You are the packaging director for a paper products company. Your company is introducing a new type of paper cups. Your design team must design a cardboard container to use when packaging the cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, your team will collect data on two types of cups found around the office.

1. Use two different types of cups to complete the tables below.

<table>
<thead>
<tr>
<th>CUP 1</th>
<th>CUP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cups</td>
<td>Height of Stack</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?
SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Guess and Check, Think/Pair/Share, Quickwrite

Use your data for Cup 1 to complete Items 3–6.

3. Make a graph of the data you collected.

4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

6. Write an equation that gives the height of a stack of cups, $h$, in terms of $n$, the number of cups in the stack.
7. Use your equation from Item 6 to find $h$ when $n = 16$ and when $n = 50$. Do your answers to this question agree with your predictions in Items 4 and 5?

8. Sketch the graph of your equation from Item 6.

9. How are the graphs you made in Items 3 and 8 the same? How are they different?
10. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

11. Find the dimensions of a container that will hold a stack of 25 cups.

12. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
   - the equation your team discovered to find the height of the stack of Cup 1 style cups
   - a description of how your team discovered the equation and the minimum number of cups needed to find it
   - an explanation of how the numbers in the equation relate to the physical features of the cup
   - an equation that could be used to find the height of the stack of Cup 2 style cups
After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The cup will be the same basic shape as described in your report. The company will use the function \( S(n) = 0.5n + 12.5 \).

13. What do \( S \), \( S(n) \), and \( n \) represent?

14. What do the numbers 0.5 and the 12.5 in the function \( S \) tell you about the physical features of the cup?

15. Evaluate \( S(1) \) to find the height of a single cup.

16. How tall is a stack of 35 cups? Show your work using function notation.
17. If you add 2 cups to a stack, how much does the height of the stack increase?

18. If you add 20 cups to a stack, how much does the height of the stack increase?

19. A member of one of the teams stated: “If you double the number of cups in a stack, then the height of the stack is also doubled.” Is this statement correct? Explain.

20. If you were to graph the linear function $S(n) = 0.5n + 12.5$ and connect the points, you would see that they lie in a straight line. The slope of a line is a measure of the steepness of a line and indicates a rate of change.
   a. What is the slope of this line?
   
   b. Interpret the slope of the line as a rate of change that compares a change in height to a change in the number of cups.
21. a. The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?

b. If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?

c. How many cups fit in a container that is 36 cm tall?

d. How many cups fit in a container that is 50 cm tall?

22. The function $S(n) = 0.5n + 12.5$ describes height $S$ in terms of the number of cups $n$.

a. Solve this equation for $n$ to describe the number of cups $n$ in terms of the height $S$. 
b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Item 21 parts (c) and (d).

c. What is the slope of the line represented by your equation in part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 20(b).

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

A consultant earns a flat fee of $75 plus $50 per hour for a contracted job. The table shows the consultant’s earnings.

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>$75</td>
<td>$125</td>
<td>$175</td>
<td>$225</td>
<td>$275</td>
</tr>
</tbody>
</table>

1. If the consultant has a 36-hour contract, how much will she earn?

2. Write an equation that shows the consultant’s earnings $E$ in terms of $h$, the number of hours of her contract.

Use this table for Items 3 and 4.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
</tr>
</tbody>
</table>

3. Write an equation for $y$ in terms of $x$.

4. Explain how the numbers in your equation relate to the numbers in the table.

5. The equation for the cost $C$ of a cab ride of $m$ miles is $C = 2.5m + 3.5$.
   a. What is the cost of a 6-mile ride?
   b. What is the cost of a 7-mile ride?
   c. How is the price difference between a 6-mile ride and a 7-mile ride related to the numbers in your equation?

6. In the equation $S = 0.25n + 8.5$, $S$ is the height in inches of a stack of jumbo cups and $n$ is the number of cups.
   a. How many cups would it take to make a stack 1 inch higher?
   b. How many cups would fit in a carton that is 18 inches high?
   c. Interpret the slope as a rate of change.

7. **MATHEMATICAL REFLECTION** What did you learn about creating a linear model? How can you recognize and interpret a constant rate of change?