Ratio and Rates

Unit Overview
In this unit you will study proportional relationships. You will learn that two equal ratios form a proportion and how to solve a proportion. You will explore and use two special ratios: rates, which compare quantities with different units, and percents, which compare a number to 100. Both rates and percents are used extensively in shopping, sports, data analysis, banking, and other aspects of everyday life.

Academic Vocabulary
As you study this unit, add these terms to your vocabulary notebook.
- percent
- rate
- unit rate

Essential Questions
Why are proportional relationships an important part of mathematics?
How is percent related to fractions and decimals, and why is it such a useful tool in everyday life?

Embedded Assessments
These assessments, following activities 4.2 and 4.5, will give you an opportunity to demonstrate your ability to work with ratios and rates and to show how well you understand and can use percent.

Embedded Assessment 1
Ratios and Rates p. 199

Embedded Assessment 2
Percents p. 233
Write your answers on notebook paper. Show your work.

1. Use the Property of One to find two equivalent fractions for each fraction.
   a. \( \frac{3}{5} \)
   b. \( \frac{3}{7} \)
   c. \( \frac{7}{8} \)
   d. \( \frac{3}{8} \)

2. Write the letter of each pair of fractions that are equal.
   a. \( \frac{2}{3} \) and \( \frac{4}{5} \)
   b. \( \frac{5}{8} \) and \( \frac{10}{16} \)
   c. \( \frac{3}{7} \) and \( \frac{7}{15} \)
   d. \( \frac{2}{5} \) and \( \frac{5}{10} \)
   e. \( \frac{3}{5} \) and \( \frac{9}{15} \)

3. Use the Property of One to write an equivalent fraction with a denominator of 100 for each fraction.
   a. \( \frac{6}{20} \)
   b. \( \frac{2}{5} \)
   c. \( \frac{1}{4} \)
   d. \( \frac{36}{25} \)
   e. \( 2 \frac{3}{5} \)

4. Use division to find an equivalent decimal. Round quotients to the nearest hundredth.
   a. \( \frac{3}{8} \)
   b. \( \frac{6}{11} \)
   c. \( \frac{6}{9} \)
   d. \( \frac{5}{7} \)

5. Multiply.
   a. \( 6.2735 \times 10 \)
   b. \( 6.2735 \times 100 \)
   c. \( 6.2735 \times 1000 \)

   Describe any pattern you notice.

6. Divide.
   a. \( 87.345 \div 10 \)
   b. \( 87.345 \div 100 \)
   c. \( 87.345 \div 1000 \)

   Describe any pattern you notice.

7. Divide.
   a. \( 40.20 \div 12 \)
   b. \( 500.50 \div 0.50 \)
   c. \( 105.3 \div 2.7 \)

8. Explain how fractions and decimals are related.
Ms. Yang’s class wants to have a Math Night. Ms. Yang agrees to plan the activities so long as the students plan the food and drinks. The Math Night theme will be *Individual Fun Facts*.

Work with a partner to sample some of the activities Ms. Yang has planned for her students. Record your data.

**Activity 1: Writing Speed**
How fast can you write the word *math* five times? Try with each hand.

a. Rate for the *dominant* hand:

b. Rate for the *non-dominant* hand:

**Activity 2: Reading Speed**
How many words can you read in one minute? You may use your own book or the passage your teacher will give you.

a. Reading rate in minutes:

b. Reading rate in seconds:

**Activity 3: Heart Rate**
Measure your heart rate by counting your pulse for 30 seconds. To do this, locate your pulse by placing your index and middle fingers on the thumb side of your wrist, then count the number of beats. What is your heart rate?

**Activity 4: Jumping Jacks**

a. How many jumping jacks can you do in 15 seconds?

b. What is your jumping jacks rate?

**Activity 5: Are You Tongue Twisted?**

a. How fast can you say “Peter Piper picked a peck of pickled peppers” 5 times?

b. What is your speaking rate?

**Activity 6: Are You a Fast Walker?**
Look at the materials your teacher has provided.

a. How could you find your walking rate? Explain.

b. What is your walking rate?
The students must work together to plan the food and drinks. Ms. Yang already has some juice concentrate, napkins, and cups they can use. First the students need to figure out how to blend the concentrate with water to make enough juice for 26 people.

The label on the can of mix gives two mixing options.

You can compare two quantities, or show a relationship between them, by writing a ratio. The numbers that are compared are called terms.

1. The directions for mixing the juice show a relationship between what 2 quantities?

2. Write a ratio in 3 different ways to show this relationship for each option.

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cups concentrate</td>
<td>3 cups concentrate</td>
</tr>
<tr>
<td>4 cups water</td>
<td>5 cups water</td>
</tr>
<tr>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>_______</td>
<td>_______</td>
</tr>
</tbody>
</table>

3. The students agree that they want to make juice with the most flavor. Which option do you think they will choose? Explain why you think they will make that choice.

☐ Option 1 is more flavorful.

☐ Option 2 is more flavorful.

☐ Options 1 and 2 are equally flavorful.
The students notice that Option 2 has one more cup of concentrate and one more cup of water than Option 1. They wondered if adding one cup of concentrate and one cup of water to Option 1 changed the flavor of the juice. To help visualize the comparison between a ratio of 2 to 4 and a ratio of 3 to 5, look at the models your teacher will show you.

4. Color the drawings above to visually compare the ratios.

5. What can you conclude about the two ratios by looking at the models?

6. The students want to determine how much water they will add to preserve the ratio of each option when they add 6 cups of the concentrate.

   a. Predict how much water they will add to preserve the ratio of Option 1.

   b. Predict how much water they will add to preserve the ratio of Option 2.
7. Using ratio tables is a way to compare the ratios $\frac{2}{4}$ and $\frac{3}{5}$.

a. Complete each ratio table to show the relationship of juice concentrate to water for each mixing option if you double the recipe, triple it, and so on.

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Juice concentrate</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Option 2</th>
<th>Juice concentrate</th>
<th>3</th>
<th>9</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>5</td>
<td>15</td>
<td>25</td>
</tr>
</tbody>
</table>

b. Highlight the column in each table that has the same number of cups of juice concentrate and write each ratio below.

c. What does this tell you about the strength of each mixture?

8. Now use a different color to highlight the column in each table that has the same number of cups of water.

a. Write each ratio.

b. What does this tell you about the strength of each mixture?
9. If the students increase the juice concentrate 1 cup at a time, find how much water they would need to use in each case.

   a. Complete the ratio tables below.

<table>
<thead>
<tr>
<th>Option</th>
<th>Concentrate (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Water (w)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Concentrate (m)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Water (w)</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

   b. What patterns do you notice in the table for Option 1?

   c. What is the rule for the Option 1 table?

   d. What patterns do you notice in the table for Option 2?

   e. What is the rule for the Option 2 table?

   Relationships that have the same ratio are called **proportional relationships** and can be represented with the algebraic rule $y = mx$. The amount, $m$, is the factor by which $y$ increases each time. It represents a **constant rate of change**.

10. Graph the data and compare the graphs.

   a. List the ratio pairs from each ratio table in Question 9 as ordered pairs.

   b. Graph the ordered pairs on the grids in the My Notes space.

   c. What do you notice about the graph of the data in each ratio table?
11. Ms. Yang’s class decides to make the juice using the concentrate and water from Option 2.

a. Explain how to determine the number of cups of juice one batch of this mixture will make.

b. To give each of the 26 people attending Math Night 2 cups of juice, how many cups do they need to make? Explain how you determined your answer.

12. To make more juice, the students double their recipe.

a. Express the ratio of concentrate to water when they revise the recipe.

b. How many cups of juice will that make? Explain why this is or is not enough juice for the party.

13. The students need to increase their mixture to make enough juice to serve two cups each to the 26 people attending.

a. Find a ratio equal to 3 cups concentrate per 5 cups water that provides enough juice, with as little extra juice as possible. Explain how you determined your answer.

b. Write a number sentence that shows the original ratio is equal to your new ratio.

The equation you just wrote shows a proportional relationship, and is called a proportion.

14. By writing an equivalent ratio, the students find out how to make more juice while keeping the same relationship between mix and water.

a. How many cups of juice will the students make using the ratio you found in Question 13b?

b. How many extra cups of juice will there be?
Now the students must decide what kind of pizza to order. They decided on pepperoni and extra cheese. They found that 16 people want pepperoni and 10 people want extra cheese.

15. Write a ratio in fraction form that shows the relationship of pepperoni slices to cheese slices.

16. The students found that the average number of slices each person would eat was 2 slices. Write a ratio equivalent to the one you wrote for Question 15 that shows the relationship of pepperoni to cheese slices, assuming each person will eat 2 slices.

17. Use this ratio to determine the total number of pizza slices they need. Show your work.

18. Another way to figure out the total slices needed is to write a ratio comparing pizza slices to people. Write the average number of slices per 1 person as a ratio in fraction form.

What you have just written is a special type of ratio known as a rate. This rate shows a relationship between quantities measured with different units (slices and people). Earlier, when pepperoni slices were compared to cheese slices, you compared different toppings (pepperoni and cheese), which had the same unit (slices). This type of ratio is also a rate.

When the rate is per 1 unit, such as slices per 1 student, it is called a unit rate. Unit rates are easy to spot because they are often written with the word per or with a slash (/) (for example, slices per person or slices/person).

19. Name at least 2 other situations where you have noticed a relationship expressed with the word per.
20. Use the unit rate to find the total number of slices needed. Set up a proportion. Fill in the values you know.

<table>
<thead>
<tr>
<th>Unit Rate</th>
<th>Rate for Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices/Person</td>
<td>Total Slices/Total People</td>
</tr>
<tr>
<td>( \frac{\text{# slices}}{\text{# person}} = \frac{? \text{ slices}}{\text{# people}} )</td>
<td></td>
</tr>
</tbody>
</table>

21. Finding equivalent rates is just like finding equivalent fractions. Rewrite the proportion and use the Property of One to solve. Think of this as finding an equivalent fraction.

22. If a large pizza is cut into 8 slices, how many pizzas must the students buy?

Two pizza places have free delivery to the school.

**Mama T’s Pizza**
- Get 2 large 1-topping pizzas for just $19.98!
- 1 large 1-topping pizza is $10.99

**Toni’s Pizza**
- 1 large 1-topping pizza for $10.99
- 3 large 1-topping pizzas: only $29.70

23. Is $19.98 for 2 pizzas at Mama T’s Pizza a good deal? To determine this, you must find the price per pizza. Find the price per pizza by finding an equivalent fraction. This time you will divide by 1 in the form of \( \frac{2}{2} \).

\[
\frac{\$19.98}{2 \text{ pizzas}} = \frac{\? \text{ \$}}{1 \text{ pizza}}
\]

24. Notice, by setting up the ratio \( \frac{\$19.98}{2 \text{ pizzas}} \), you treat it as a fraction and divide 19.98 by 2. You also divide the unit dollars (or price) by the unit pizzas to get “_______/_______” or “_____ per _______”.
25. Another way to find the cost of each pizza is to reason this way: “If $19.98 is the cost of 2 pizzas, then how many dollars does 1 pizza cost?”

\[
\frac{19.98}{2 \text{ pizzas}} \rightarrow \frac{\text{dollars}}{1 \text{ pizza}}
\]

For this way of reasoning, you think about how much for one. Think of this as finding the unit rate. When a problem involves working with money, the unit rate is called the unit price.

26. How much do the students save by using this deal instead of buying 2 pizzas at regular price?

27. What is the price per pizza for the deal at Toni’s Pizza?

28. To decide where they will get the better deal, the students cannot simply compare rates. They need a specific number of pizzas, so the better deal may depend on how many pizzas they are buying.

   a. Determine how much it would cost to buy 7 pizzas from Mama T’s Pizza. The students can use the deal for every 2 pizzas they buy, but the seventh pizza will be at regular price. Show how to use a proportion to help determine your answer.

   b. Determine how much it would cost to buy 7 pizzas from Toni’s Pizza. Show your work.

   c. Where should the students buy their pizza? Explain.
Now that the juice and pizza are figured out, the students must purchase paper plates. They are not concerned about buying a specific number of each this time, because they will use the extra plates in the future. They are searching for the best deals.

29. The table shows rates for the cost of paper plates at three different stores. Each store has two options.

<table>
<thead>
<tr>
<th>Paper Supplies</th>
<th>Party!</th>
<th>Local Grocer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.99/100 plates</td>
<td>$3.39/15 plates</td>
<td>$3.39/15 plates</td>
</tr>
<tr>
<td>$6.90/100 plates</td>
<td>$3.39/25 plates</td>
<td>$6.39/24 plates</td>
</tr>
</tbody>
</table>

a. Decide which is the better rate at Paper Supplies. Explain your thinking.

b. Decide which is the better rate at Party! Explain your thinking.

c. Can you easily figure out the better rate at Local Grocer? Why or why not?

30. Now consider only the rates at Local Grocer.
   a. Write each rate at Local Grocer in fractional form.

   b. Can you multiply 15 plates by something to get 24, or $3.39 by something to get $6.39? How does this affect your ability to compare these two rates as you did for the Paper Supplies and Party! stores?
c. When it is not easy to find an equivalent fraction to compare quantities, find the unit rate for each deal to find the unit price (price per plate).

\[
\frac{\$3.39}{15 \text{ plates}} = \frac{\text{____}}{1 \text{ plate}}
\]

\[
\text{____ per plate or $\text{____}$/plate}
\]

d. Use this unit price (price per plate) to find the cost for 24 plates. Is that more or less than the other rate of $6.39/24 plates?

\[
\frac{\$0.23}{1 \text{ plate}} = \frac{\text{____}}{24 \text{ plate}}
\]

31. Another way to find the cost of a pack of 24 plates that has the same rate as $3.39/15 plates is to write a proportion. Let \(c\) represent the unknown cost of the 24 plates.

\[
\frac{\$3.39}{15 \text{ plates}} = \frac{c}{24 \text{ plates}}
\]

To figure out a rule you can use to solve for \(c\), think about some procedures you already know. Finish solving the equations for \(c\), but do not simplify the terms. Explain what steps you use.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solve for (c). Do not simplify.</th>
<th>What are the steps?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (3 = \frac{c}{5})</td>
<td>(5 \times 3 = c)</td>
<td>(5 \times 3 = c)</td>
</tr>
<tr>
<td>b. (\frac{c}{7} = 4)</td>
<td>(\text{____})</td>
<td>(\text{____})</td>
</tr>
<tr>
<td>c. (\frac{3}{5} = \frac{c}{9})</td>
<td>(\frac{3}{5} \times 9 = c) or (\frac{3 \times 9}{5} = c)</td>
<td>(\frac{3}{5} \times 9 = c) or (\frac{3 \times 9}{5} = c)</td>
</tr>
<tr>
<td>d. (\frac{c}{4} = \frac{2}{7})</td>
<td>(\text{____})</td>
<td>(\text{____})</td>
</tr>
</tbody>
</table>

e. What are the similarities in Parts a–d?
You can also use an **algorithm** to solve a proportion. Look at the following worked-out proportions.

**Proportion 1:** \( \frac{2}{5} = \frac{a}{7} \)  
Steps:  
(1) \( (7 \times 5) \frac{2}{5} = (7 \times 5) \frac{a}{7} \) Multiply both sides by \( 7 \times 5 \).  
(2) \( 7 \times 2 = 5 \times a \) Simplify.  
(3) \( \frac{7 \times 2}{5} = \frac{5 \times a}{5} \) Divide both sides by \( 5 \).  
(4) \( \frac{7 \times 2}{5} = a \) Simplify.

**Proportion 2:** \( \frac{3}{11} = \frac{a}{2} \)  
Steps:  
(1) \( 3 \times 2 = 11 \times a \) Multiply both sides of the equation by \( 11 \times 2 \).  
(2) \( \frac{3 \times 2}{11} = a \) Divide both sides of the equation by \( 11 \).

**Proportion 3:** \( \frac{a}{9} = \frac{5}{8} \)  
Steps:  
(1) \( a \times 8 = 9 \times 5 \) Cross-multiply the fractions.  
(2) \( a = \frac{9 \times 5}{8} \) Divide to isolate the variable.

32. What patterns do you see in the solutions to these proportions?

The process shown in solving these proportions is the cross products algorithm for solving proportions.

33. In question 31, you wanted to solve \( \frac{\$3.39}{15 \text{ plates}} = \frac{c}{24 \text{ plates}} \).  
   a. Use the cross products algorithm to find the cost of 24 plates.  
   b. How does this compare to the answer when you used the unit price in Question 30d?
34. The second rate at Local Grocer is $6.39/24 plates.
   
   a. Find the unit price for the rate of $6.39/24 plates.

   b. Which is the better rate at Local Grocer? Explain.

35. The students decide to buy the packages of plates priced $3.39/15 plates. They set up the proportion \( \frac{3.39}{15} = \frac{c}{30} \).

   a. What do the numbers and the variable in the proportion represent?

   b. Solve the proportion for \( c \).

36. The students want to know what a pack of 26 plates would cost if the unit price were the same as 15 plates for $3.39.

   a. Use the unit price to find the cost of a pack of 26 plates at that rate.

   b. Write a proportion to find the cost of a pack of 26 plates at that rate. Let \( c \) represent the cost.

   c. Use the algorithm to determine the total cost.

   d. Did you get the same answers for Parts a and c? Explain.
37. The students figure the party is going to cost $77.71. If all 26 people are going to share the cost equally, how much do they each need to contribute?

Now that you know about rates and unit rates, use what you have learned and your results from the first page of activities to find your personal rates for each of the tasks!

38. For each activity, use your data to set up a proportion. Next, use one of the three methods you have learned to solve the proportion. Then enter your rate for that activity. As you do this, use each method at least once.

Activity 1: Writing Speed

a. How many times can you write *math* in 1 minute with your dominant hand?
   - Set up a proportion:
   - Solve it:
   - Express your result using a complete sentence.

b. What is your rate per minute with your non-dominant hand?

c. About how many times faster are you with your dominant hand?

Activity 2: Reading Speed

If there are about 250 words on a page, how many pages could you read in an hour based on the data you collected earlier?
Activity 3: Heart Rate
Heart rate is calculated in beats per minute. Express your heart rate in beats per minute.

Activity 4: Jumping Jacks
How many jumping jacks can you do in 90 seconds?

Activity 5: Are You Tongue Twisted?
About how many times can you say, “Peter Piper picked a peck of pickled peppers” in a minute?

Activity 6: Are You a Fast Walker?
How many feet can you walk in an hour? How many miles per hour can you walk? (There are 5,280 ft in a mile.)

Summary: How Do You Solve Proportions?
Look back at the ways you used to find the rates for the six activities. Find the method you used most often and explain why you chose this method.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

<table>
<thead>
<tr>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

2. Write a ratio for each situation.
   a. 310 heartbeats per 5 minutes
   b. $68 for 8 hours of work
   c. Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

<table>
<thead>
<tr>
<th>Brown</th>
<th>Yellow</th>
<th>Red</th>
<th>Blue</th>
<th>Orange</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>14</td>
<td>13</td>
<td>24</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

   a. The numbers for two colors form a ratio that is equal to \( \frac{7}{12} \). What are the colors? What is their ratio?
   b. If there were 500 pieces, about how many would be red?

4. Kate made lemonade using a powder mix. She used 4 scoops of mix to a gallon of water.
   a. Write a proportion to determine the amount of water to mix with 12 scoops of mix.
   b. If Kate mixes less lemonade powder with more water, how will her mixture be affected?

5. Jaden travels 520 miles in 8 hours.
   a. Use a proportion to find his average rate per hour.
   b. Show why the formula \( d = rt \) is actually a rate problem.

6. It is about 2508 miles from Orange County in California to Orange County in Florida. With an average speed of 70 miles per hour, about how long will it take to drive from one to the other?

7. Which is the better cell phone deal if you consider only cost per minute: 450 minutes for $69.99 or 900 minutes for $89.99? Show how you know.

8. **MATHEMATICAL REFLECTION** Why does the cross-products method work when solving a proportion? Use an example to explain your reasoning.
People use ratios and proportions to solve all kinds of problems in the real world. For example, ratios and proportions are used in cooking to double recipes, by travelers to find distances on maps, and by architects when making scale models.

Some people have even investigated the relationship between numbers of stacked pennies and their height in millimeters.

1. Without using pennies or rulers, predict the height of a stack of 150 pennies, and explain why you made this prediction.
   a. Make your prediction below. Be sure to include units.
      
      A stack of 150 pennies is ____________ tall.
   b. What reasons did you have for making your prediction? Clearly explain your thinking.

2. It has been found that on average the ratio of the number of pennies in a stack to the height of the stack in millimeters is 5:7. In your own words, explain what this means.

3. Explore this finding by measuring and recording the height of a stack of each number of pennies.

<table>
<thead>
<tr>
<th>Number of Pennies</th>
<th>10</th>
<th>15</th>
<th>25</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Stack</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(in mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Write a ratio in fraction form that relates the number of pennies to the height of a stack that has:
   a. 10 pennies  b. 15 pennies  c. 25 pennies  d. 35 pennies
5. What is true about the ratios in Question 4? Does this confirm or disprove the original finding? Explain.

6. This time use the ratio 5:7 and proportional reasoning to complete the table below.

<table>
<thead>
<tr>
<th>Number of Pennies</th>
<th>10</th>
<th>15</th>
<th>25</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Stack (in mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. Write two ratios in fraction form relating the number of pennies to the height of the stacks.
   b. Write these ratios as an equation.
   c. What is your equation called? Explain.

8. How could you find the height of a stack of 60 pennies without measuring? Find a reasonable estimate and explain your method.

9. How could you find the height of a stack of 372 pennies? Find a reasonable estimate and explain your method.

11. Why might the value you determined for height in Questions 8 and 9 be different than the actual measured height of a stack of 60 pennies or 372 pennies?

12. Write and solve a proportion to determine the number of pennies, \( p \), in a stack that is 100 mm high. Use numbers, words, or both to explain your method.

13. Graph the data from Question 6 on the grid in My Notes.

14. Use the graph to predict the height of a stack of 20 pennies. Explain.

15. Use the graph to predict the number of pennies in a stack that is 60 mm high. Explain.

16. How accurate are your predictions in Question 14 and Question 15? Why?
17. Look back at your original prediction for the height of a stack of 150 pennies.

a. Use a proportion to make a better estimate than your original prediction. Explain how you determined your estimate.

b. Is your new estimate an exact height for the stack of 150 pennies? Why or why not?

c. How close was your original prediction at the beginning of the activity to your new estimate?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. The commonly accepted ratio of a human’s hand span to total body height is 1 to 8.5.
   a. Use a proportion to predict Tia’s height. She has a hand span of 7 inches.
   b. When actually measuring, Tina found her height to be 60 inches. How close is the actual ratio to the commonly accepted ratio? Does using a commonly accepted ratio give a reasonable estimate of height?
   c. Why are the values found in Parts a and b not the same?

2. Solve the proportion \( \frac{4}{5} = \frac{28}{x} \) using two different strategies. Explain each.

3. Complete the ratio table to show ratios equivalent to 9:33.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>330</td>
<td>132</td>
</tr>
</tbody>
</table>

4. Use the following graph to make predictions.

\[ y \]
\[ x \]

\[ 300 \]
\[ 250 \]
\[ 200 \]
\[ 150 \]
\[ 100 \]
\[ 50 \]

\[ 1 \]
\[ 2 \]
\[ 3 \]
\[ 4 \]
\[ 5 \]
\[ 6 \]
\[ 7 \]

\[ \text{Number of Miles} \]
\[ \text{Hours} \]

a. Use the graph to predict the number of miles driven in 5 hours. Explain.

b. Use the graph to predict the number of hours it would take to drive 300 miles. Explain.

5. Is the ratio 35 to 10 proportional to the ratio 7 to 5? Explain.

6. **Mathematical Reflection** Why are proportions useful? Give an example.
Ratios and Rates

A SUMMER JOB

Write your answers on notebook paper. Show your work.

Wendy has a summer job. She will paint houses Monday through Friday, 8 hours each day. She is surprised by the many decisions she has to make as part of this job.

- She has to choose from two different pay options.
- She must decide how much paint to purchase.
- She must mix paints to get the right colors.

Help her make these important decisions.

1. The two different pay options she may choose from are: either $62 per day or $304 per week. Which is the better deal for Wendy? Use unit rates to explain your decision.

2. Using the option you chose in Question 1, determine how much money Wendy will earn by working 4 weeks.

3. To get the right color to paint the house, Wendy must mix 1 gallon of green paint with 3 gallons of white paint.
   a. Write a ratio in 3 different ways to show the relationship between green paint and white paint.
   b. How many gallons of paint will her mixture make?

4. Wendy is told ahead of time that she will need to purchase about 12 gallons of paint in order to cover the entire house. Write a proportion to determine the amount of green and white paint she will need to purchase.

5. How many gallons of green paint would be needed if Wendy had 10 gallons of white paint? Explain your reasoning.

6. How many gallons of white paint would Wendy need to mix with 0.5 gallon of green paint? Explain your answer.

7. Suppose that Wendy accidentally mixed 2 gallons of green paint with 3 gallons of white paint.
   a. How would her color change? Would it be darker or lighter? Explain.
   b. Without starting over, how could she fix her mistake to get the right color to paint the house?
## Ratios and Rates
### A SUMMER JOB

**Math Knowledge #2–6**
- **Exemplary:** Student accurately calculates pay (2) and finds the amounts of paint (3–6).
- **Proficient:** Student attempts at least 4 calculations and accurately determines at least 3 of these calculations.
- **Emerging:** Student attempts at least 3 calculations and accurately determines 2 of these calculations.

**Problem Solving #1, 7a, 7b**
- **Exemplary:** Student correctly selects the better deal for pay using unit rates (1), determines how the paint color will change (7a), and finds a way to correct the mistake (7b).
- **Proficient:** Student correctly solves two of the three problems.
- **Emerging:** Student correctly solves at least one of the three problems.

**Representation #3a, 4**
- **Exemplary:** Student writes three correct ratios (3a) and gives a correct proportion to determine paint quantities (4).
- **Proficient:** Student writes at least 2 correct ratios and sets up a proportion.
- **Emerging:** Student provides at least one correct ratio and sets up a proportion.

**Communication #1, 5, 6, 7a**
- **Exemplary:** Student clearly communicates the better pay option (1), gives clear explanations of the methods used to find the amount of green paint (5) and white paint needed (6), and gives clear reasoning for whether the paint would be darker or lighter (7a).
- **Proficient:** Student clearly communicates logical reasoning for three of the items.
- **Emerging:** Student clearly communicates logical reasoning for two of the items.
Understanding Percent
Coloring Creations

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share, Quickwrite, Vocabulary Organizer

1. Color the grid. The table at the right tells how many squares to fill with each color. Make any design you want.

2. For each color use a colon to write a ratio of the number of squares of that color to the total number of squares. Then write each ratio in fraction, decimal, and word form.

<table>
<thead>
<tr>
<th>Color</th>
<th>Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>40</td>
</tr>
<tr>
<td>Orange</td>
<td>8</td>
</tr>
<tr>
<td>Yellow</td>
<td>13</td>
</tr>
<tr>
<td>Green</td>
<td>17</td>
</tr>
<tr>
<td>Blue</td>
<td>22</td>
</tr>
</tbody>
</table>

3. Another way to represent a part to whole relationship is by using another type of ratio called a percent. A percent is a ratio that is always a number out of 100.

   a. Consider the parts of the word percent. Why do you think a number out of 100 is called a percent?

   b. The symbol % is used to represent the term percent. Make a connection between the symbol and its meaning.

ACADEMIC VOCABULARY

Percent means parts per hundred. It can be expressed as a fraction, such as $\frac{87}{100}$ or with a percent sign, 87%.
4. Use the grid in Question 1 to answer the following questions.
   a. How many squares out of 100 are red? _________ out of 100
   b. Replace “out of 100” with the word percent: ______________
   c. Replace “percent” with its symbol: ______________

5. Explain why the fraction and the decimal representing the ratio for red both also represent a number out of 100.

6. In the table below, write the percent of the grid that is covered by each color.

<table>
<thead>
<tr>
<th>Percent of Grid</th>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
</table>

7. Use the tables from Questions 2 and 6 to answer each question.
   a. What is the sum of the percents?
   b. What is the sum of the fractions?
   c. What is the sum of the decimals?
   d. What relationships do you find among your answers to Parts a–c?

8. Consider the colors you used in the grid.
   a. List the colors and percents from Question 6 in order from the color most used to the color least used.
   b. What representations other than the percents could you have used to order the colors?
9. What about the grid in Question 1 made it easy to find percents?

10. How many tiles make up the message, \textit{Hi}?\n
11. To find the percent of the tiles in \textit{Hi} that are in the \textit{H}, first find either a fraction or a decimal.
   a. Which is easier to find in this situation, a decimal or a fraction? Explain and write your answer.

   b. Write your answer in hundredths, since percent is a number out of 100. Then convert the hundredths to a percent.

   c. Write this percent as a decimal.

12. Consider the tiles that are in the letter \textit{i}.
   a. What percent of the tiles in \textit{Hi} are in the \textit{i}?

   b. Write this percent as a decimal.
13. Use your answers to Questions 11 and 12 to determine what percent of the tiles in Hi! are in the !. Explain how you found your answer.

14. Write the percent from Question 13 as a decimal and a fraction.

15. To write percents, you need a fraction or a decimal written in hundredths. Convert each fraction, decimal, or ratio to a percent. If not already in hundredths, first convert to hundredths and then write as a percent.

   a. 0.45
   b. \( \frac{34}{100} \)
   c. 0.9
   d. \( \frac{7}{10} \)
   e. \( \frac{11}{25} \)
   f. 15 blue dots: 50 total dots

Use the grid below to answer Questions 16–21.

16. What percent of the grid is shaded? Explain.

17. Color 36% of the grid blue.
   a. What fraction of the grid is now blue?
   b. Write a decimal to represent the blue boxes.
18. Color \( \frac{2}{5} \) of the grid red.
   a. Write a decimal to represent the number of red boxes.
   b. Write this decimal as a percent.

19. Color 0.16 of the grid yellow.
   a. Write this amount as a fraction.
   b. Convert your fraction to a percent.

20. What percent of the grid is now shaded? Write this percent as a decimal and a fraction.

21. Use the grid to order 36%, \( \frac{2}{5} \), and 0.16 from least to greatest.

22. If you did not have a shaded model to look at, you could use a number line to compare percents, fractions, and decimals. Place 36%, \( \frac{2}{5} \), and 0.16 on the number line.

23. Consider your answers to Parts a and b of Question 8.
   a. Why were the amounts so easy to order?
   b. Show how you could order the amounts from Question 21 without using a model or a number line.

24. Consider the figure:  ⬿⬤⬤⬤⬤⬤
   What percent of the figure is shaded? Explain how you determined your answer.

Math Tip
Recall that percent means a number out of a hundred. For example, think about how many hundredths are in the decimal 0.485. This should help you see that 0.485 = 48.5%.
25. Find what percent of each figure is shaded. Write your answer to the nearest tenth of a percent.

a. 
\[
\text{\includegraphics{figure_a.png}}
\]

b. 
\[
\text{\includegraphics{figure_b.png}}
\]

c. 
\[
\text{\includegraphics{figure_c.png}}
\]

26. Compare each amount.

a. \[
\frac{5}{7} \quad \text{71%}
\]

b. \[
0.5625 \quad \text{56.4%}
\]

c. \[
27\% \quad 0.3
\]

d. \[
10\% \quad 0.01
\]

27. Other than creating designs, name at least three real world uses of percent.
28. Percents are commonly used in trivia or fun facts. Convert each percentage in the facts below to decimals and fractions.

- About 50.8% of the U.S. population is female.
- More than 90% of plane crashes have survivors.
- In the U.S., 32.4% of households own a cat.

29. Use the figures below to answer Parts a and b.

![Figure 1](image1)
![Figure 2](image2)
![Figure 3](image3)

a. Estimate what percent of each figure is shaded. Give reasons for your estimates.

b. Why is it difficult to write a percent to represent the shaded amounts of each figure? Explain your reasoning.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write the shaded part of each figure as a fraction, decimal, and percent.
   a. 
   ![Shaded square]
   b. 
   ![Shaded triangle]

2. Write each amount as a percent.
   a. 0.37
   b. \(\frac{21}{100}\)
   c. 79 out of 100

3. Write each percent in the form specified.
   a. 48%; ratio
   b. 1%; decimal
   c. 99%; fraction

4. Abby spelled \(\frac{8}{10}\) of the words correct on her quiz. What percentage was that?

5. Kate shot 25 free throws at basketball practice and scored on 13 of them. What percent of free throws did she make?

6. Put the following amounts in order from greatest to least: 60%, \(\frac{2}{3}\), 0.599. Explain your reasoning.

7. Write each amount as a percent. If necessary, round to the nearest tenth of a percent.
   a. \(\frac{5}{8}\)
   b. 0.2189
   c. \(\frac{6}{7}\)
   d. \(\frac{13}{15}\)

8. Estimate the percentage of each figure that is shaded.

9. **Mathematical Reflection** Explain why a percent is a type of ratio, and how percents relate to decimals and fractions.
The U.S. Department of Agriculture found from much research that two big factors—exercising and eating right—can help keep people healthy. The USDA created the food pyramid as a visual reminder of how to choose healthy types of food to eat.

Sections are bigger at the bottom and narrower at the top to show that even within a food group, some foods should be eaten more often than others.

1. Each food group in the pyramid is labeled with a number written below it. Use Table 1 to predict which food group goes with each number. Use Table 2 to match the amounts of food that go with each group number.

<table>
<thead>
<tr>
<th>Food Group</th>
<th>Number</th>
<th>Amount</th>
<th>Food Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruits</td>
<td></td>
<td>1.5 cups</td>
<td></td>
</tr>
<tr>
<td>Oils</td>
<td></td>
<td>3 cups</td>
<td></td>
</tr>
<tr>
<td>Vegetables</td>
<td></td>
<td>5 ounces</td>
<td></td>
</tr>
<tr>
<td>Milk</td>
<td></td>
<td>5–6 ounces</td>
<td></td>
</tr>
<tr>
<td>Meat &amp; Beans</td>
<td></td>
<td>2–2.5 cups</td>
<td></td>
</tr>
<tr>
<td>Grains</td>
<td></td>
<td>≤ 5 teaspoons</td>
<td></td>
</tr>
</tbody>
</table>
Bob likes to eat fast food but has heard it is not healthy. After hearing about the food pyramid, Bob is curious about how healthy his food is. His teacher tells him to use the food Nutrition Facts labels.

He begins by looking at the label for the cheeseburger he ate for lunch.

Section 1 of the label tells how much food is in one serving, and how many servings are included. The information on the rest of the label is based on one serving.

2. According to this label, what is considered one serving?

Section 2 tells how much of each nutrient is in one serving. The labels also show % Daily Value (%DV). The Daily Value is the amount of each nutrient that health experts recommend you eat in one day. These percents are based on intake of 2,000 calories, the number of calories for an average person each day. The number of calories best for you may be a little higher or lower depending on your age, gender, activity level, and so on.
Percent of a Number
Eating by the Numbers

3. What percent represents the total amount of each nutrient you should consume each day?

4. After eating just one cheeseburger, what percent of total fat is left for Bob for the rest of the day? Explain.

5. If Bob ate two cheeseburgers, what percent of his daily fat would he be consuming? Explain.

According to nutrition experts, Bob should have only 65g of total fat and 20g of saturated fat per day. However, the part of the label showing fat grams is covered with grease so he cannot see how much total fat and saturated fat are in his cheeseburger.

6. First help him determine the amount of saturated fat.
   a. Use the label to complete the expression to show the percent of saturated fat that is in the cheeseburger.
      
      
      % of total grams
   b. Rewrite the expression replacing the percent with a fraction in simplest form.
   c. Divide and shade the model to represent this expression.
      
      
   d. There are grams of saturated fat in the cheeseburger.

7. Now help Bob find the number of grams of total fat in the cheeseburger.
   a. Use the label to complete the expression to show the percent of total fat that is in the cheeseburger.
      
      
      % of total grams
   b. Would using a model be the best way to solve this expression? Why or why not?
8. Another way to find the number of grams of total fat is by using a dual number line. This is a vertical number line with two scales drawn side by side.

\[
\begin{array}{c|c}
\text{Grams} & \text{Percent} \\
0g & 0\% \\
\end{array}
\]

\[a.\] On the top end, write the total percent on the right and the total grams on the left for the whole day.

\[b.\] Next, place a box for the percent of fat in the cheeseburger on the percent side about where that percent would be on the vertical number line.

\[c.\] To find the number of grams of fat for that percent, use the dual number line to write and solve a proportion. Let \(g\) = the number of grams of total fat in the cheeseburger.

9. Use estimation to check your answer.

\[a.\] 47% is about \(\_\_\_\_\_\%\) (choose a percent that is a benchmark number).

\[b.\] Convert the percent to a fraction in simplest terms.

\[c.\] Find this fraction of 65g.

\[d.\] Compare your estimate to your actual answer.

Instead of using proportions, you could apply another strategy you already know.
10. You know that percents can be written as fractions or decimals in expressions such as $\frac{3}{4}$ of 12 and 0.75 of 12.

a. What operation would you use to evaluate these expressions?

b. Which expression is easier for you to evaluate? Explain.

c. How could you apply this same process to finding the percent of a number?

11. Bob remembers he should have only 65g of total fat per day. Use what you have learned about finding percent of a number to find the number of fat grams in each food item Bob ate.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Fat (g)</th>
<th>%DV Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Breakfast</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ham, egg, and cheese bagel</td>
<td></td>
<td>24%</td>
</tr>
<tr>
<td>Hash browns</td>
<td></td>
<td>14%</td>
</tr>
<tr>
<td>Medium orange juice</td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td><strong>Lunch</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grilled chicken sandwich</td>
<td></td>
<td>15%</td>
</tr>
<tr>
<td>Fruit salad</td>
<td></td>
<td>12%</td>
</tr>
<tr>
<td>Medium soda</td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td><strong>Dinner</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheeseburger</td>
<td></td>
<td>47%</td>
</tr>
<tr>
<td>Medium fries</td>
<td></td>
<td>29%</td>
</tr>
<tr>
<td>Medium chocolate milkshake</td>
<td></td>
<td>22%</td>
</tr>
</tbody>
</table>

12. Total the grams of fat that Bob ate. What do you conclude?

13. Total the percent of fat that Bob ate. What do you conclude?
Bob decides to consider total calories and carbohydrates as well as fats, in order to see the bigger picture.

14. Look at the cheeseburger label from Question 2. Bob is trying to eat the standard 2000 calories a day.
   a. Write a ratio to compare the number of calories in the cheeseburger to the standard total number of calories.
   b. Convert this ratio to a percent.

15. Find the %DV of each fast food that Bob ate today.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Calories</th>
<th>%DV Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ham, egg, and cheese bagel</td>
<td>430</td>
<td></td>
</tr>
<tr>
<td>Hash browns</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Medium orange juice</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>Grilled chicken sandwich</td>
<td>420</td>
<td></td>
</tr>
<tr>
<td>Fruit salad</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Medium soda</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>560</td>
<td></td>
</tr>
<tr>
<td>Medium fries</td>
<td>380</td>
<td></td>
</tr>
<tr>
<td>Medium chocolate milkshake</td>
<td>580</td>
<td></td>
</tr>
</tbody>
</table>

16. Total the number of calories that Bob ate. What do you conclude?

17. What is the total percent of recommended calories that Bob ate today? What can you conclude?
When Bob starts to look at carbohydrates, he does not know the total number of grams recommended per day. However, he does have this table.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Carbs (g)</th>
<th>%DV Carbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakfast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ham, egg, and cheese bagel</td>
<td>35</td>
<td>12%</td>
</tr>
<tr>
<td>Hash browns</td>
<td>15</td>
<td>5%</td>
</tr>
<tr>
<td>Medium orange juice</td>
<td>42</td>
<td>14%</td>
</tr>
<tr>
<td>Lunch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grilled chicken sandwich</td>
<td>51</td>
<td>17%</td>
</tr>
<tr>
<td>Fruit salad</td>
<td>31</td>
<td>10%</td>
</tr>
<tr>
<td>Medium soda</td>
<td>58</td>
<td>19%</td>
</tr>
<tr>
<td>Dinner</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>47</td>
<td>16%</td>
</tr>
<tr>
<td>Medium fries</td>
<td>48</td>
<td>16%</td>
</tr>
<tr>
<td>Medium chocolate milkshake</td>
<td>102</td>
<td>34%</td>
</tr>
</tbody>
</table>

18. Bob decides to use the medium chocolate milkshake to find the total carbohydrates recommended per day. Work through his method.

   a. Use the dual number line/proportion method. Work in the My Notes space.

   b. Now write an algebraic equation showing percent of a number. Let $g =$ total grams of carbohydrates.

   c. Convert the percent to a decimal or fraction and then solve for $g$.

   d. Which method do you like better and why?

19. What answer should you get for the recommended daily amount using any of the food groups? Explain.
20. Work with the information given for each food item to compute a value for the recommended daily total grams of carbohydrates. Use the dual number line method or an algebraic equation. Round to the nearest whole number.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Carbs (g)</th>
<th>%DV Carbs</th>
<th>Total Carbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakfast</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ham, egg, and cheese bagel</td>
<td>35</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td>Hash browns</td>
<td>15</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Medium orange juice</td>
<td>42</td>
<td>14%</td>
<td></td>
</tr>
<tr>
<td>Lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chicken sandwich</td>
<td>51</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td>Fruit salad</td>
<td>31</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Medium soda</td>
<td>58</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>Dinner</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>47</td>
<td>16%</td>
<td></td>
</tr>
<tr>
<td>Medium fries</td>
<td>48</td>
<td>16%</td>
<td></td>
</tr>
<tr>
<td>Med. choc. milkshake</td>
<td>102</td>
<td>34%</td>
<td></td>
</tr>
</tbody>
</table>

21. Compare your answers. What did you find? Use your answer to Question 19 and the grams of carbohydrates in the food items to explain what happened.

22. Total the grams of carbohydrates. What do you conclude?

23. Find the total percent of carbohydrates. What do you conclude?
24. Based on Bob’s study of fats, calories and carbohydrates, is eating fast food all day a healthy nutritional choice? Explain.

Bob decides to change his eating habits and start making healthier meals at home. Help him track his nutrition the following day to see if there is a positive difference.

For lunch, Bob made soup and a salad.

25. Use the Nutrition Facts label to determine the percent daily values for each nutrient.

**Nutrition Facts**
Serving Size 1 Salad

<table>
<thead>
<tr>
<th>Amount Per Serving</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories: 90</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% Daily Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Fat 3.9g</td>
</tr>
<tr>
<td>Saturated Fat 2.4g</td>
</tr>
<tr>
<td>Cholesterol 12mg</td>
</tr>
<tr>
<td>Total Carbohydrate 9g</td>
</tr>
<tr>
<td>Dietary Fiber 3.25g</td>
</tr>
</tbody>
</table>

* Percent Daily Values (DV) are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

<table>
<thead>
<tr>
<th>Calories: 2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Fat</td>
</tr>
<tr>
<td>Saturated Fat</td>
</tr>
<tr>
<td>Cholesterol</td>
</tr>
<tr>
<td>Sodium</td>
</tr>
<tr>
<td>Total Carbohydrate</td>
</tr>
<tr>
<td>Dietary Fiber</td>
</tr>
</tbody>
</table>

a. From earlier work with Nutrition Facts labels, Bob knows the %DV for *total fats, saturated fats,* and *total carbohydrates.* List these and use the data at the top of the label to confirm them.

b. Find the %DV for the remaining nutrients.

c. Compare your % Daily Values to the ones shown at the bottom of a real label, based on a 2,000 calorie diet.
26. Now that you know the %DV for each nutrient, calculate the number of grams/milligrams of each nutrient listed on this soup label.

<table>
<thead>
<tr>
<th>Nutrition Facts</th>
<th>Serving Size 1/2 c soup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serving Per Container about 2.5</td>
<td></td>
</tr>
<tr>
<td>Amount Per Serving</td>
<td></td>
</tr>
<tr>
<td>Calories: 60</td>
<td></td>
</tr>
<tr>
<td>% Daily Value</td>
<td></td>
</tr>
<tr>
<td>Total Fat g</td>
<td>10%</td>
</tr>
<tr>
<td>Saturated Fat g</td>
<td>15%</td>
</tr>
<tr>
<td>Cholesterol mg</td>
<td>1%</td>
</tr>
<tr>
<td>Sodium mg</td>
<td>40%</td>
</tr>
<tr>
<td>Total Carbohydrate g</td>
<td>10%</td>
</tr>
<tr>
<td>Dietary Fiber g</td>
<td>5%</td>
</tr>
<tr>
<td>Sugars g</td>
<td></td>
</tr>
<tr>
<td>Protein g</td>
<td></td>
</tr>
</tbody>
</table>

Bob notices that all of the percents listed are benchmark percents. He decides to look for patterns to see if there is a quicker way to find these percents by using mental math.

27. First, he considers total fat.

a. To find 10% of 65g of total fat, first write an expression showing percent as a fraction.

b. Instead of multiplying by this fraction, you could divide by ________.

c. How can you use your knowledge of decimal place value to divide by this number using mental math?

28. Use what you have learned about finding 10% of a number to explain how you could use mental math to find 40% of a number, or 5% of a number.
29. Use mental math to find the number of grams/milligrams of each nutrient listed on the label. Do you get the same answers as you did in Question 26? Without doing calculations on paper, explain your thinking.

30. When calculating %DV, it is important to pay special attention to the serving size.

   a. How many total cups of soup are in the can? Explain.

   b. Bob actually consumes two servings. How many cups of soup does he eat?

   c. What %DV of saturated fat is this? Of sodium?

31. Notice that by eating 1 serving of soup, Bob is consuming 10% of his total fat and 10% of his total carbohydrates. Is he eating the same amount of each nutrient? Explain.
32. Help Bob complete his own label for the turkey sub he made for dinner.

Nutrition Facts
Serving Size 1 sub
Servings Per Container 1

<table>
<thead>
<tr>
<th>Amount Per Serving</th>
<th>Calories: 254</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Daily Value</td>
<td></td>
</tr>
<tr>
<td>Total Fat</td>
<td>g</td>
</tr>
<tr>
<td>Saturated Fat</td>
<td>g</td>
</tr>
<tr>
<td>Cholesterol</td>
<td>mg</td>
</tr>
<tr>
<td>Sodium</td>
<td>mg</td>
</tr>
<tr>
<td>Total Carbohydrate</td>
<td>g</td>
</tr>
<tr>
<td>Dietary Fiber</td>
<td>g</td>
</tr>
<tr>
<td>Sugars</td>
<td>g</td>
</tr>
<tr>
<td>Protein</td>
<td>16g</td>
</tr>
</tbody>
</table>

33. Based on a 2000 calorie diet, what percent of his daily calories is this?

34. This time the percentages are not benchmarks. Help him use estimation to determine the reasonableness of his answers by comparing given amounts to benchmarks. Explain your thinking for each.

Total Fat:

Saturated Fat:

Cholesterol:

Sodium:

Total Carbohydrate:

Dietary Fiber:
35. So far, what percent of his total fat and carbohydrates has Bob eaten this day?

Look at the table showing Bob’s food consumption for the whole day.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>%DV Fat</th>
<th># Cal</th>
<th>%DV Carb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakfast</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bagel w/ cream cheese</td>
<td>11</td>
<td>340</td>
<td>22</td>
</tr>
<tr>
<td>banana</td>
<td>1</td>
<td>200</td>
<td>17</td>
</tr>
<tr>
<td>2c orange juice</td>
<td>0</td>
<td>180</td>
<td>14</td>
</tr>
<tr>
<td>Lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bean soup</td>
<td>10</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>salad</td>
<td>6</td>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>16 oz water</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dinner</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>turkey sub</td>
<td>6</td>
<td>254</td>
<td>13</td>
</tr>
<tr>
<td>apple</td>
<td>0</td>
<td>130</td>
<td>11</td>
</tr>
<tr>
<td>2c milk</td>
<td>16</td>
<td>240</td>
<td>8</td>
</tr>
</tbody>
</table>

36. Analyze his nutrient intake this day to determine if cooking these foods at home is healthier than eating fast food all day as he did yesterday.

37. Considering the food groups listed in Question 1, did Bob eat a variety of foods? Explain.
Eating by the Numbers

Write your answers on notebook paper. Show your work.

1. Draw a group of 15 identical circles, boxes, or other simple figures. Shade 60% of the figures. Explain how you know that you shaded 60%.

2. If you took a test with 60 questions and scored 85%, how many questions did you answer correctly? Show how you know.

3. In a survey of 398 students, 52% said their favorite sport is baseball. Use estimation to explain about how many students chose baseball as their favorite sport.

4. 14,389 people run a race to raise money. 7005 of them are boys. What percent of the runners are boys? What percent are girls? Explain.

5. 71% of the times Emily batted she got a hit. She had a total of 28 hits. How many times did she bat?

6. Use mental math to find each missing amount.
   a. 10% of 70
   b. 5% of 100
   c. 30% of 5,000
   d. 50% of ____ = 15

7. On the first day of winter in Juneau, Alaska, there is about 6 hours and 21 minutes of sunlight. About what percent of the day is sunlight?

8. **MATHEMATICAL REFLECTION** Explain why 30% of two different quantities is not the same amount. Use an example in your explanation.
Kim’s older brother, Lee, had a summer job and earned $6.25 an hour while working 8 hours a day, 5 days a week, for a total of 8 weeks.

1. How much money did Lee earn? Show your work.

2. Lee’s parents said, “If you put 85% of your earnings in the bank as savings, then you can spend the rest the way you want.”

   a. How much money does Lee have to put into savings?

   b. How much money does Lee have left to spend?

Lee and Kim decide to go shopping together. Lee has his hard-earned money and Kim has saved some money for back-to-school shopping. It is around lunchtime, so they decide to have lunch first. They both have $20 gift cards for a restaurant, The Lunch Spot, and they do not want to spend more than their cards will cover.

Look at the menu below.
3. Kim orders a grilled chicken sandwich, pasta salad, and lemonade.

   a. Estimate the cost of Kim’s meal.

   b. Find the actual cost and complete Part 1 of the receipt below.

   c. How close was your estimate?

   The food total gives an idea of what the bill will be. It is not the exact amount because the tax and tip have not been included.

   After the food is totaled, the restaurant adds sales tax, and a customer usually then leaves a tip.
The tax is calculated on items purchased and must be exact.

4. To find the cost of sales tax, take that percent of your food total.
   a. According to the menu, what is the sales tax at The Lunch Spot?
   b. Using this percent what is the tax on Kim's food total? Round the amount to the nearest hundredth.
   c. Add this amount to the food total to find the subtotal.
   d. Complete Part 2 of the receipt on the previous page.

Customers usually leave a tip. The tip is a percent of the bill left as a thank-you for the service provided. The customer decides what percent to leave based on personal satisfaction with the service.

5. Kim is happy with the service and decides to leave a 20% tip. Tips do not have to be exact, and are most commonly determined using estimation.
   a. Use estimation to find 20% of the subtotal. Explain.
   b. Check your answer to Part a by calculating the tip using paper and pencil. Is your estimate close to the actual amount?
   c. Add the tip amount from Part b to the bill to find Kim's total.
   d. Complete Part 3 of the receipt above.

6. Will Kim have enough money to pay for lunch with only the gift card? If so, how much money will be left on the card? If not, how much money will Kim owe?
7. Use the same menu to order lunch for Lee. Fill in the blank receipt below. Do not order exactly the same foods as Kim did. Order 3 or 4 items, and then leave a 15% tip. Show your work.

![Receipt Image]

8. Use estimation to check the reasonableness of your food total. Show your estimate and explain how you found it.

9. Explain how you could use mental math to estimate the tip amount.

10. Will Lee have enough money to pay for lunch with only the gift card? If so, how much money will be left on the card? If not, how much money will Lee owe?
### SUGGESTED LEARNING STRATEGIES: Think/Pair/Share

11. The amounts below are the subtotals of the bills of people sitting near Lee and Kim in the restaurant. Use mental math to estimate each tip. Explain how you made each estimate.

- **a.** subtotal = $29.87; 20% tip
- **b.** subtotal = $42.00; 15% tip
- **c.** subtotal = $31.46; 10% tip

12. A waitress receives a tip of $5.50 on a bill subtotalling $30.56. What percent tip did her customer leave?

After lunch, Lee and Kim go shopping. Their favorite store is having a big back-to-school sale. Look at the sales flyer below.

<table>
<thead>
<tr>
<th>ELECTRONICS</th>
<th>CLOTHING AND SHOES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Camera</strong></td>
<td><strong>Sweatshirt</strong></td>
</tr>
<tr>
<td>Originally: $127.77</td>
<td>Originally: $35.95</td>
</tr>
<tr>
<td>30% off</td>
<td>Now: $23.37</td>
</tr>
<tr>
<td><strong>Video Game</strong></td>
<td><strong>Sneakers</strong></td>
</tr>
<tr>
<td>Originally: $95.99</td>
<td>Originally: $67.99</td>
</tr>
<tr>
<td>40% off</td>
<td>Now: $47.59</td>
</tr>
<tr>
<td><strong>DVD Player</strong></td>
<td><strong>Shorts</strong></td>
</tr>
<tr>
<td>Originally: $134.15</td>
<td>Originally: $28.65</td>
</tr>
<tr>
<td>35% off</td>
<td>Now: $17.19</td>
</tr>
<tr>
<td><strong>MP3 Player</strong></td>
<td><strong>Jeans</strong></td>
</tr>
<tr>
<td>Originally: $189.25</td>
<td>Originally: $49.99</td>
</tr>
<tr>
<td>25% off</td>
<td>Now: $39.99</td>
</tr>
<tr>
<td><strong>Cell Phone</strong></td>
<td><strong>T-shirt</strong></td>
</tr>
<tr>
<td>Originally: $119.79</td>
<td>Originally: $22.50</td>
</tr>
<tr>
<td>15% off</td>
<td>Now: $9.00</td>
</tr>
<tr>
<td><strong>Calculator</strong></td>
<td><strong>Wind Pants</strong></td>
</tr>
<tr>
<td>Originally: $149.99</td>
<td>Originally: $55.89</td>
</tr>
<tr>
<td>20% off</td>
<td>Now: $47.51</td>
</tr>
</tbody>
</table>

After lunch, Lee and Kim go shopping. Their favorite store is having a big back-to-school sale. Look at the sales flyer below.
13. Kim wants to purchase a calculator and uses mental math to figure out about how much it will cost.

   a. First, find the **discount**, the amount taken off the original price. To do this, estimate 20% of $149.99. Use numbers, words, or both to show how to do this mentally.

   b. Is this the sales price of the calculator? Explain. If not, show how to estimate the sales price.

14. After estimating, Kim used a price scanner in the store to find the actual sales price.

   a. What will the scanner display? Show your work.

   b. Was your estimate reasonable? Why or why not?

15. As Kim was scanning the calculator, a salesperson was handing out coupons for an additional 10% off the discounted price of electronics items.

   a. What is the new price of the calculator when the coupon is scanned?

   b. Kim whispers to Lee, “Wow, I just do not believe I will get 30% off the original price!” Is Kim correct? Why or why not?
Kim decides also to purchase a sweatshirt and jeans.

16. Help Kim determine the percent discount on the sweatshirt.
   a. Find the amount of money Kim saves. Show your work.
   b. What percent of the total is this amount?
   c. How could you use the percent in Part b, the original price, and the sale price to check your work?

17. Find the percent of the discount on the jeans.

18. Which is the better deal, the sweatshirt or the jeans? Explain.

19. At checkout, the cashier first determines the subtotal for the calculator, sweatshirt, and jeans. Find the subtotal. Show your work.

20. After the discounts have been taken, a sales tax of 6.25% is added to the subtotal.
   a. What is the cost of the calculator, sweatshirt, and jeans before tax?
   b. How much money must Kim pay in sales tax? Show your work.
   c. What is the total cost of the bill? Explain.

CONNECT TO GOVERNMENT
Sales tax varies from state to state, and even from county to county, or town to town in some places. Do you know your sales tax percent?
21. Now help Lee shop. Choose four items from the store sale flyer on page 227 without going over the amount of spending money you found for Lee in Question 2b. Show your work for each part.

   a. List the four items you chose for Lee and estimate the cost.

   b. Find the actual cost before tax.

   c. Calculate the total bill including the sales tax of 6.25%.

   d. How much spending money does Lee have left?

After they finish shopping, Lee heads to the bank to deposit the savings amount found in Question 2a. The bank teller explains that the bank offers simple interest at a rate of 6.1% per year.

Lee asks her what this means and she responds, “Interest is money the bank pays you for letting them use your money. The interest rate tells you how much money the bank will give you. The bank pays you 6.1% of whatever amount you let us hold for a year. So at the end of a year, you can get back the amount you deposited, called the principal, plus the 6.1% the bank pays you.”

22. In your own words, explain what each term means:

   a. Interest

   b. Interest rate

   c. Principal
23. How much interest will Lee get at the end of one year as long as no other money is deposited or withdrawn during the year? Show your work.

24. How much money will Lee have in that bank account at the end of one year?

The teller further explains, “Every year you leave your money in the bank, you receive an additional 6.1% of the original principal. Although you have more money than you originally deposited in your account, after the first year, your interest is still based on your original deposit. This is what is meant by simple interest.”

25. How much money will Lee have in the bank after two years?

26. Complete this table to show how much simple interest the bank will pay Lee after a given number of years.

<table>
<thead>
<tr>
<th>Years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Interest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

27. Lee thinks there must be a formula for finding simple interest.

   a. What operation is used to calculate interest?

   b. Write an equation that represents the amount of simple interest, \( I \), earned after \( t \) years. Let \( r \) = interest rate and \( P = \) principal.

28. If \( t \) represents the number of whole years, show how you would find simple interest after 6 months? After 3 months?

29. What is the least number of whole years Lee must keep the money in the bank to make at least $1,000 in simple interest?
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Jack has $50 to spend on a basketball. The one he wants costs $46.99. If sales tax is 7%, will he have enough money to purchase the basketball? Explain.

2. Blake and his family eat dinner at a restaurant and the bill comes to $81.67. Use estimation to determine how much money they should leave to tip the server 15%. Explain.

3. Calculate the total amount due on the restaurant bill shown below.

4. Bella wants to buy a tennis racket that was originally $74.49, but is on sale for 30% off. Determine the cost of the tennis racket before tax is added.

5. Toby tells his mom about a great deal he found on sneakers. “They were originally $110.60 but now they are on sale for $88.48!” Use percents to determine whether this is a good deal. Explain your answer.

6. Latricia deposits $650 in a bank account that pays simple interest of 4.8%. Abby deposits $600 in a bank account that pays 6.9% simple interest. How much money will they each have after 5 years if they do not deposit or withdraw any money?

7. You buy a shirt for $22.97 and a pair of jeans for $58.80. Everything in the store is 25% off. Plus, you have a coupon for an additional 10% off the total. Find the subtotal before tax. Show your work.

8. **MATHEMATICAL REFLECTION** How are finding tips, discounts, taxes, and simple interest similar and different?
A SPECIAL BIRTHDAY

Write your answers on notebook paper. Show your work.

Aiden’s grandparents put $2,000 in a bank savings account when he was born. The bank account earns simple interest at a rate of 4.8% a year. They told Aiden that he could have the money on his sixteenth birthday. He plans to use the money to buy a used car.

1. How much money will be in the account when he turns sixteen?

2. Aiden found ads for used cars on the internet. Look at three ads that he found. The ads are from dealers in different states, so the sales tax rates are different.

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4015 15% off</td>
<td>Originally $4350 Now $3915</td>
<td>18% off You Save $750!</td>
</tr>
<tr>
<td>Sales Tax: 5.6%</td>
<td>Sales Tax: 7.25%</td>
<td>Sales Tax: 4.5%</td>
</tr>
</tbody>
</table>

a. Which offer has the best discount? Explain.

b. Calculate the total cost of each car including sales tax.

On his birthday, Aiden’s family and relatives go out to dinner to celebrate. The three families decide to get separate checks.

3. Copy and complete each receipt. Show your work.

<table>
<thead>
<tr>
<th>Pasta House Receipt</th>
<th>Pasta House Receipt</th>
<th>Pasta House Receipt</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 12 6:30 PM</td>
<td>March 12 6:30 PM</td>
<td>March 12 6:30 PM</td>
</tr>
<tr>
<td></td>
<td>Food Total</td>
<td>Food Total</td>
</tr>
<tr>
<td></td>
<td>$72.50</td>
<td>$49.87</td>
</tr>
<tr>
<td></td>
<td>Tax (6%)</td>
<td>Tax (6%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
<td>Subtotal</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tip Amount (15%)</td>
<td>Tip Amount (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10.57</td>
</tr>
<tr>
<td></td>
<td>Total Due</td>
<td>Total Due</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thank you! Please come again!</td>
<td>Thank you! Please come again!</td>
<td>Thank you! Please come again!</td>
</tr>
</tbody>
</table>

4. Explain how each family could have used mental math to estimate the tip.
# Percents

## A SPECIAL BIRTHDAY

<table>
<thead>
<tr>
<th>Math Knowledge #1, 2b, 3, 4</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student accurately determines the amount of money in the account (1), correctly calculates the cost of all three cars (2b), correctly completes all three receipts (3), and provides three logical estimates for tips (4). Student makes no more than one minor error in calculations.</td>
<td>Student completely and correctly solves three of the problems. OR The student solves all the problems using correct methods, but the calculations may contain minor errors.</td>
<td>Student completely and correctly solves at least two of the problems.</td>
<td></td>
</tr>
</tbody>
</table>

| Problem Solving #2a | Student accurately determines which of the three offers gives the best discount, providing appropriate supporting calculations (2a). | Student accurately determines two of the three discounts but makes an incorrect conclusion. OR The best discount may be chosen, but the supporting calculations provided are incomplete. | Student is able to determine the best discount without any appropriate supporting calculations. OR Student logically determines the best discount based on incorrect work. |

| Communication #2a, 4 | Student clearly explains his or her reasoning for the discount chosen (2a) and for each of the three mental math strategies used (4). | Student gives explanations for the discount and the three mental math strategies. At most one explanation may be incomplete or unclear. | In providing the explanations for the discount and 3 mental math strategies, at least two of the four must be clear and complete. |
ACTIVITY 4.1

1. In Britain, about 7.5 million people have cats as pets, and 6.1 million have dogs. Write 2 different ratios that could be made from this information. Explain your ratios.

2. On a class field trip, there are 5 teachers, 117 students, and 16 parents. What does the ratio 21:138 represent?

3. Which of the following are equal ratios?
   A. 3:5 and 6:10
   B. 2:4 and 6:8
   C. 3:4.5 and 5:7.5
   D. 2:3 and 4:5

4. Use the graph to write a ratio showing the relationship between number of hours Sam babysits and the money he earns.

   ![Graph](EmbeddedImage)

   - Money Earned Babysitting
   - x: Number of Hours
   - y: Dollars Earned

   ![Graph](EmbeddedImage)

5. For working 11 hours you are paid $93.50. How much money do you make per hour?

6. Jim drove from Exit 32 on the highway to Exit 170 in 2 hours. Exits 32 and 170 are 138 miles apart. Did Jim follow the speed limit of 65 mph? Explain how you know.

7. Which is the better deal, 8 songs for $7.52 or 5 songs for $4.75? Explain how you know.

8. In April 1897, John J. McDermott won the first Boston Marathon, which was only 24.5 miles instead of 26 miles as it is today. He ran it in 175 minutes. Find his unit rate.

ACTIVITY 4.2

9. At Lake Middle School, the average ratio of boys to girls in a classroom is 3:2.
   a. Use a proportion to predict the number of girls in a classroom that has 15 boys.
   b. Hunter actually counted the number of boys and girls in his class at Lake Middle School, and found there were 14 boys and 9 girls. How close is the actual ratio to the ratio found using proportions? Does using proportions give a reasonable estimate of boys and girls?
   c. Why are the values found in Parts a and b not the same?

10. Solve \( \frac{x}{42} = \frac{3}{7} \) using two different strategies. Explain each strategy.

11. Complete the ratio table to show ratios equivalent to 16:10.

<table>
<thead>
<tr>
<th>48</th>
<th>160</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>90</td>
<td></td>
</tr>
</tbody>
</table>

12. Use the following graph to make predictions.

   ![Graph](EmbeddedImage)

   - Chocolate Chips in Pancakes
   - x: Number of Pancakes
   - y: Number of Chocolate Chips

   a. Use the graph to predict the number of chocolate chips in 9 pancakes. Explain.
b. Use the graph to predict the number of pancakes that would have 48 chocolate chips. Explain.

13. Is the ratio 4.2:1.5 proportional to the ratio 12.6:4.5? Explain.

**ACTIVITY 4.3**

14. A group of 100 apples and bananas are shown below. Express the number of bananas in the group using a fraction, decimal, and percent.

15. Copy and complete the table below by filling in missing amounts, shading figures, and plotting on the number line. Write ratios using a colon (:) to represent part to whole relationships.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Ratio</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:4</td>
<td>0.25</td>
<td></td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>7:10</td>
<td>0.3</td>
<td></td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

16. Replace each bold number in the fun facts below with a percent.

a. 0.95 of a jellyfish is water.

b. \( \frac{1}{4} \) of all the bones in your body are in your feet.

c. About 0.183 of people let their pets sleep in their bed.

d. About \( \frac{8}{21} \) of America is wilderness.

e. Pizzerias make up about \( \frac{8}{47} \) of all restaurants.

17. Put the following amounts in order from greatest to least: 43%, \( \frac{3}{7} \), 0.453. Explain your reasoning.

18. Estimate the percent of the smiley face that is NOT shaded.

**ACTIVITY 4.4**

19. Draw 24 identical simple figures. Shade 75% of the figures. Explain how you knew how many to shade.

20. Jack played a video game 18 times and won about 62% of the games. How many games did he win?

21. 22% of the 55 boys in the club wanted to play kickball. Use estimation to tell about how many boys wanted to play kickball.

22. 12 of Derek’s 15 pets are fish. What percent of his pets are fish?

23. The news reporter stated that, “About 8% of Americans have diabetes, which is about 24 million people.” At the time this statistic was on the news, how many people were there in the U.S.?
24. Use mental math to find each missing amount.
   a. 10% of 1,843
   b. 40% of 120
   c. 15% of 200
   d. 10% of _____ = 17

25. 29% of the 61 shells that Tia found were broken. About how many of the shells were broken?

26. Kay and Lisa spent a day shopping and going to a museum with their mom. At lunch, Kay offered to pay 50% of the lunch bill, which was $28.98, and their mom paid the rest. At dinner, Lisa paid for 50% of the bill, which was $59.50, and their mom paid the rest. Did the girls each pay the same amount? Explain.

ACTIVITY 4.5

27. A picture frame has a price tag of $23.77. At checkout, the total cost of the frame comes to $24.96. What percent sales tax was added? Show how you found your answer.

28. A server receives a tip of $13 on a bill subtotaling $56.90. Was her customer happy with her service? Use percents to explain your thinking.

29. Calculate the total amount due on the restaurant bill shown below.

   RECEIPT
   January 22 7:15PM
   Pasta with Meatballs $11.35
   Milk $1.70
   Clam Chowder (bowl) $5.15
   Food Total
   Tax (5.9%)  
   Subtotal 
   Tip Amount (15%) 
   Total Due

   Thank you!
   CUSTOMER COPY

30. Weston has a gift card for $15 to download songs with his computer. Each song costs $0.99 and he wants to buy 16 songs. He has a coupon for 10% off the total and the tax will be 6%. Will the gift card cover the cost of the 16 songs? Explain.

31. Leah borrowed $3,700 at a simple interest rate of 6.9% to buy a used car. If the loan is for 2 years, how much money must she repay?

32. Merry deposits $1,200 in a bank account and does not add to it or make any withdrawals. After 3 years she withdraws all of her money and has $1,430.40. What percent simple interest did the bank give? Show your work.

33. Juan deposits $500 in a simple interest account offering 5.9%. After 6 months, he deposits another $350. How much money will Juan earn in interest at the end of 1 year?

34. Which is the better option for a customer, to have 25% taken off each item purchased, or to have 25% taken off the total cost? Explain.
Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - Why are proportional relationships an important part of mathematics?
   - How is percent related to fractions and decimals, and why is it such a useful tool in everyday life?

Academic Vocabulary

2. Look at the following academic vocabulary words:
   - percent
   - unit rate
   - rate

Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. Susan drove 545.5 miles on 32 gallons of gas. At this rate, how many miles per gallon (to the nearest hundredth) does her car get?

   F. 17 mi/gal   H. 18.00 mi/gal  
   G. 17.05 mi/gal   I. 17,456.00 mi/gal

2. *SUPERCALIFRAGILISTICEXPIALIDOCIOUS* is one of the longest words in English. This word is a nonsense word used to describe something that is fantastic. What is the ratio of vowels to constants written as a fraction reduced to its lowest terms?

3. Kellen purchased three bags of different kinds of nuts.
   - a 12-ounce bag of pecans for $4.08
   - a 16-ounce bag of walnuts for $4.32
   - a 14-ounce bag of macadamia nuts for $4.20

   **Part A:** Find the unit price for each kind of nut. Then put the nuts in order by price per ounce from lowest to highest.

   **Solve and Explain**

   ____

   ____

   ____

   **Part B:** Write an expression you can use to determine the cost in dollars of $p$ bags of pecans, $w$ bags of walnuts, and $m$ bags of macadamia nuts. Let $c$ stand for the total cost. What is the cost of 1 bag of pecans, 3 bags of walnuts, and 2 bags of macadamia nuts?

   **Solve and Explain**

   ____

   ____

   ____
4. A sport watch is on sale at the Time To Tell Store. The original price was $450. The watch has been marked down three times as shown on the tag.

**Part A:** Find the final sale price of the watch. What percent of the original price is that?

**Solve and Explain**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

**Part B:** Suppose the sale percents had been reversed so that the price tag looked like this one. What would the final sale price be? What percent of the original price is that?

**Solve and Explain**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

**Part C:** Compare the answers to Parts A and B. Explain any patterns you notice and any patterns that may apply.

**Solve and Explain**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________