it had to meet the following qualifications:

- contain traditional ingredients in the correct proportions,
- be prepared in the same manner as a normal-sized cake,
- be prepared according to appropriate hygiene standards, and
- be totally edible and safe to eat.

To create the world's largest cake, the organizers had to begin with a recipe for one cake and scale the ingredients accordingly. A sample recipe for one cake is shown below.

2 cups flour

$1\frac{1}{3}$ tablespoon baking powder

$\frac{1}{4}$ teaspoon salt

$\frac{2}{3}$ cup sugar

2 eggs

$\frac{3}{4}$ cup milk

$\frac{1}{2}$ teaspoon vanilla

1. If a volunteer prepared for 15 cakes, determine the following:

a. The sum of the needed cups of sugar and milk.

b. The difference between the number of teaspoons of vanilla needed and the number of teaspoons of salt needed.

c. Describe the step-by-step process you used to determine your answer for part a.

Rational Numbers

A RECIPE FOR SUCCESS

2. Express the amount of baking powder needed for one cake as each of the following:

a. An improper fraction.

b. A decimal.

c. A percent.

d. A graphical representation.

3. Does it make sense, in the real world, to convert the amount of baking powder used to a percent? Explain why or why not.

4. Are the amounts of the ingredients used rational or irrational numbers? Explain your reasoning.

5. The finished cake weighed in at 130,000 pounds, breaking the world record of 128,000 pounds.

a. Express each of these values in scientific notation.

b. Explain the process you would use to find the product of the numbers in scientific notation.

6. If cake A weighs $3 \cdot 10^6$ ounces and cake B weighs $7 \cdot 10^7$ ounces, how many times as heavy as cake A is cake B?

7. The world record cake used approximately 60,000 eggs. If the previous world record cake was $\frac{8}{9}$ the size of the Las Vegas cake, explain why you could multiply 60,000 by $\frac{8}{9}$ or divide 60,000 by $\frac{9}{8}$ to determine the number of eggs used in the smaller cake.

Rational Numbers

A RECIPE FOR SUCCESS

Embedded Assessment 2

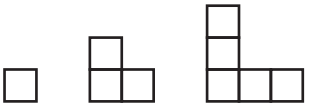
Use after Activity 1.5.

	Exemplary	Proficient	Emerging
Math Knowledge 1a, b, 2a, b, c, 4, 5a	• Correctly determines the sum of the measurements of sugar and milk (1a) and the difference between measurements of vanilla and salt (1b) • Correctly converts a mixed number to an improper fraction (2a), a decimal (2b), a percent (2c) • Correctly determines numbers as rational or irrational (4) • Correctly expresses the weight of the cakes in scientific notation (5a)	• Attempts both calculations but is able to correctly determine only the sum or the difference • Converts the mixed number into two of the three required forms • Determines four of the seven measurements as rational or irrational • Expresses the weight of one of the cakes in scientific notation correctly	• Attempts both calculations but is unable to find the sum or the difference. • Provides one correct conversion of the mixed number • Determines fewer than four of the measurements as rational or irrational • Does not express the weight of the cakes in scientific notation correctly
Representation 2d	Represents the measurement of baking powder as a visual model (2d)		Is unable to represent the baking powder measurement using a visual model.
Communication 1c, 3, 4, 5b, 7	• Correctly describes the step-by-step process used to find the answer (1c) • Correctly explains why a percent is not an appropriate form of measurement in a recipe (3) • Correctly explains whether the measurements are rational or irrational (4) • Correctly explains a process for finding the product of two numbers written in scientific notation.(5b) • Correctly explains why multiplying 60,000 by $\frac{8}{9}$ or dividing 60,000 by $\frac{9}{8}$ produces the same value. (7)	Communicates four of the five explanations clearly and accurately.	Communicates three of the five explanations clearly and accurately.

Practice

ACTIVITY 1.1

1. Determine the next two terms in the following patterns.

a. 

b. $0, \frac{1}{3}, \frac{2}{3}, \ldots$

c. 14, 13, 11, 8, …

d. 64, 16, 4, …

2. Express the fifth term in the pattern in two different ways: 100, 20, 4, …

3. Explain how you could use either addition, subtraction, multiplication, or division to generate the next term in the pattern −1, −4, …

4. Describe the following pattern, and continue it as long as you can:

a, by, cat, dove, early, …

ACTIVITY 1.2

5. $6^4 \cdot 6^3$

6. $12^6 \cdot 12^9$

7. $x^{14} \cdot x^5$

8. $x^1 \cdot x^5 \cdot x^7$

9. $\frac{8^7}{8^3}$

10. $\frac{15^6}{15^5}$

11. $\frac{x^{19}}{x^{10}}$

12. 150^0

13. $x^0 \cdot 8^0$

14. $\frac{x^2}{x^6}$

15. $\frac{x}{x^3}$

ACTIVITY 1.3

16. 25.7 + 34.6

17. 12.2 + 14.03

18. 155.68 − 24.3

19. 46 − 18.14

20. 2.8 • 10.45

21. 18 • 10.4

22. 100 − 3(2.3) + 5.6

23. Express in scientific notation:

a. 234,000

b. Eighty-seven million

c. 303

d. 7,062,010.2

24. Express in standard form:

a. $3.5 \cdot 10^8$

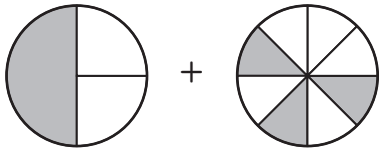
b. $4.62 \cdot 10^{-4}$

c. $3.5 \cdot 10^{-5}$

d. $4.62 \cdot 10^3$

25. Express your answer in scientific notation:
$(1.35 \cdot 10^6) \cdot (8.1 \cdot 10^5)$

ACTIVITY 1.4

26. 

27. $\frac{4}{3} + \frac{3}{4}$

28. $5\frac{2}{5} + \frac{4}{5}$

29. $\frac{8}{9} - \frac{7}{12}$

30. $8 - \frac{7}{12}$

31. $\frac{4}{9} \cdot \frac{6}{11}$

32. $\frac{14}{20} \cdot \frac{1}{6}$

33. $\frac{10}{11} \div \frac{1}{3}$

34. $14 \div \frac{3}{2}$

35. A roll of quarters contains ten dollars. Explain how you could determine the number of quarters in a roll by using either multiplication or division.

ACTIVITY 1.5

36. Express each of the following as a quotient of two integers.

a. 38%

b. 0.074

c. 2.9

d. 6

e. $5\frac{3}{8}$

37. Identify each of the following as either rational or irrational.

a. 5

b. $\frac{3}{10}$

c. 0.85

d. $2.\overline{3}$

e. 140%

f. $\sqrt{49}$

g. $\sqrt{120}$

38. Determine a reasonable estimate for the following and give the approximate location of each on a number line.

a. $\sqrt{14}$

b. $\sqrt{27}$

c. $\sqrt{118}$

d. $\sqrt{97}$

39. If *Pi* is considered an irrational number, explain why using 3.14 or $\frac{22}{7}$ to find the area of a circle would only give you an estimate of the actual answer.

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How are fractions, decimals, and percents related?
   - Why is it important to understand the procedures for working with different kinds of numbers?

Academic Vocabulary

2. Look at the following academic vocabulary words:
   - hypothesis
   - power
   - reciprocal
   - scientific notation

   Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

   a. What will you do to address each weakness?

   b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

1. When pouring concrete for a rectangular patio, a contractor will make a "diagonal check" to make sure that the corners are right angles. One contractor wrote down the four common diagonal measurements she uses on a regular basis. Which list below represents the measurements ordered from **least to greatest**?

$$2\sqrt{10},\ 10\sqrt{5},\ 12\sqrt{2},\ 2\sqrt{41}$$

A. $2\sqrt{10},\ 2\sqrt{41},\ 10\sqrt{5},\ 12\sqrt{2}$

B. $2\sqrt{10},\ 2\sqrt{41},\ 12\sqrt{2},\ 10\sqrt{5}$

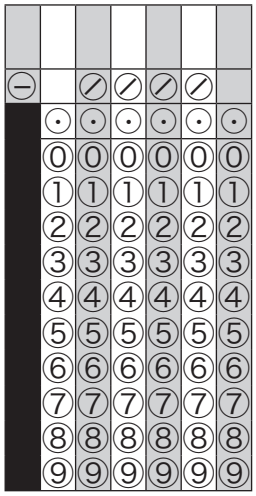
C. $12\sqrt{2},\ 10\sqrt{5},\ 2\sqrt{10},\ 2\sqrt{41}$

D. $2\sqrt{41},\ 10\sqrt{5},\ 2\sqrt{41},\ 12\sqrt{2}$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. There is evidence that some microscopic life forms existed as long as 3,700 to 3,800 million years ago. This evidence was found in Isua greenstone in Greenland. What power of ten would you use when naming these two numbers using scientific notation?

2.



Read
Solve
Explain

3. ABC Cellular offers a plan to purchase international text messages. The basic plan is to pay \$0.25 per text message. Upgrade 1 allows you to send 100 text messages a month for \$9.95 with additional messages charged at \$0.15 each. Upgrade 2 allows unlimited international texting for \$29.25 a month. In one month Evyn sent 150 text messages to her friend in Costa Rica.

Part A: Calculate Evyn's charge for each plan. Which plan would be the best for Evyn? Justify your answer.

Answer and Explain

Part B: The following month, Evyn makes 240 calls to Costa Rica. Does this change the plan that should be selected? Justify your answer.

Answer and Explain

Read
Solve
Explain

4. The formula $t = \frac{\sqrt{h}}{4}$ represents the time (t) in seconds that it takes an object to fall from a height of h feet.

Part A: If a ball is dropped from a height of 200 ft, calculate how long it will take to reach the ground to the nearest tenth.

Answer and Explain

Part B: If $t = 1\frac{1}{4}$ seconds, find h. Explain what the number you calculated represents.

Answer and Explain

Part C: What is the height from which an object must be dropped to land on the ground after one second?

Answer and Explain