

Properties of Real Numbers

Properties by the Pound

ACTIVITY

1.3

SUGGESTED LEARNING STRATEGIES: Create Representations, Activating Prior Knowledge, Think/Pair/Share, Interactive Word Wall

My Notes

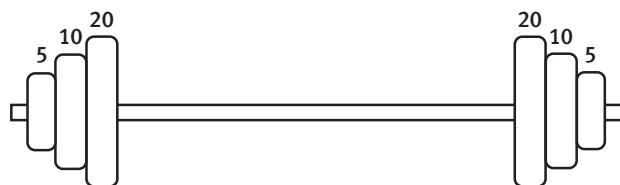
The local girls' track team is strength training by lifting weights. One of the runners puts weights on each end of the weightlifting bar while she is getting ready to bench press.

1. On one end she places a ten-pound plate and then a five-pound plate. On the other end, she places first a ten-pound plate and then a five-pound plate. Does the bar have the same amount of weight at each end? Explain using a diagram in the My Notes section.

In symbols, this situation described above is written as $a + b = b + a$, where both a and b are real numbers. This is a statement of the Commutative Property.

2. What does the commutative property state?
3. Demonstrate the commutative property of addition by writing an example using:
 - a. negative integers
 - b. rational numbers represented as fractions

Another runner needs to increase the amount of weight on the bar for another lift. On each side she places three different plates: one five pound plate, one ten pound plate, and one twenty pound plate as shown in the diagram below.



To ensure that the weight on both sides is the same, she wants to find the total weight on both sides. For the left side of the bar, she used the expression $(5 + 10) + 20$ and on the right side of the bar, she used the expression $5 + (10 + 20)$.

4. Use the order of operations to evaluate each expression and compare the results.

MATH TIP

Order of Operations:

- 1st: Evaluate expressions within grouping symbols.
- 2nd: Evaluate powers and radicals.
- 3rd: Do all multiplication and division from left to right.
- 4th: Do all addition and subtraction from left to right.

My Notes

MATH TERMS

A **counterexample** is an example or case that proves a conjecture or theory wrong.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations, Interactive Word Wall, Group Presentation

The expression $(5 + 10) + 20 = 5 + (10 + 20)$ can be written in symbols as $(a + b) + c = a + (b + c)$, where a , b , and c are real numbers. This is an example of the associative property.

5. What does the associative property state?
6. Demonstrate this property of addition by writing an example using:
 - a. negative integers
 - b. rational numbers represented as decimals
7. The properties you have been using were described as properties of *addition*. Determine if these properties also apply for the operations that follow. If the property applies, then write an example that demonstrates it. If not, write a **counterexample**.

	Commutative Property	Associative Property
Subtraction		
Multiplication		
Division		

The **identity properties** apply to both addition and multiplication.

8. Use what you know about identity properties to complete these statements. Then state the identity properties in words.
 - a. $a + \underline{\hspace{1cm}} = a$, and $\underline{\hspace{1cm}} + a = a$.
Identity Property of Addition:
 - b. $x \cdot \underline{\hspace{1cm}} = x$, and $\underline{\hspace{1cm}} \cdot x = x$.
Identity Property of Multiplication:
 - c. Explain whether the identity properties apply to division and subtraction. Include an example or counterexample with each explanation.

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continued

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation, Activating Prior Knowledge, Quickwrite, Think/Pair/Share

Each real number has an associated number called its **additive inverse**. All but one real number has an associated number called its **multiplicative inverse** or **reciprocal**.

The **Additive Inverse Property** states that a number added to its additive inverse gives a sum of zero. In symbols, the additive inverse of a is $-a$: $a + (-a) = 0$, and $-a + a = 0$

9. Write an equality showing a number and its additive inverse.

The **Multiplicative Inverse Property** states that a number multiplied by its multiplicative inverse yields a product of one. In symbols, the multiplicative inverse of a is $\frac{1}{a}$: $a \cdot \left(\frac{1}{a}\right) = 1$ and $\frac{1}{a} \cdot a = 1$

10. Write an equality showing a number and its multiplicative inverse.

11. Which real number does not have a multiplicative inverse? Explain.

TRY THESE A

Complete the following number sentences by finding each:

a. Additive inverse

$$5 + \underline{\hspace{2cm}} = 0 \quad -b + \underline{\hspace{2cm}} = 0 \quad \frac{4}{5} + \underline{\hspace{2cm}} = 0$$

b. Multiplicative inverse

$$6 \cdot \underline{\hspace{2cm}} = 1 \quad \frac{2}{t} \cdot \underline{\hspace{2cm}} = 1 \quad 0.23 \cdot \underline{\hspace{2cm}} = 1$$

12. The **distributive property** can be written in symbols as $a(b + c) = a(b) + a(c)$. The factor a is said to be “distributed” to both addends in the parentheses.

- Describe the order of operations used on the left side of the equation $a(b + c) = a(b) + a(c)$ in terms of a , b , and c .
- Describe the order of operations used on the right side of the equation $a(b + c) = a(b) + a(c)$ in terms of a , b , and c .
- What is true about the two sides of the equation?

My Notes

ACADEMIC VOCABULARY

distributive property

MATH TIP

The distributive property can also be written as $(b + c)a = ba + ca$.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Activating Prior Knowledge, Group Presentation

TRY THESE B

Use the distributive property to rewrite and evaluate each of the following expressions.

- a. $5(-7 + 12)$ b. $\frac{1}{2}\left(\frac{7}{3} + \frac{5}{3}\right)$ c. $-3(-24 + 16)$
 d. $(2 + 9)5$ e. $4(x + 2y)$

13. Does the distributive property apply to operations other than multiplication and addition? Write an example or a counterexample for each of the following.

- a. Does multiplication “distribute” over subtraction?
 b. Does division “distribute” over addition?

A set has **closure** under an operation if, when an operation is performed on members of that set, the result also is included in the set.

14. The set of whole numbers is closed under addition because the sum of any two whole numbers is also a whole number. Write an equality that demonstrates the closure of the whole numbers under addition.

15. Not all number sets are closed under an operation. Write an equality that demonstrates that natural numbers are not closed under subtraction.

16. Determine for which operations the sets of numbers listed are closed. Use whole numbers in the first table and integers in the second table.

Operation	Example or Counterexample	Closed or Not Closed
Addition	$1 + 4 = 5$	Closed
Subtraction		
Multiplication		
Division		

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Addition		
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continued

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation

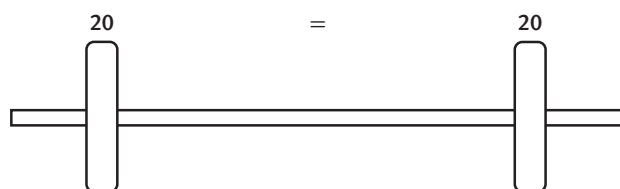
My Notes

17. Think of some examples of rational and irrational numbers.

- Is the sum of a rational number and an irrational number rational or irrational? Explain.
- Is the product of a rational number and irrational number rational or irrational? Explain.

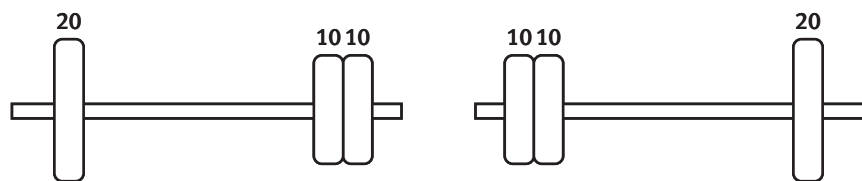
18. There are properties of equality that can make it easier to solve equations and to work with proofs in geometry. The weights that the track team is lifting can be used to illustrate these.

- The **reflexive property** states that $a = a$.



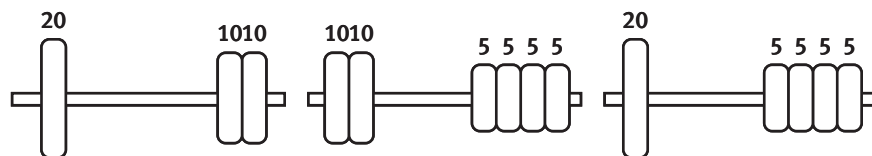
Tell what the reflexive property means in your own words using at least one example.

- The **symmetric property** states if $a = b$, then $b = a$.



Tell what the symmetric property means in your own words using at least one example.

- The **transitive property** states if $a = b$ and $b = c$, then $a = c$.



What does the transitive property mean? Explain in your own words using at least one example.

CONNECT TO FITNESS

The illustrations of different kinds of plates on a barbell in this activity are meant to demonstrate the properties of equality. If you were actually placing plates on a barbell at a gym, however, you would use exactly the same plates, placed in the same order from each end.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation

19. If $3x + 2 = y$ and $y = 8$ what property allows you to conclude that $3x + 2 = 8$?
20. If $10 = y$, which property allows you to conclude that $y = 10$?
21. Do each of these properties apply to inequalities? Use a numerical example or counterexample to explain.
 - a. reflexive
 - b. symmetric
 - c. transitive:

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Identify the property illustrated in each of the following equations.
 - a. $5(2 + 3) = (2 + 3)5$
 - b. $25 \cdot (0.04) = 1$
 - c. $4 + (9 + 3) = (9 + 3) + 4$
 - d. $(x + 3)4 = 4x + 12$
2. Write an equality that illustrates the associative property of multiplication.
3. Write an equality that illustrates the additive inverse of 3.
4. The expressions below are the result of distributing multiplication over addition. Rewrite each to show the expression prior to distributing.
 - a. $3(x) + 3(8)$
 - b. $-10y + xy$
 - c. $6w + 9z$

5. Using the table below, determine for which operations the set of natural numbers is closed or not closed.

Operation	Example or Counterexample	Closed or Not Closed
Addition		
Subtraction		
Multiplication		
Division		

6. Explain why the sum of $\sqrt{2}$ and 3 is irrational.

Identify the property illustrated in each of the following equations.

7. $y = y$
8. $10 \cdot 5 = 5 \cdot 10$
9. If $y = 11$ and $y = 3x + 2$, then $11 = 3x + 2$.
10. If $7 + x = 12$, then $12 = 7 + x$
11. **MATHEMATICAL REFLECTION** Which of the properties of operations of real numbers is most useful to you? Explain using more than one example.