Jonas and Margo’s dad explained that since their grandparents were moving in with them, he needed to make it easier for them to get in and out of the house. Their dad asked Jonas and Margot to research the specifications for building stairs and wheelchair ramps. They decided to look at the government website that gave the Americans with Disabilities Act (ADA) accessibility guidelines for wheelchair ramps and found the following diagram:

The chart below gives information from the ADA website about the slope of wheelchair ramps.

<table>
<thead>
<tr>
<th>Slope</th>
<th>Maximum rise in.</th>
<th>Maximum run mm</th>
<th>Maximum run ft</th>
<th>Maximum run m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{16} &lt; m \leq \frac{1}{12} )</td>
<td>30</td>
<td>760</td>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>( \frac{1}{20} \leq m &lt; \frac{1}{16} )</td>
<td>30</td>
<td>760</td>
<td>40</td>
<td>12</td>
</tr>
</tbody>
</table>

Then, they decided to look for the requirements for building stairs and found the following diagram:
1. What do you think is meant by the terms *rise* and *run* in this context?

Consider the line in the graph below:

2. What is the vertical change between:
   a. points A and B?
   b. points A and C?
   c. points C and D?

3. What is the horizontal change between:
   a. points A and B?
   b. points A and C?
   c. points C and D?

The ratio of the vertical change to the horizontal change determines the slope of the line.

\[ \text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} \]

4. Find the slope of the segment of the line connecting:
   a. points A and B
   b. points A and C
   c. points C and D

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations
5. What do you notice about the slope of the line in parts (a), (b), and (c) of item 4?

6. Slope is sometimes referred to as \( \frac{\text{rise}}{\text{run}} \). Explain how the ratio \( \frac{\text{rise}}{\text{run}} \) relates to the ratios for finding slope mentioned above.

7. Would the slope change if you counted the run (horizontal change) before you counted the rise (vertical change)? Explain your reasoning.

8. Vertical change can be represented as a change in \( y \), and horizontal change can be represented by a change in \( x \).

These movements can also be written as the ratio \( \frac{\Delta y}{\Delta x} \). Using this new terminology, explain how to move along the grid to get from one point to another.

From A to C: \( \Delta y = \) _____ and \( \Delta x = \) _____ Ratio \( \frac{\Delta y}{\Delta x} = \) _____

From B to D: \( \Delta y = \) _____ and \( \Delta x = \) _____ Ratio \( \frac{\Delta y}{\Delta x} = \) _____

From A to D: \( \Delta y = \) _____ and \( \Delta x = \) _____ Ratio \( \frac{\Delta y}{\Delta x} = \) _____

9. What do you notice about these ratios?

**WRITING MATH**

In mathematics the Greek letter \( \Delta \) (delta) represents a change or difference between mathematical values.
10. Describe the movement along the grid to get from $B$ to $A$ and then from $D$ to $B$.
   
   From $B$ to $A$: $\Delta y = \underline{}$ and $\Delta x = \underline{}$
   
   From $D$ to $B$: $\Delta y = \underline{}$ and $\Delta x = \underline{}$

11. What kind of number represents the change in $y$, $\Delta y$, described in Item 10?

12. What kind of number represents the change in $x$, $\Delta x$, described in Item 10?

13. Write ratios for $\frac{\Delta y}{\Delta x}$ using these numbers for the movement from $B$ to $A$ and then from $D$ to $B$.
   
   $B$ to $A$:
   
   $D$ to $B$:

14. How do these ratios compare to those you found in Item 8?

15. When the points in a scatter plot lie on a line, a ratio $\frac{\Delta y}{\Delta x}$ tells you the slope of a line through those points. What do you think is true about slope ratios between any two points on a line?
Jonas and Margo have drawn this design for the wheelchair ramp.

Recall the chart from the ADA website about the slope of wheelchair ramps.

<table>
<thead>
<tr>
<th>Slope</th>
<th>Maximum rise in.</th>
<th>Maximum run mm</th>
<th>Maximum run ft</th>
<th>Maximum run m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{16} &lt; m \leq \frac{1}{12}$</td>
<td>30</td>
<td>760</td>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>$\frac{1}{20} \leq m \leq \frac{1}{16}$</td>
<td>30</td>
<td>760</td>
<td>40</td>
<td>12</td>
</tr>
</tbody>
</table>

16. Jonas and Margo need to check that the ramp that they designed meets the ADA recommendations.

a. In feet, what is the rise of the ramp they designed?

b. What is the slope of the ramp that they designed?

c. Which row of the chart do Jonas and Margo need to check? Explain.

d. Does the ramp they designed meet the recommendations for rise? Explain.

e. Does the ramp they designed meet the recommendation for run? Explain.

f. Does the ramp they designed meet the ADA recommendations?
17. Determine the slope of the line graphed below.

![Graph of a line](image)

Although the slope of a line can be calculated by looking at a graph and counting the vertical and horizontal change, it can also be calculated numerically.

18. Recall that the slope of a line is found by the ratio \( \frac{\text{change in } y}{\text{change in } x} \).

   a. Find two points on the graph above and record the coordinates of the two points that you selected.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )-coordinate</td>
<td>( y )-coordinate</td>
</tr>
<tr>
<td>( 1^{st} ) point</td>
<td></td>
</tr>
<tr>
<td>( 2^{nd} ) point</td>
<td></td>
</tr>
</tbody>
</table>

   b. Which coordinates relate to the vertical change on a graph?

   c. Which coordinates relate to the horizontal change on a graph?
d. Find the vertical change by subtracting the $y$-coordinate of the first point from the $y$-coordinate of the second point.

e. Find the horizontal change by subtracting the $x$-coordinate of the first point from the $x$-coordinate of the second point.

f. Calculate the slope of the line. How does this slope compare to the slope that you found in Item 17?

g. If other students in your class selected different points for this problem, should they have gotten different values for the slope of this line? Explain.

19. It is customary to label the coordinates of the first point $(x_1, y_1)$ and the coordinates of the second point $(x_2, y_2)$.

a. Write an expression to calculate the vertical change, $\Delta y$, of the line through these two points.

b. Write an expression to calculate the horizontal change, $\Delta x$, of the line through these two points.

c. Write an expression to calculate the slope of the line through these two points.
20. Margo and Jonas went to the lumber yard to buy supplies to build the wheelchair ramp. They know that they will need many pieces of wood. Each piece of wood costs $3.

**a.** Write a function, \( f(x) \), for the total cost of the wood pieces if Jonas and Margo buy \( x \) pieces of wood.

**b.** Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Pieces of Wood, ( x )</th>
<th>Total cost, ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Total Cost of Wood

\[
f(x) = 3x
\]

**c.** What is the slope of the line that you graphed? Use the formula that you developed in Item 19 and then use the graph to check your work.

**d.** How does the slope of this line relate to the situation with the pieces of wood?

**e.** Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.
21. Margo is going to work with a local carpenter during the summer. Each week she will earn $10.00 plus $2.00 per hour.

a. Write a function, \( f(x) \), for Margo's total earnings if she works \( x \) hours in one week.

b. Make an input/output table of ordered pairs and then graph the function. Label your axes.

<table>
<thead>
<tr>
<th>Hours, ( x )</th>
<th>Earnings, ( f(x) ) (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

\[ f(x) = 10 + 2x \]

\[ \text{Graph on coordinate plane with axes labeled.} \]

\[ x \): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \]

\[ f(x) \): 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 \]

\[ \text{Graph line with slope formula developed in Item 19.} \]

c. What is the slope of the line that you graphed? Use the formula you developed in Item 19 and then use the graph to check your work.

d. How does the slope of this line relate to Margot's job?

e. Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.

f. How much will Margot earn if she works for 8 hours in one week?
22. By the end of the summer Margot has saved $375. Recall that each of the small pieces of wood costs $3.

a. Write a function, $f(x)$, for the amount of money that Margo still has if she buys $x$ pieces of wood.

b. Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Pieces of wood, $x$</th>
<th>Money remaining, $f(x)$ (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>


c. How does this line differ from the other lines you have seen in this activity?

d. What is the slope of the line that you graphed? Use the formula you developed in Item 19.

e. How does this slope differ from the other slopes that you have seen in this activity?

f. How does the slope of this line relate to Margot’s savings?
23. Consider the graph of the line below:

![Graph of a line with points A and B]

a. What is the vertical change from point A to point B?

b. What is the horizontal change from point A to point B?

c. What is the slope of the line?

24. Describe the slope of any line that rises as you view it from left to right. An example is shown below.

![Graph of a rising line]

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Think/Pair/Share
25. Describe the slope of any line that falls as you view it from left to right. An example is shown below.

26. What is the slope of a horizontal line? Choose any two points on the line and calculate the rise and the run.

27. What is the slope of a vertical line? Choose any two points on the line and calculate the rise and the run.
28. One point that is on the graph of \( y = -\frac{2}{3}x + 3 \) is shown on the grid below. Use your knowledge of slope to place three more points on the graph and then give the coordinates of your points.

29. Margot and Jonas used the diagram from the first page of this activity as a model for the staircase that they are going to build.

a. If the height the entire staircase needs to cover is 56.7 inches, what should the height of each step be?

b. If the length the entire staircase needs to cover is 67.9 inches, what should the length of each tread be?
Suggested Learning Strategies: RAFT

My Notes

Writing Math
When writing your response to Item 30 you can use a RAFT.
• Role
• Audience
• Format—a letter
• Topic—wheelchair ramps and stairs

Check Your Understanding

1. Find \( \Delta x \) and \( \Delta y \) for the points \((7, -2)\) and \((9, -7)\).
2. Find the slope given a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>-1</td>
</tr>
</tbody>
</table>

3. Find the slope of a line that passes through the points \((0, 4)\) and \((3, 9)\).
4. Find the slope of a line that passes through the points \((-2, 4)\) and \((3, -3)\).
5. Find two points that make a slope of \(-\frac{5}{7}\).
6. Determine the slope of the given line.

7. Sketch a line that has a positive slope.
8. Sketch a line that has a negative slope.
9. Sketch a line that has a zero slope.
10. Given the linear equation \( y = -\frac{2}{5}x + 1 \), determine the slope.
11. One point on the graph of \( y = \frac{3}{4}x - 2 \) is given. Place two more points on the graph of the equation.

12. Mathematical Reflection Explain three different ways to find the slope of a line and how these methods are the same and different.

C. What is the slope of each step? What is the slope of entire staircase?

30. Write a letter to Jonas and Margot explaining how rise and run relate to slope and why these topics are an important aspect of building wheelchair ramps and stairs.