Chapter 10

Prerequisite Skills (p. 648)

1. Two similar triangles have congruent corresponding angles and proportional corresponding sides.
2. Two angles whose sides form two pairs of opposite rays are called vertical angles.
3. The interior of an angle is all of the points between the sides of the angle.
4. $c^2 = a^2 + b^2$
   \[
   0.9^2 = 0.6^2 + 0.8^2 \\
   0.81 < 1 \\
   289 > 265
   \]
   The triangle is acute.
5. $c^2 = a^2 + b^2$
   \[
   17^2 < 11^2 + 12^2 \\
   289 < 121 + 144 \\
   289 > 265
   \]
   The triangle is obtuse.
6. $c^2 = a^2 + b^2$
   \[
   2.5^2 < 1.5^2 + 2^2 \\
   6.25 < 2.25 + 4 \\
   6.25 = 6.25
   \]
   The triangle is right.
7. $(8x - 2) + (2x + 2) = 180$
   \[
   10x = 180 \\
   x = 18
   \]

Lesson 10.1

Investigating Geometry Activity 10.1 (p. 650)

1. Answers will vary.
2. Tangent segments from a common external point are congruent.
3. $MQ = MP = 5.5$
   \[
   MN = LM = 7 \\
   LQ = LM + MQ = 7 + 5.5 = 12.5 \\
   PN = PM + MN = 5.5 + 7 = 12.5
   \]
4. Because $AC = EC$ and $BC = DC$, it follows that $AB = ED$ which makes $AB \equiv ED$.
10.1 Guided Practice (pp. 651–654)

1. $\overline{AC}$ is a chord because its endpoints are on the circle.
   $\overline{CB}$ is a radius because $C$ is the center and $B$ is a point on the circle.
2. A tangent is $\overline{DE}$ and a tangent segment is $\overline{DB}$.
3. The radius of $\odot C$ is 3 units.
   The diameter of $\odot C$ is 6 units.
   The radius of $\odot D$ is 2 units.
   The diameter of $\odot D$ is 4 units.
4. 2 common tangents
5. 1 common tangent
6. 0 common tangents
7. \[
   CE^2 \neq CD^2 + DE^2 \\
   (3 + 2)^2 \neq 3^2 + 4^2 \\
   25 \neq 9 + 16 \\
   25 = 25
   \]
   Yes, $\overline{DE}$ is tangent to $\odot C$.
8. \[
   QT^2 = QS^2 + ST^2 \\
   (r + 18)^2 = r^2 + 24^2 \\
   r^2 + 36r + 324 = r^2 + 576 \\
   36r = 252 \\
   r = 7
   \]
9. $x^2 = 9$
   \[
   x = \pm 3
   \]

10.1 Exercises (pp. 655–658)

Skill Practice

1. The points $A$ and $B$ are on $\odot C$. If $C$ is a point on $\overline{AB}$, then $\overline{AB}$ is a diameter.
2. When referring to a segment, “a radius” and “a diameter” is used. When referring to a length, “the radius” and “the diameter” is used.
3. $G; \overline{BD}$ is a point of tangency.
4. $H; \overline{BH}$ is a common tangent.
5. $C; \overline{AB}$ is a chord.
6. $E; \overline{AB}$ is a secant.
7. $F; \overline{AE}$ is a tangent.
8. $A; G$ is the center.
9. $B; \overline{CD}$ is a radius.
10. $D; \overline{BD}$ is a diameter.
11. The error is that $\overline{AB}$ is not a secant, but rather a chord. The length of chord $\overline{AB}$ is 6 units.
12. The radius of $\odot C$ is 9 units.
   The diameter of $\odot C$ is 18 units.
13. The radius of $\odot D$ is 6 units.
   The diameter of $\odot D$ is 12 units.
14. [Diagram of circles and tangents]
Chapter 10, continued

15. 4 common tangents

16. 0 common tangents

17. 1 common tangent

18. Use the converse of the Pythagorean Theorem. Because $3^2 + 4^2 = 5^2$, $\triangle ABC$ is a right triangle and $AB \perp AC$. By Theorem 10.1, $AB$ is tangent to $\odot C$.

19. Use the converse of the Pythagorean Theorem. Because $9^2 + 15^2 \neq 18^2$, $AB$ is not perpendicular to $BC$ and $AB$ is not tangent to $\odot C$.

20. The diameter of $\odot C$ is 20. Using the converse of the Pythagorean Theorem, $20^2 + 48^2 = 52^2$ and the triangle is a right triangle. This implies that $CB \perp BA$. Using Theorem 10.1, $AB$ is tangent to $\odot C$.

21. $(r + 10)^2 = r^2 + 24^2$
   $r^2 + 2r + 100 = r^2 + 576$
   $2r = 476$
   $r = 238$

22. $(r + 6)^2 = r^2 + 9^2$
   $r^2 + 12r + 36 = r^2 + 81$
   $12r = 45$
   $r = 3.75$

23. $(r + 7)^2 = r^2 + 14^2$
   $r^2 + 14r + 49 = r^2 + 196$
   $14r = 147$
   $r = 10.5$

24. $3x + 10 = 7x - 6$
   $16 = 4x$
   $x = 4$

25. $2x^2 + 5 = 13$
   $2x^2 = 8$
   $x^2 = 4$
   $x = \pm 2$

26. $3x^2 + 4x - 4 = 4x - 1$
   $3x^2 = 3$
   $x^2 = 1$
   $x = \pm 1$

27. The common tangents are external because they do not intersect the segment that joins the centers of the two circles.

28. The common tangents are internal because they intersect the segment that joins the centers of the circles.

29. C; Using Theorem 10.1, $RS \perp QR$. Draw a congruent, parallel segment $XP$ as shown.

![Diagram](image)

Using Pythagorean Theorem, $(XQ)^2 + (YP)^2 = (QP)^2$, $2^2 + (YP)^2 = 8^2$. $XP = 2\sqrt{15}$, so $RS = 2\sqrt{15}$.

30. Using Theorem 10.2, $PA \equiv PB$ and $PB \equiv PC$. Using the transitive property of segment congruence, $PA \equiv PB \equiv PC$.

31. Sample Answer: Two lines tangent to the same circle will not intersect when the lines are tangent at opposite endpoints of the same diameter. Using Theorem 10.1, the two lines are perpendicular to the same line, so they are parallel.

32. C is in the interior of $\angle ABD$ and $AC = DC$. By Theorem 5.6, $BC$ bisects $\angle ABD$.

33. For any point outside of a circle, there is never only one or more than two tangents to the circle that passes through the point. There will always be two tangents.

34.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AB = AC = 12$, $BC = 8$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Radius $r = PD = PE = PF$</td>
<td>2. Def. of radius</td>
</tr>
<tr>
<td>3. $AB = AD + BD$, $AC = AF + CF$, $BC = BE + CE$</td>
<td>3. Postulate 2, Segment Addition</td>
</tr>
<tr>
<td>4. $AB = AC$</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. $AD + BD = AF + CF$</td>
<td>5. Substitution for $AB$ and $AC$</td>
</tr>
<tr>
<td>6. $BE = BD$, $CE = CF$, $AD = AF$</td>
<td>6. Theorem 10.2</td>
</tr>
<tr>
<td>7. $AD + BE = AD + CE$</td>
<td>7. Substitution for $BD$, $AF$, and $CF$</td>
</tr>
<tr>
<td>8. $BE = CE$</td>
<td>8. Subtract $AD$</td>
</tr>
<tr>
<td>9. $E$ and $P$ are on the angle bisector of $\angle A$.</td>
<td>9. Theorem 5.6</td>
</tr>
<tr>
<td>10. $AE = AP + PE$</td>
<td>10. Postulate 2, Segment Addition</td>
</tr>
<tr>
<td>11. $BC = 8$</td>
<td>11. Given</td>
</tr>
<tr>
<td>12. $BE + CE = 8$</td>
<td>12. Substitution for $BC$</td>
</tr>
<tr>
<td>13. $CE + CE = 8$</td>
<td>13. Substitution for $BE$</td>
</tr>
<tr>
<td>14. $CE = 4$</td>
<td>14. Divide by 2</td>
</tr>
<tr>
<td>15. $PE \perp CE$</td>
<td>15. Theorem 10.1</td>
</tr>
</tbody>
</table>
Chapter 10, continued

41. Statements | Reasons
--- | ---
1. \( SR \) and \( ST \) are tangent to \( \odot P \). | 1. Given
2. \( SR \perp RP, ST \perp TP \) | 2. Tangent and radius are perpendicular.
3. \( RP = TP \) | 3. Def. of circle
4. \( RP \equiv TP \) | 4. Def. of congruence
5. \( PS \equiv PS \) | 5. Reflexive Property
6. \( \triangle PRS \equiv \triangle PTS \) | 6. HL Congruence Theorem
7. \( SR \equiv ST \) | 7. Corresponding parts of congruent triangles are congruent.

42. a. The slope of the line perpendicular to line \( l \) through \( C \) is \( -\frac{3}{4} \), so the slope of line \( l \) is \( \frac{4}{3} \).
   b. \( y = mx + b \)
      \[ 3 = \frac{4}{3}(-4) + b \]
      \[ 3 = -\frac{16}{3} + b \]
      \[ \frac{25}{3} = b \]
      The equation for \( l \) is \( y = \frac{4}{3}x + \frac{25}{3} \).
   c. \( 3^2 + (-4)^2 = r^2 \)
      \[ 25 = r^2 \]
      \[ 5 = r \]
   d. The \( y \)-intercept is \( \frac{25}{3} \) and the radius is 5. So, the distance from the \( y \)-intercept to \( \odot C \) is \( \frac{25}{3} - 5 = \frac{10}{3} \).

Mixed Review
43. \( m\angle ABD + m\angle DBC = m\angle ABC \)
   \( 25^\circ + m\angle DBC = 70^\circ \)
   \( m\angle DBC = 45^\circ \)
44. \( x^\circ + 50^\circ = 180^\circ \)
   \( x = 130 \)
   \( x^\circ + 3y^\circ = 180^\circ \)
   \( y = 130 \)
   \( 102 + 3y = 180 \)
   \( 3y = 78 \)
   \( y = 26 \)
46. \( (2x + 3)^\circ + 137^\circ = 180^\circ \)
   \( 2x + 140 = 180 \)
   \( 2x = 40 \)
   \( x = 20 \)
   \( (4y - 7)^\circ = 137^\circ \)
   \( 44 = 144 \)
   \( 36 \)

Problem Solving
35. The wheel has radial spokes because each spoke has endpoints that are the center and a point on the circle.
36. The wheel has tangential spokes because each spoke intersects the circle in exactly one point.
37. \( BE^2 = EC^2 + CB^2 \)
   \( (11,000 + 3959)^2 = 3959^2 + CB^2 \)
   \( 14,959^2 = 3959^2 + CB^2 \)
   \( 208,098,000 = CB^2 \)
   \( 14,426 = CB \)
   \( BA = BC = 14,426 \) miles
38. \( m\angle ARC = m\angle BSC = 90^\circ \), so \( \angle ARC \equiv \angle BSC \). Also, \( \triangle RAC \equiv \triangle SCB \) because vertical angles are congruent. Therefore, \( \triangle ARC \sim \triangle BSC \) by the AA Similarity Postulate. So, \( \frac{AC}{BC} = \frac{SC}{SC} \) because the ratio of corresponding sides is the same.
39. a. Because \( R \) is exterior to \( \odot Q \) and \( P \) is on \( \odot Q \), \( QR > QP \).
   b. Because \( QR \) is perpendicular to line \( m \) it must be the shortest distance from \( Q \) to line \( m \). Thus, \( QR < QP \).
   c. It was assumed \( QR \) was not perpendicular to line \( m \) but \( QR \) is perpendicular to line \( m \). Since \( R \) is outside of \( \odot Q \) you know that \( QR > QP \) but part (b) tells you that \( QR < QP \) which is a contradiction. Therefore, line \( m \) is perpendicular to \( QP \).
40. Assume line \( m \) is not tangent to \( \odot Q \). So, there is another point \( X \) on line \( m \) that is also on \( \odot Q \). \( X \) is on \( \odot Q \), so \( QX = QP \). But the perpendicular segment from \( Q \) to line \( m \) is the shortest such segment, so \( QX > QP \).
\( QX \) cannot be both equal to and greater than \( QP \). The assumption that point \( X \) exists must be false. Therefore, line \( m \) is tangent to \( \odot Q \).
Chapter 10, continued

Lesson 10.2

10.2 Guided Practice (pp. 660–661)

1. $\overparen{TO}$ is a minor arc.
   $m\overparen{TO} = 120^\circ$

2. $\overparen{QR}$ is a major arc.
   $m\overparen{QR} = m\overparen{OR} + m\overparen{RT} = 60^\circ + 180^\circ = 240^\circ$

3. $\overparen{TQR}$ is a semicircle.
   $m\overparen{TQR} = 180^\circ$

4. $\overparen{QS}$ is a minor arc.
   $m\overparen{QS} = m\overparen{QR} + m\overparen{RS} = 60^\circ + 100^\circ = 160^\circ$

5. $\overparen{TS}$ is a minor arc.
   $m\overparen{TS} = 80^\circ$

6. $\overparen{RST}$ is a semicircle.
   $m\overparen{RST} = 180^\circ$

7. $AB \cong CD$ because they are in congruent circles and $m\overparen{AB} = m\overparen{CD}$.

8. $MN$ and $PQ$ have the same measure but they are not congruent because they are arcs of circles that are not congruent.

10.2 Exercises (pp. 661–663)

Skill Practice

1. If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $AB$ and $DE$ are congruent.

2. You need to know that the radii of two circles are the same in order to show that the two circles are congruent.

3. $BC$ is a minor arc.
   $m\overparen{BC} = 70^\circ$

4. $\overparen{DC}$ is a minor arc.
   $m\overparen{DC} = 180^\circ - m\overparen{DE} - m\overparen{BC}$
   $= 180^\circ - 45^\circ - 70^\circ = 65^\circ$

5. $\overparen{DB}$ is a minor arc.
   $m\overparen{DB} = m\overparen{DC} + m\overparen{CB} = 65^\circ + 70^\circ = 135^\circ$

6. $\overparen{AE}$ is a minor arc.
   $m\overparen{AE} = m\overparen{BC} = 70^\circ$

7. $\overparen{AD}$ is a minor arc.
   $m\overparen{AD} = m\overparen{AE} + m\overparen{ED} = 70^\circ + 45^\circ = 115^\circ$

8. $\overparen{ABC}$ is a semicircle.
   $m\overparen{ABC} = 180^\circ$

9. $\overparen{ACD}$ is a major arc.
   $m\overparen{ACD} = m\overparen{ABC} + m\overparen{CD} = 180^\circ + 65^\circ = 245^\circ$

10. $\overparen{EAC}$ is a major arc.
    $m\overparen{EAC} = m\overparen{EB} + m\overparen{BC} = 180^\circ + 70^\circ = 250^\circ$

11. $\overparen{CQ}$ is a semicircle because you know that $\overparen{QS}$ is a diameter.

12. $m\overparen{CD} = 180^\circ - 70^\circ - 40^\circ = 70^\circ$
    $\overparen{AB} \cong \overparen{CD}$ because they are in the same circle and $m\overparen{AB} = m\overparen{CD}$.

13. $\overparen{LP}$ and $\overparen{MN}$ have the same measure but they are not congruent because they are arcs of circles that are not congruent.

14. $\overparen{VW} \cong \overparen{XY}$ because they are in congruent circles and $m\overparen{VW} = m\overparen{XY}$.

15. The statement is incorrect because you can tell that the circles are congruent. The circles have the same radius, $CD$.

16. $\overparen{ACD} = 360^\circ - m\overparen{AD} = 360^\circ - 20^\circ = 340^\circ$
    $m\overparen{AC} = 180^\circ - m\overparen{AD} = 180^\circ - 20^\circ = 160^\circ$

17. $\triangle ABP$ is a right triangle.
    $AP^2 + PB^2 = AB^2$
    $x^2 + 2^2 = AB^2$
    $18 = AB^2$
    $3\sqrt{2} = AB$

18. $\overparen{GE} = 360^\circ - 100^\circ - 120^\circ = 140^\circ$
   If $m\overparen{GH} = 150^\circ$, point $H$ must be $10^\circ$ beyond point $E$, placing it on $EF$. Or, point $H$ is $30^\circ$ beyond point $F$, placing it on $EF$.

19. $\overparen{AE} = m\overparen{AB} + m\overparen{BC} + m\overparen{CD} = m\overparen{DE}$
   $= 60^\circ + 25^\circ + 70^\circ + 20^\circ = 175^\circ$
   $m\overparen{AE} = 360^\circ - (m\overparen{AB} + m\overparen{BC} + m\overparen{CD} + m\overparen{DE})$
   $= 360^\circ - 175^\circ = 185^\circ$

20. $\triangle APQ$ is a right triangle with $\angle APQ = 30^\circ$ and $m\angle AQP = 60^\circ$. $\triangle PBQ$ is a right triangle with $m\angle BPQ = 30^\circ$ and $m\angle BQP = 60^\circ$. So, $m\angle AQB = m\angle AQP + m\angle BQP = 60^\circ + 60^\circ = 120^\circ$.
    Therefore, $m\angle AQB = 120^\circ$. 

Geometry

Worked-Out Solution Key

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21. a. \( \tan X = \frac{3}{4} \)
\[ m\angle X = 36.9^\circ \]
\[ m\angle BD = 36.9^\circ \]
b. \( \tan Y = \frac{4}{3} \)
\[ m\angle Y = 53.1^\circ \]
\[ m\angle AD = 53.1^\circ \]
c. \( m\angle AB = m\angle AD - m\angle BD \]
\[ = 53.1^\circ - 36.9^\circ = 16.2^\circ \]

Problem Solving

22. The measure of the arc is 60°.

23. 360° ÷ 20 = 18°

The measure of each arc in the outermost circle of the dartboard is 18°.

24. a. 360° – 90° = 270°

The measure of the arc surveyed by the camera is 270°.

b. 270° ÷ 10° per minute = 27 minutes

It takes the camera 27 minutes to survey the area once.

c. The camera must go 270° – 85° = 185° counterclockwise and another 185° clockwise to return to the same position. So, 185° + 185° = 370°, 370° ÷ 10° per minute = 37 minutes. It will take the camera 37 minutes.

d. In 15 minutes, the camera can go 10°(15) = 150°. The camera will go 50° counterclockwise and then 100° clockwise. So, the camera will be 100° from wall A after 15 minutes.

25. a. \( \frac{360^\circ}{60 \text{ min}} = 6^\circ \text{ per minute} \)

After 20 minutes, the minute hand will move 20(6°) = 120°.

\[ \frac{360^\circ}{12 \text{ h}} = 30^\circ \text{ per hour} \]

The hour hand moves 30° per hour.

\[ \frac{30^\circ}{60 \text{ min}} = 0.5^\circ \text{ in 10 minutes} \]

At 1:20, the hour hand is 30° + 10° = 40° from the 12. The minute hand is 120° from 12. The minor arc between the hour and minute hand is 120° – 40° = 80°.

b. You want the arc between the hour and minute hand to be 180°. Use guess, check, and revise. At 1:40, the minute hand is 40(6°) = 240° from 12. The hour hand moves \( \frac{30^\circ}{60 \text{ min}} = 0.5^\circ \text{ in 40 minutes} \). The hour hand is 30° + 20° = 50° from 12. The arc is 240° – 50° = 190°. At 1:38, the minute hand is 38(6°) = 228° from 12. The hour hand moves \( \frac{30^\circ}{60 \text{ min}} = 0.5^\circ \text{ in 38 minutes} \). The hour hand is 30° + 19° = 49° from 12. The arc is 228° – 49° = 179°.

Mixed Review

26. \( y = 5x + 2 \)
\[ y = 5(1 - x) = 5 - 5x \]

The slopes of the lines are different, so they are not parallel.

27. \( 2y + 2x = 5 \)
\[ 2y = -2x + 5 \]
\[ y = -x + \frac{5}{2} \]
\[ y = 4 - x \]

The slopes are the same, so the lines are parallel.

28. \( y \)

Problem Solving

29. \( (x + 2)(x + 3) = x^2 + 2x + 3x + 6 \)
\[ = x^2 + 5x + 6 \]

30. \( (2y - 5)(y + 7) = 2y^2 - 5y + 14y - 35 \)
\[ = 2y^2 + 9y - 35 \]

31. \( (x + 6)(x - 6) = x^2 + 6x - 6x - 36 = x^2 - 36 \)

32. \( (z - 3)^2 = (z - 3)(z - 3) \)
\[ = z^2 - 3z - 3z + 9 = z^2 - 6z + 9 \]

33. \( (3x + 7)(5x + 4) = 15x^2 + 12x + 35x + 28 \)
\[ = 15x^2 + 47x + 28 \]

34. \( (z - 1)(z - 4) = z^2 - 4z - z + 4 = z^2 - 5z + 4 \)

Lesson 10.3

10.3 Guided Practice (pp. 664–666)

1. \( m\angle BC = 110^\circ \)

2. \( 2m\angle AB + m\angle AC = 360^\circ \)
\[ 2m\angle AB + 150^\circ = 360^\circ \]
\[ 2m\angle AB = 210^\circ \]
\[ m\angle AB = 105^\circ \]

3. \( 9x^2 = (80 - x)^2 \)
\[ 10x = 80 \]
\[ x = 8 \]
\[ m\angle CD = 9x^2 = 9(8)^2 = 72^\circ \]

4. \( 9x^2 = (80 - x)^2 \)
\[ 10x = 80 \]
\[ x = 8 \]
\[ m\angle DE = (80 - x)^2 = (80 - 8)^2 = 72^\circ \]

5. \( m\angle CE = m\angle CD + m\angle DE = 72^\circ + 72^\circ = 144^\circ \)

6. \( QR = ST = 32 \)

7. \( QU = \frac{1}{2} QR = \frac{1}{2}(32) = 16 \)
Chapter 10, continued

8. \[ QC^2 = QO^2 + UC^2 \]
   \[ QC^2 = 16^2 + 12^2 \]
   \[ QC^2 = 256 + 144 \]
   \[ QC^2 = 400 \]
   \[ QC = 20 \]

10.3 Exercises (pp. 667–670)

Skill Practice

1. Sample answer: Point \( Y \) bisects \( \overline{XZ} \) if \( XY \equiv \overline{YZ} \).
2. If two chords of a circle are perpendicular and congruent, one of them does not have to be a diameter. A square inscribed in a circle is an example of two chords that are congruent and perpendicular.
3. \( m\overline{AB} = m\overline{ED} = 75^\circ \)
4. \[ 2m\overline{AB} + m\overline{AD} = 360^\circ \]
   \[ 2m\overline{AB} + 128^\circ = 360^\circ \]
   \[ 2m\overline{AB} = 232^\circ \]
   \[ m\overline{AB} = 116^\circ \]
5. \( \overline{EG} = \overline{EJ} = 8 \)
6. By Theorem 10.5, \( \overline{BD} \) bisects \( \overline{AC} \).
   \[ 4x = 3x + 7 \]
   \[ x = 7 \]
7. By Theorem 10.5, \( \overline{LN} \) bisects \( \overline{PM} \).
   \[ 5x - 6 = 2x + 9 \]
   \[ 3x = 15 \]
   \[ x = 5 \]
8. Because \( \overline{SQ} \) and \( \overline{TQ} \) are radii, they have the same measure.
   \[ 6x + 9 = 8x - 13 \]
   \[ 22 = 2x \]
   \[ 11 = x \]
9. \( \overline{AB} \) and \( \overline{CD} \) are equidistant from the center, so by Theorem 10.6, they are congruent.
   \[ 5x - 7 = 18 \]
   \[ 5x = 25 \]
   \[ x = 5 \]
10. \( \overline{AD} \) and \( \overline{BC} \) are equidistant from the center, so by Theorem 10.6, they are congruent. By Theorem 10.5, \( AD = 2(3x + 2) = 6x + 4 \).
   \[ 6x + 4 = 22 \]
   \[ 6x = 18 \]
   \[ x = 3 \]
11. \( \overline{EF} \) and \( \overline{HG} \) are congruent, so by Theorem 10.6, they are equidistant from \( Q \).
   \[ 4x + 1 = x + 8 \]
   \[ 3x = 7 \]
   \[ x = \frac{7}{3} \]
12. Because \( \overline{AB} \) is a perpendicular bisector of \( \overline{CD} \), it is a diameter by Theorem 10.4.
13. Because \( \overline{HZ} \) is a diameter and it is perpendicular to \( \overline{FG} \), it bisects \( \overline{FG} \) and \( \overline{FG} \) by Theorem 10.5.

Geometry

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Chapter 10, continued

c. Using the converse of Theorem 10.5, you know that \( PT \cong R^{T} \) and \( QT \perp PR \). By the Reflexive Property, \( QT \cong QT \). Using the SAS Congruence Postulate, \( \triangle PQT \cong \triangle RQT \). \( PQ \cong RQ \) because corresponding parts of congruent triangles are congruent. So, \( PQ = RQ \).

23. From the diagram, \( m\overset{\frown}{AB} = x^\circ \). By Theorem 10.3, \( m\overset{\frown}{BC} = m\overset{\frown}{CA} \). Let \( y = m\overset{\frown}{BC} = m\overset{\frown}{CA} \). Then
\[
m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CA} = 360^\circ
\]
x + y + y = 360
x = 360 - 2y

\( x \) is an even number because \((360 - 2y)\) is even.

24. \[
\begin{array}{c}
\text{In } \triangle PAT, PA = 10 \text{ and } AT = 8.
\\
m\angle APT = \sin^{-1}\left(\frac{8}{10}\right) = 53.13^\circ.
\\
\text{In } \triangle PBR, PB = 10 \text{ and } BR = 6.
\\
m\angle BPR = \sin^{-1}\left(\frac{6}{10}\right) = 36.86^\circ.
\\
m\overset{\frown}{AB} = m\overset{\frown}{AR} \Rightarrow m\overset{\frown}{BR} = 53.13^\circ - 36.86^\circ = 16.26^\circ
\end{array}
\]

Problem Solving

25. In order for \( \overset{\frown}{AB} \cong \overset{\frown}{BC} \), \( AB \) should be congruent to \( BC \).

26. To find the center of the cross section, you can (1) construct the perpendicular bisector of the control panel, extend this segment to the other control panel, and then find the midpoint of the segment. Or you can (2) draw two diagonals from the top of one control panel to the bottom of the other. The center will be at their intersection.

27. | Statements | Reasons |
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1. ( AB \cong CD )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( PA, PB, PC, ) and ( PD ) are radii of ( \odot P ).</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( PA \cong PB \cong PC \cong PD )</td>
<td>3. All radii of same circle are congruent</td>
</tr>
<tr>
<td>4. ( \triangle PCD \cong \triangle PAB )</td>
<td>4. SSS Congruence Postulate</td>
</tr>
<tr>
<td>5. ( \angle CPD \cong \angle APB )</td>
<td>5. Corr. parts of ( \cong ) s are ( \cong )</td>
</tr>
<tr>
<td>6. ( m\overset{\frown}{CPD} = m\overset{\frown}{APB} )</td>
<td>6. Def. of ( \cong )</td>
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<td>7. ( \angle CPD ) and ( \angle APB ) are central ( \overset{\frown}{s} ).</td>
<td>7. Given</td>
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<tr>
<td>8. ( m\overset{\frown}{CD} = m\overset{\frown}{AB} )</td>
<td>8. Def. of arc measure</td>
</tr>
<tr>
<td>9. ( \overset{\frown}{CD} \cong \overset{\frown}{AB} )</td>
<td>9. Def. of ( \cong )</td>
</tr>
</tbody>
</table>

28. Because \( \overset{\frown}{AB} \cong \overset{\frown}{CD} \), \( \overset{\frown}{APB} \cong \overset{\frown}{CPD} \) by the definition of congruent arcs. \( PA, PB, PC, \) and \( PD \) are all radii of \( \odot P \), so \( PA \cong PB \cong PC \cong PD \). Then \( \overset{\frown}{APB} \cong \overset{\frown}{CPD} \) by the SAS Congruence Postulate, so corresponding sides \( AB \) and \( CD \) are congruent.

29. a. The longer chord is closer to the center.

b. The length of a chord in a circle increases as the distance from the center of the circle to the chord decreases.

c. Let \( c \) be the radius.
Let \( x < y \).

By the Pythagorean Theorem,
\[
c^2 = b^2 + x^2 \quad \text{and} \quad c^2 = d^2 + y^2
\]

So, \( b^2 + x^2 = d^2 + y^2 \). Since \( x < y \), \( b > d \). Therefore, the length of a chord in a circle increases as the distance from the center decreases.

30. a. \( r^2 = 160^2 + (r - 80)^2 \)
\[
r^2 = 25,600 + r^2 - 160r + 6400
\]
\[
160r = 32,000
\]
\[
r = 200
\]

The radius of the circle is 200 feet.

b. \( S = \frac{3.86 \sqrt{fr}}{3.86(0.7)(200)} = 45.67 \)

The car’s speed is about 45.7 miles per hour.

31. Given \( QS \) is the perpendicular bisector of \( RT \) in \( \odot L \). Suppose \( L \) is not on \( QS \). Since \( LT \) and \( LR \) are radii of the circle, they are congruent. With \( PL \cong PL \) by the Reflexive Property, you now have \( \triangle RLP \cong \triangle TLP \) using the SSS Congruence Postulate. Corresponding angles \( \angle RPL \) and \( \angle TPL \) are congruent and they form a linear pair. This makes them right angles and leads to \( QL \) being perpendicular to \( RT \). Using the Perpendicular Postulate, \( L \) must be on \( QS \) and thus \( QS \) must be a diameter.

32. Draw radii \( LD \) and \( LF \). So, \( LD \cong LF \), \( LC \cong EC \) (Reflexive Property) and since \( EG \perp DF \), \( \angle LDC \cong \angle LFC \) by the HL Congruence Theorem. Then, corresponding sides \( DC \) and \( FC \) are congruent, as are corresponding angles \( \angle DLC \) and \( \angle FLC \). By the definition of congruent arcs, \( DG \cong FG \).

33. Case 1: Given: \( EF \perp AB \), \( EG \perp DC \), \( EG \parallel EF \)

Prove: \( AB \parallel DC \)

Draw radii \( EB \) and \( EC \). \( EB \parallel EC \) and \( EF \parallel EG \). Also, since \( EF \perp AB \) and \( EG \perp DC \), \( \triangle EFB \) and \( \triangle EGC \) are right triangles and are congruent by the HL Congruence Theorem. Corresponding sides \( BF \) and \( CG \) are congruent, so \( BF = CG \) and, by the Multiplication Property of Equality, \( 2BF = 2CG \). By Theorem 10.5, \( EF \) bisects \( AB \) and \( EG \) bisects \( CD \), so \( AB = 2BF \) and \( CD = 2CG \). Then by the Substitution Property, \( AB = CD \) or \( AB \parallel CD \).
Chapter 10, continued

Case 2: Given: $EF \perp AB$, $EG \perp DC$, $AB \equiv DC$
Prove: $EF \equiv EG$
Draw radii $EB$ and $EC$, $EB \equiv EC$. By Theorem 10.5, $EF$ bisects $AB$ and $EG$ bisects $CD$, so $AB = 2BF$ and $CD = 2CG$. We know $AB = CD$, so by the Substitution Property, $2BF = 2CG$. By the Division Property of Equality, $BF = CG$, or $BF \equiv CG$. $\triangle EFB$ and $\triangle EGC$ are right triangles and are congruent by the HL Congruence Theorem. It follows that corresponding sides $EF$ and $EG$ are congruent.

34. 

The point where the tire touches the ground is a point of tangency. By Theorem 10.1, line $g$ is perpendicular to radius of the circle, $TP$. Because $AB \parallel g$, $PT \perp AB$. By Theorem 10.5, $PQ$ bisects $AB$ and $AB$.

Mixed Review

35. $100^\circ + 140^\circ + (x + 20)^\circ + (2x + 10)^\circ = 360^\circ$
$3x + 270 = 360$
$3x = 90$
$x = 30$

36. 

$JKLM$ is a rectangle.

37. 

$JKLM$ is a rhombus.

Quiz 10.1–10.3 (p. 670)

1. $CA^2 = CB^2 + BA^2$
$15^2 = 9^2 + 12^2$
$225 = 81 + 144$
$225 = 225$
$AB$ is tangent to $O_C$ at $B$ because $m\angle ABC = 90^\circ$ and radius $CB$ is perpendicular to $AB$.

2. $CB^2 = CA^2 + AB^2$
$14^2 = 5^2 + 12^2$
$196 = 25 + 144$
$196 = 169$
$AB$ is not tangent to $O_C$ at $A$ because $m\angle BAC \neq 90^\circ$ and radius $CA$ is not perpendicular to $AB$.

3. $m\angle EFG = m\angle EFB + m\angle FEG$
$195^\circ = 80^\circ + m\angle FEG$
$115^\circ = m\angle FEG$
$m\angle EFG = 360^\circ - m\angle EFG = 360^\circ - 195^\circ = 165^\circ$

4. 

$m\angle ABD = m\angle AB + m\angle BD$
$m\angle ABD = m\angle AB + m\angle AB$
$m\angle ABD = 2m\angle AB$
$194^\circ = 2m\angle AB$
$97^\circ = m\angle AB$

Lesson 10.4

Investigating Geometry Activity 10.4 (p. 671)

1. Answers will vary.
2. Answers will vary.
3. The measure of an inscribed angle is one half the measure of the corresponding central angle.

10.4 Guided Practice (pp. 673–675)

1. $m\angle HGF = \frac{1}{2} m\angle HF = \frac{1}{2}(90^\circ) = 45^\circ$
2. $m\angle TUV = 2m\angle TUV = 2(38^\circ) = 76^\circ$
3. $m\angle ZXW = m\angle ZYW = 72^\circ$
4. To frame the front and left side of the statue in your picture, make the diameter of your circle the diagonal of the rectangular base. This diagonal connects the upper left corner to the bottom right corner.
5. $m\angle B + m\angle D = 180^\circ$
$x^\circ + 82^\circ = 180^\circ$
$x = 98$
$m\angle C + m\angle A = 180^\circ$
$68^\circ + y^\circ = 180^\circ$
$y = 112$
6. $m\angle S + m\angle U = 180^\circ$
$e^\circ + (2c - 6)^\circ = 180^\circ$
$3e = 186$
$c = 62$
$m\angle T + m\angle V = 180^\circ$
$10x^\circ + 8x^\circ = 180^\circ$
$18x = 180$
$x = 10$
Chapter 10, continued

10.4 Exercises (pp. 676–679)

Skill Practice

1. If a circle is circumscribed about a polygon, then the polygon is **inscribed** in the circle.

2. The diagonals of a rectangle create two right triangles. Theorem 10.9 tells you the hypotenuse of each of these triangles is a diameter of the circle.

3. \( m \angle A = \frac{1}{2} m \angle BC = \frac{1}{2} (84^\circ) = 42^\circ \)

4. \( m \angle D = 360^\circ - 120^\circ - 70^\circ = 170^\circ \)

5. \( m \angle G = \frac{1}{2} m \angle FD = \frac{1}{2} (170^\circ) = 85^\circ \)

6. \( m \angle N = \frac{1}{2} m \angle LM = \frac{1}{2} (20^\circ) = 10^\circ \)

7. \( m \angle R = 2m \angle Q = 2 (67^\circ) = 134^\circ \)

8. \( m \angle T = 2m \angle U = 2 (30^\circ) = 60^\circ \)

9. From the diagram, the measure of \( \widehat{RS} \) is 90°. So, the measures of the arcs add up to 370°. You can either change the measure of \( \angle Q \) to 40° or change the measure of \( \angle Q \) to 90°.

10. \( \angle ADB \cong \angle ACB \) because they intercept the same arc, \( \widehat{AB} \).

11. \( \angle JMK \cong \angle KLM \) because they intercept the same arc, \( \widehat{JK} \).

12. \( \angle WXZ \cong \angle ZYW \) because they intercept the same arc, \( \widehat{WX} \).

13. \( m \angle R + m \angle T = 180^\circ \)

\( x^\circ + 80^\circ = 180^\circ \)

\( x = 100 \)

14. \( m \angle S + m \angle Q = 180^\circ \)

\( y^\circ + 95^\circ = 180^\circ \)

\( y = 85 \)

15. \( m \angle J = \frac{1}{2} m \angle KLM = \frac{1}{2} (130^\circ + 110^\circ) = 120^\circ \)

\( m \angle J + m \angle L = 180^\circ \)

\( 120^\circ + 3a^\circ = 180^\circ \)

\( a = 60 \)

\( m \angle K = \frac{1}{2} m \angle M = \frac{1}{2} (54^\circ + 130^\circ) = 92^\circ \)

\( m \angle K + m \angle M = 180^\circ \)

\( 92^\circ + 4b^\circ = 180^\circ \)

\( b = 88 \)

16. \( B; m \angle AC = 60^\circ \)

\( m \angle B = \frac{1}{2} m \angle AC = \frac{1}{2} (60^\circ) = 30^\circ \)

17. a. There are 5 congruent arcs. \( 360^\circ \div 5 = 72^\circ \). The measure of each inscribed angle is \( \frac{1}{2} \) the measure of the arc. \( \frac{1}{2} (72^\circ) = 36^\circ \). There are 5 inscribed angles. Sum = 5(36°) = 180°.

b. There are 7 congruent arcs. \( 360^\circ \div 7 = 51.4^\circ \). The measure of each inscribed angle is \( \frac{1}{2} \) the measure of the arc. \( \frac{1}{2} (51.4^\circ) = 25.7^\circ \). There are 7 inscribed angles. Sum = 7(25.7°) = 180°.

c. There are 9 congruent arcs. \( 360^\circ \div 9 = 40^\circ \). The measure of each inscribed angle is \( \frac{1}{2} \) the measure of the arc. \( \frac{1}{2} (40^\circ) = 20^\circ \). There are 9 inscribed angles. Sum = 9(20°) = 180°.

18. A: \( 2m \angle FEG = m \angle G \)

\[ 2(8x + 10)^\circ = (12x + 40)^\circ \]

\[ 16x + 20 = 12x + 40 \]

\[ 4x = 20 \]

\[ x = 5 \]

19. In a parallelogram, the opposite angles are congruent. In an inscribed parallelogram, opposite angles are supplementary. A rectangle is a parallelogram with congruent supplementary opposite angles. The \( m \angle R \) is 90°.

20. A square can always be inscribed in a circle because its opposite angles are 90° and thus are supplementary.

21. A rectangle can always be inscribed in a circle because its opposite angles are 90° and thus are supplementary.

22. A parallelogram cannot always be inscribed in a circle because its opposite angles are not always supplementary.

23. A kite cannot always be inscribed in a circle because its opposite angles are not always supplementary.

24. A rhombus cannot always be inscribed in a circle because its opposite angles are not always supplementary.

25. An isosceles trapezoid can always be inscribed in a circle because its opposite angles are supplementary.
Chapter 10, continued

26. \[ JK \] is a diameter. The altitude from \( C \) to \( AB \) is also a
diameter. \( \triangle ABC \) is a right triangle, so \( \sin A = \frac{4}{5} \). Using
the smaller triangle, \( \sin A = \frac{\text{alt}}{3} \). So, \( \frac{4}{3} = \frac{\text{alt}}{3} \to \text{alt} = \frac{12}{5} \).
Therefore, the diameter \( JK \) is also \( \frac{12}{5} \).

**Problem Solving**

27. \[ \begin{align*}
\angle AC &= 100,000 + 20,000 + 100,000 = 220,000 \\
\text{Moon } A \text{ is } 220,000 \text{ km from moon } C.
\end{align*} \]

28. Place the carpenter’s square so the endpoints
of the square and the vertex of the square are
on the circumference of the circle, then connect the endpoints.

29. The hypotenuse of the right triangle inscribed
in the circle is the diameter of the circle. So, double the length
of the radius to find the length of the hypotenuse.

30. By the Arc Addition Postulate, \( m\angle FG + m\angle DE = 360^\circ \)
and \( m\angle DE + m\angle EF = 360^\circ \). Using the Measure of an
Inscribed Angle Theorem, \( m\angle EDG = 2m\angle F \),
\( m\angle EFG = 2m\angle D \), \( m\angle DEG = 2m\angle G \), and
\( m\angle FGD = 2m\angle E \). By the Substitution Property,
\( 2m\angle D + 2m\angle F = 360^\circ \), so \( m\angle D + m\angle F = 180^\circ \).
Similarly, \( m\angle E + m\angle G = 180^\circ \).

31. Let \( m\angle B = x \). Because \( QA \) and \( QB \) are both radii
of \( \odot Q \), \( QA \equiv QB \) and \( \triangle AQB \) is isosceles. Because \( \angle A \) and
\( \angle B \) are base angles of an isosceles triangle, \( \angle A \equiv \angle B \).
So, by substitution, \( m\angle A = x \). By the Exterior Angles
Theorem, \( m\angle AQ C = m\angle A + m\angle B = 2x \). So, by
the definition of the measure of a minor arc, \( m\angle AC = 2x \).
Divide each side by 2 to show that \( x = \frac{1}{2} m\angle AC \). Then,
by substitution, \( m\angle B = \frac{1}{2} m\angle AC \).

32. Given: \( \angle ABC \) is inscribed in \( \odot Q \). Point \( Q \) is in the
interior of \( \angle ABC \).

Prove: \( m\angle ABC = \frac{1}{2} m\angle AC \)

Plan for Proof: Construct the diameter \( BD \) of \( \odot Q \) and
show \( m\angle ABD = \frac{1}{2} m\angle AD \) and \( m\angle DBC = \frac{1}{2} m\angle DC \). Use
the Arc Addition Postulate and the Angle Addition
Postulate to show \( 2m\angle ABD = m\angle AD + m\angle DC \).

33. Given: \( \angle ABC \) is inscribed in \( \odot Q \). Point \( Q \) is in the
exterior of \( \angle ABC \).

Prove: \( m\angle ABC = \frac{1}{2} m\angle AC \)

Plan for Proof: Construct the diameter \( BD \) of \( \odot Q \) and
show \( m\angle ABD = \frac{1}{2} m\angle AD \) and \( m\angle DBC = \frac{1}{2} m\angle DC \). Use
the Arc Addition Postulate and the Angle Addition
Postulate to show \( 2m\angle ABD = m\angle AD + m\angle DC \).

34. Given: \( \angle ACB \) and \( \angle ADB \) are
inscribed angles.

Prove: \( \angle ADB = \angle ACB \)

**Paragraph Proof:**

By Theorem 10.7, \( m\angle ADB = \frac{1}{2} m\angle ACB \) and \( m\angle ADB = \frac{1}{2} m\angle ACB \).
By substitution, \( m\angle ADB = m\angle ACB \). By the definition of congruence, \( \angle ADB \cong \angle ACB \).

35. Case 1: Given: \( \odot T \) with inscribed
\( \triangle ABC \). \( AC \) is a diameter of \( \odot T \).

Prove: \( \triangle ABC \) is a right triangle.

Plan for Proof:

Use the Arc Addition Postulate to show
that \( m\angle AEC = m\angle ABC \) and thus \( m\angle B = 180^\circ \). Then use
the Measure of an Inscribed Angle Theorem to show
\( m\angle B = 90^\circ \), so that \( \angle B \) is a right angle and \( \angle ABC \) is a right triangle.

Case 2: Given: \( \odot T \) with inscribed \( \triangle ABC \). \( \angle B \) is a right
angle.

Prove: \( AC \) is a diameter of \( \odot T \).

Plan for Proof: Use the Measure of an Inscribed Angle
Theorem to show the inscribed right angle intercepts an arc with measure \( 2(90^\circ) = 180^\circ \). Since \( AC \) intercepts an arc
that is half of the measure of the circle, it must be a
diameter.

36. In the figure, \( \triangle ABC \) is a right triangle with \( \angle ABC \) being the right angle. Using Theorem 10.1, since \( AB \) is
perpendicular to radius \( BC \), it is tangent to \( \odot C \) at point \( B \).

37. \( \frac{HJ}{GJ} = \frac{GJ}{FJ} \): in a right triangle, the altitude from the right
angle to the hypotenuse divides the hypotenuse into two
segments. The length of the altitude is the geometric
mean of the lengths of these two segments.

38. \( FJ = 6 \text{ in.}, \ JH = 2 \text{ in.} \)

\[
\frac{HJ}{GJ} = \frac{GJ}{FJ} \]

\[
\frac{x}{x} = \frac{6}{6} \]

\[
x^2 = 12 \]

\[
x = 2\sqrt{3} \]

\[
GJ = 2\sqrt{3} \text{ in.} \]

\[
GK = 2GJ = 2(2\sqrt{3}) = 4\sqrt{3} \text{ in.} \]
Chapter 10, continued

39. Show that $\angle APB$ is less than or equal to $45^\circ$. The diagram shown represents $\frac{1}{11}$ of the fuel booster design. $\angle ACB$ is equal to $\frac{1}{11}$ of $360^\circ$ or about $32.7^\circ$. $CP$ bisects $\angle ACB$ and $\angle APB$, so $m\angle ACP = 16.35^\circ$. $CP$ is also the perpendicular bisector of $AB$, and contains point $D$. $\triangle ACD$ and $\triangle ADP$ are right triangles with $\angle D = 90^\circ$ in both triangles.

$$\sin 16.35^\circ = \frac{AD}{2} \Rightarrow AD = 0.563$$

$$\cos \angle PAD = \frac{AD}{1.5} = 0.375$$

$$m\angle PAD = \cos^{-1} 0.375 = 68^\circ$$

Since $\triangle APB$ is isosceles, the base angles are equal.

$$m\angle PAB = m\angle PBA = 68^\circ$$

$$m\angle APB = 180^\circ - m\angle PAB - m\angle PBA$$

$$= 180^\circ - 68^\circ - 68^\circ = 44^\circ$$

So $m\angle APB$ is less than $45^\circ$.

Mixed Review

40. $x^2 = 55^2 + 60^2$

$x^2 = 3025 + 3600$

$x^2 = 6625$

$x = 81.4$

$x = 90.4$

41. $x^2 = 38^2 + 82^2$

$x^2 = 1444 + 6724$

$x^2 = 8168$

$x = 90.4$

42. $x^2 = 26^2 + 16^2$

$x^2 = 676 + 256$

$x^2 = 932$

$x = 30.5$

Lesson 10.5

10.5 Guided Practice (pp. 680–682)

1. $m\angle 1 = \frac{1}{2}(210^\circ) = 105^\circ$

2. $m\angle TST = 2m\angle T = 2(98^\circ) = 196^\circ$

3. $m\angle XXY = 2m\angle X = 2(80^\circ) = 160^\circ$

4. $180^\circ - 102^\circ = 78^\circ$

$$78^\circ = \frac{1}{2}(95^\circ + y')$$

$$156 = 95 + y$$

$$61 = y$$

5. $m\angle FJG = \frac{1}{2}(m\angle FG - m\angle HH)$

$$30^\circ = \frac{1}{2}(a^2 - 44^\circ)$$

$$60 = a - 44$$

$$104 = a$$

6. $\sin TQS = \frac{3}{5}$

$m\angle TQS = 36.87^\circ$

$m\angle TQR = 2(m\angle TQS) = 2(36.87^\circ) = 73.74^\circ$

$m\angle TQR = \frac{1}{2}(m\angle TR - m\angle TR)$

$$73.74^\circ = \frac{1}{2}(x^2 - (360 - x^2))$$

$$147.48 = x - 360 + x$$

$$507.48 = 2x$$

$$253.74 = x$$

10.5 Exercises (pp. 683–686)

Skill Practice

1. The points $A$, $B$, $C$, and $D$ are on a circle and $\overline{AB}$ intersects $\overline{CD}$ at $P$. If $m\angle APC = \frac{1}{2}(m\overline{BD} - m\overline{AC})$ then $P$ is outside the circle.

2. If $m\overline{AB} = 0^\circ$, then the two chords intersect on the circle.

By Theorem 10.12, $m\angle 1 = \frac{1}{2}(m\overline{DC} + m\overline{AB})$

$$= \frac{1}{2}(m\overline{DC} + 0^\circ) = \frac{1}{2}m\overline{DC}.$$  

This is consistent with the measure of an Inscribed Angle Theorem (Lesson 10.4).

3. $m\angle A = \frac{1}{2} m\overline{AB}$

$65^\circ = \frac{1}{2} m\overline{AB}$

$117^\circ = \frac{1}{2} m\overline{DEF}$

$130^\circ = m\overline{AB}$

$234^\circ = m\overline{DEF}$

4. $m\angle D = \frac{1}{2} m\overline{DEF}$

$120^\circ = \frac{1}{2} m\overline{DEF}$

$240^\circ = m\overline{AB}$

5. $m\angle 1 = \frac{1}{2}(260^\circ) = 130^\circ$

6. $D$; Because $\overline{AB}$ is not a diameter, $m\angle A \neq 180^\circ$.

$m\angle A = \frac{1}{2} m\overline{AB} = \frac{1}{2}(180^\circ) = 90^\circ$
Chapter 10, continued

7. \(x^\circ = \frac{1}{2}(145^\circ + 85^\circ) = \frac{1}{2}(230^\circ) = 115^\circ\)

8. \(180^\circ - 122.5^\circ = 57.5^\circ\)

9. \((180 - x)^\circ = \frac{1}{2}(30^\circ + (2x - 30)^\circ)\)

\(2(180 - x) = 2x\)

\(360 - 2x = 2x\)

\(360 = 4x\)

\(90 = x\)

10. \(360^\circ - 247^\circ = 113^\circ\)

\(x^\circ = \frac{1}{2}(247^\circ - 113^\circ)\)

\(x = \frac{1}{2}(134)\)

\(x = 67\)

11. \(29^\circ = \frac{1}{2}(114^\circ - x^\circ)\)

\(58 = 114 - x\)

\(x = 56\)

12. \(34^\circ = \frac{1}{2}(3x - 2)^\circ - (x + 6)^\circ\)

\(68 = (3x - 2) - (x + 6)\)

\(68 = 2x - 8\)

\(76 = 2x\)

\(38 = x\)

13. \(D; m\angle 4 = \frac{1}{2}(80^\circ + 120^\circ) = \frac{1}{2}(200^\circ) = 100^\circ\)

14. The error is that the given measurements imply two different measures for \(BE\). Using Theorem 10.12,

\(50^\circ = \frac{1}{2}(m\widehat{BE} + 60^\circ)\)

\(100^\circ = m\widehat{BE} + 60^\circ\)

\(40^\circ = m\widehat{BE}\)

Using Theorem 10.13,

\(15^\circ = \frac{1}{2}(60^\circ - m\widehat{BE})\)

\(30^\circ = 60^\circ - m\widehat{BE}\)

\(m\widehat{BE} = 30^\circ\)

15. If \(KL\) is perpendicular to \(JK\) at \(K\), then \(m\angle LPJ = 90^\circ\), otherwise it would measure less than \(90^\circ\). So, \(m\angle LPJ \leq 90^\circ\).

16. \(a. \ \frac{1}{2}(110^\circ) = 55^\circ\)

\(55^\circ = \frac{1}{2}(x^\circ - 40^\circ)\)

\(110^\circ = x^\circ - 40^\circ\)

\(150 = x\)

\(\frac{1}{2} \cdot 5^\circ = \frac{1}{2}(b^\circ - a^\circ)\)

\(c = b - a\)

17. \(m\angle Q = \frac{1}{2}(m\widehat{EF} - m\widehat{F})\)

\(60^\circ = \frac{1}{2}[(360 - x)^\circ - x^\circ]\)

\(120 = 360 - 2x\)

\(2x = 240\)

\(x = 120\)

So, \(m\widehat{EF} = 120^\circ\).

\(m\angle R = \frac{1}{2}(m\widehat{EF} - m\widehat{F})\)

\(80^\circ = \frac{1}{2}[(360 - x)^\circ - x^\circ]\)

\(160 = 360 - 2x\)

\(2x = 200\)

\(x = 100\)

So, \(m\widehat{EF} = 100^\circ\).

\(m\widehat{GE} = 360^\circ - m\widehat{EF} - m\widehat{F}\)

\(= 360^\circ - 120^\circ - 100^\circ = 140^\circ\)

So, \(m\widehat{GF} = 140^\circ\).

18. \(m\angle B = \frac{1}{2}(m\widehat{AD} - m\widehat{AC})\)

\(40^\circ = \frac{1}{2}(7x^\circ - 3x^\circ)\)

\(80 = 4x\)

\(20 = x\)

\(m\widehat{AD} = 7x^\circ = 7(20)^\circ = 140^\circ\)

\(m\widehat{AC} = 3x^\circ = 3(20)^\circ = 60^\circ\)

\(m\widehat{CD} = 360^\circ - m\widehat{AD} - m\widehat{AC}\)

\(= 360^\circ - 140^\circ - 60^\circ = 160^\circ\)

19. \(a.\)

\(b.\) For the diagram on the left,

\(m\angle BAC = \frac{1}{2}m\widehat{AB}\)

\(2m\angle BAC = m\widehat{AB}\)

For the diagram on the right,

\(m\angle BAC = \frac{1}{2}(360^\circ - m\widehat{BA})\)

\(2m\angle BAC = 360^\circ - m\widehat{BA}\)

\(m\widehat{BA} = 360^\circ - 2m\angle BAC\)

\(m\widehat{BA} = 2(180^\circ - m\angle BAC)\)
Chapter 10, continued

c. \(2m\angle BAC = 2(180^\circ - m\angle BAC)\)
\[m\angle BAC = 180^\circ - m\angle BAC\]
\[2m\angle BAC = 180^\circ\]
\[m\angle BAC = 90^\circ\]

So, these equations give the same value for \(m\overline{AB}\) when \(AB\) is perpendicular to \(t\) at point \(A\).

20. Let \(x^\circ = m\overline{WX} = m\overline{ZY}\).

Then \(m\overline{WZ} = (200 - x)^\circ\) and
\[m\overline{XY} = (360^\circ - (200 + x)^\circ) = (160 - x)^\circ\]
\[\angle P = \frac{1}{2}(m\overline{WZ} - m\overline{XY})\]
\[= \frac{1}{2}[(200 - x)^\circ - (160 - x)^\circ] = \frac{1}{2}(40) = 20^\circ\]

21. \(m\angle CHD = 180^\circ - 115^\circ = 65^\circ\)
\[m\angle CHD = \frac{1}{2}(m\overline{CD} + m\overline{EA})\]
\[65^\circ = \frac{1}{2}(85^\circ + m\overline{EA})\]
\[130^\circ = 85^\circ + m\overline{EA}\]
\[45^\circ = m\overline{EA}\]
\[m\overline{AF} = m\overline{EA} - m\overline{EF} = 45^\circ - 20^\circ = 25^\circ\]
\[m\angle J = \frac{1}{2}(m\overline{AB} - m\overline{AF})\]
\[30^\circ = \frac{1}{2}(m\overline{AB} - 25^\circ)\]
\[60^\circ = m\overline{AB} - 25^\circ\]
\[85^\circ = m\overline{AB}\]
\[m\overline{FGH} = \frac{1}{2}(m\overline{AB} + m\overline{ED})\]
\[90^\circ = \frac{1}{2}(85^\circ + (20^\circ + m\overline{ED}))\]
\[180^\circ = 105^\circ + m\overline{ED}\]
\[75^\circ = m\overline{ED}\]

Problem Solving

22. \(m\angle A = 80^\circ\)
\[m\angle B = \frac{1}{2}(180^\circ - 30^\circ) = \frac{1}{2}(150^\circ) = 25^\circ\]
\[m\angle B = \frac{1}{2}(80^\circ) = 40^\circ\]

23. \(x^\circ = \frac{1}{2}(180^\circ - 80^\circ) = \frac{1}{2}(100^\circ) = 50^\circ\)

24. \(m\angle B = \frac{1}{2}(80^\circ - x^\circ)\)
\[30^\circ = \frac{1}{2}(80^\circ - x^\circ)\]
\[60^\circ = 80^\circ - x^\circ\]
\[x^\circ = 20^\circ\]

Camera \(B\) will have a 30° view of the stage when the arc measuring 30° is reduced to an arc measuring 20°. You should move the camera closer to the stage.

25.

\[
\sin BCA = \frac{4000}{4001.2} \rightarrow m\angle BCA = 88.6^\circ
\]
\[m\angle BCD = 2(88.6^\circ) = 177.2^\circ\]

Let \(m\overline{BD} = x^\circ\).
\[m\angle BCD = \frac{1}{2}(m\overline{DEB} - m\overline{BD})\]
\[177.2^\circ = \frac{1}{2}[(360^\circ - x^\circ) - x^\circ]\]
\[x = 2.8\]

The measure of the arc from which you can see is about 2.8°.

26.

\[
14^\circ = \frac{1}{2}[(180 - x)^\circ - x^\circ]
\]
\[28 = 180 - 2x\]
\[2x = 152\]
\[x = 76\]

\[(180 - x)^\circ = (180 - 76)^\circ = 104^\circ\]

The measures of the arcs between the ground and the cart are 76° and 104°.

27. Case 1: Given: tangent \(\overline{AC}\) intersects chord \(\overline{AB}\) at point \(A\) on \(\odot Q\). \(\overline{AB}\) contains the center of \(\odot Q\).

Prove: \(m\angle CAB = \frac{1}{2}m\overline{AB}\)

\[\boxed{\begin{align*}
\overline{AC} & \parallel \overline{AB} \\
\text{Case 1:} & \quad m\angle CAB = \frac{1}{2}m\overline{AB}
\end{align*}}\]

Paragraph proof: By Theorem 10.1, \(\overline{CA} \perp \overline{AB}\). By the definition of perpendicular, \(m\angle CAB = 90^\circ\). Because \(\overline{AB}\) is a diameter, \(m\overline{AB} = 180^\circ\). So, \(m\angle CAB = \frac{1}{2}m\overline{AB}\).

Case 2: Given: tangent \(\overline{AC}\) intersects chord \(\overline{AB}\) at point \(A\) on \(\odot Q\). The center of the circle, \(Q\), is in the interior of \(\angle CAB\).

Prove: \(m\angle CAB = \frac{1}{2}m\overline{APB}\)

\[\boxed{\begin{align*}
\overline{AC} & \parallel \overline{AB} \\
\text{Case 2:} & \quad m\angle CAB = \frac{1}{2}m\overline{APB}
\end{align*}}\]

Geometry
Worked-Out Solution Key
Chapter 10, continued

Plan for proof: Draw diameter $AP$. Use the Angle Addition Postulate and Theorem 10.7 to show that $m\angle CAB = 90^\circ + \frac{1}{2} mPB$. Use the Arc Addition Postulate to show that $mPB = 180^\circ + mPB$.

Case 3: Given: Tangent $AC$ intersects chord $AB$ at point $A$ on $\odot Q$. The center of the circle, $Q$, is in the exterior of $\angle CAB$.

Prove: $m\angle CAB = \frac{1}{2} mAB$

Plan for Proof: Draw diameter $AP$. Use the Angle Addition Postulate and Theorem 10.7 to show that $m\angle CAB = 90^\circ - \frac{1}{2} mPB$. Use the Arc Addition Postulate to show that $mAB = 180^\circ - mPB$.


Prove: $m\angle 1 = \frac{1}{2}(mDC + mAB)$

<table>
<thead>
<tr>
<th>Statements</th>
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<tr>
<td>1. Chords $AC$ and $BD$ intersect.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw $BC$.</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $m\angle 1 = m\angle DBC + m\angle ACB$</td>
<td>3. Exterior Angle Theorem</td>
</tr>
<tr>
<td>4. $m\angle DBC = \frac{1}{2} mDC$</td>
<td>4. Theorem 10.7</td>
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<tr>
<td>5. $m\angle ACB = \frac{1}{2} mAB$</td>
<td>5. Theorem 10.7</td>
</tr>
<tr>
<td>6. $m\angle 1 = \frac{1}{2} mDC + \frac{1}{2} mAB$</td>
<td>6. Substitution Property</td>
</tr>
<tr>
<td>7. $m\angle 1 = \frac{1}{2}(mDC + mAB)$</td>
<td>7. Distributive Property</td>
</tr>
</tbody>
</table>

30. Given: $PQ$ and $PR$ are tangents to a circle.

Prove: $QR$ is not a diameter.

Paragraph Proof: Assume $QR$ is a diameter. Then $m\angle QTR = m\angle R = 180^\circ$. By Theorem 10.13, $m\angle P = \frac{1}{2}(m\angle QTR - m\angle R) = \frac{1}{2}(180^\circ - 180^\circ) = 0^\circ$

The $m\angle P$ cannot equal $0^\circ$, so $QR$ cannot be a diameter.

31. $DE = 113, EC = 15$

$DE^2 = DC^2 + EC^2$

$113^2 = DC^2 + 15^2$

$DC = 12, 544 = DC^2$

$112 = DC$

$m\angle DEC = \sin^{-1}\left(\frac{112}{113}\right) = 82.4^\circ$

$m\angle B = m\angle DEC + m\angle DEC$

$= 90^\circ + 82.4^\circ = 172.4^\circ$

$172.4^\circ / 360^\circ = 48\%$

About 48% of the circumference of the bottom pulley is not touching the rope.

Geometry

Worked-Out Solution Key

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Chapter 10, continued

Mixed Review

32. Because \( \frac{CP'}{CP} = \frac{12}{16} = \frac{3}{4} \), the scale factor is \( K = \frac{3}{4} \). This is a reduction.

33. Because \( \frac{CP'}{CP} = \frac{15}{9} = \frac{5}{3} \), the scale factor is \( K = \frac{5}{3} \). This is an enlargement.

34. \( x^2 + 7x + 6 = 0 \)
   \[
x = \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}
   \]
   \[
   = \frac{-7 \pm \sqrt{25}}{2}
   \]
   \[
   = \frac{-7 \pm 5}{2} \rightarrow x = -1 \text{ or } x = -6
   \]

35. \( x^2 - x - 12 = 0 \)
   \[
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}
   \]
   \[
   = \frac{1 \pm \sqrt{49}}{2} = \frac{1 \pm 7}{2} \rightarrow x = 4 \text{ or } x = -3
   \]

36. \( x^2 + 16 = 8x \)
   \[
x^2 - 8x + 16 = 0
   \]
   \[
x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}
   \]
   \[
   = \frac{8 \pm \sqrt{0}}{2} = 4
   \]

37. \( x^2 + 6x = 10 \)
   \[
x^2 + 6x - 10 = 0
   \]
   \[
x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-10)}}{2(1)}
   \]
   \[
   = \frac{-6 \pm \sqrt{36 + 40}}{2} \rightarrow x = 1.36 \text{ or } x = -7.36
   \]

38. \( 5x + 9 = 2x^2 \)
   \[
   -2x^2 + 5x + 9 = 0
   \]
   \[
x = \frac{-5 \pm \sqrt{5^2 - 4(-2)(9)}}{2(-2)}
   \]
   \[
   = \frac{-5 \pm \sqrt{89}}{4} \rightarrow x = -3.71 \text{ or } x = 3.71
   \]

39. \( 4x^2 + 3x - 11 = 0 \)
   \[
x = \frac{-3 \pm \sqrt{3^2 - 4(4)(-11)}}{2(4)}
   \]
   \[
   = \frac{-3 \pm \sqrt{185}}{8} \rightarrow x = 1.33 \text{ or } x = -2.08
   \]

Quiz 10.4–10.5 (p. 686)

1. \( m\angle D + m\angle B = 180^\circ \)
   \( x^2 + 85^\circ = 180^\circ \)
   \( x = 95 \)

2. \( m\angle C + m\angle A = 180^\circ \)
   \( y^\circ + 75^\circ = 180^\circ \)
   \( y = 105 \)

3. \( m\angle ABC = 2m\angle D = 2(95^\circ) = 190^\circ \)
   So, \( z = 190^\circ \)

2. \( m\angle E + m\angle G = 180^\circ \)
   \( x^\circ + 112^\circ = 180^\circ \)
   \( x = 68 \)

3. \( m\angle F + m\angle H = 180^\circ \)
   \( 2m\angle F = 180 \)
   \( m\angle F = 90^\circ \)

4. \( \angle GHE = 2m\angle F = 2(90^\circ) = 180^\circ \)
   So, \( z = 180^\circ \)

5. \( m\angle K + m\angle M = 180^\circ \)
   \( 7x^\circ + 131^\circ = 180^\circ \)
   \( x = 7 \)

6. \( m\angle L + m\angle J = 180^\circ \)
   \( (11x + y)^\circ + 99^\circ = 180^\circ \)
   \( (11(7) + y) + 99 = 180 \)
   \( y = 4 \)

7. \( m\angle F = 2m\angle M = 2(131^\circ) = 262^\circ \)
   So, \( z = 262^\circ \)

8. \( x^\circ = \frac{1}{2}(107^\circ + 83^\circ) = \frac{1}{2}(180^\circ) = 95^\circ \)

9. \( x^\circ = \frac{1}{2}(74^\circ - 22^\circ) = \frac{1}{2}(52^\circ) = 26^\circ \)

11. \( \angle K = 2(\frac{1}{2}(180^\circ - 87^\circ)) = 2(72^\circ) = 144^\circ \)

12. \( \angle E = \frac{1}{2}(360^\circ - x^\circ - x^\circ) = \frac{1}{2}(360^\circ - 2x^\circ - 2x^\circ) \)

The measure of the arc from which you can see is about 3°.
Chapter 10, continued

Mixed Review of Problem Solving (p. 687)

1. a. \( r^2 + 3^2 = (r + 2)^2 \)
   \[ r^2 + 9 = r^2 + 4r + 4 \]
   \[ 0 = 4r - 5 \]
   \[ 5 = 4r \]
   \[ 1.25 = r \]
   The radius of the discus circle is 1.25 meters.

   b. \( 1.25 + 2 = 3.25 \)
   The official is 3.25 meters form the center of the discus circle.

2. \( m\angle XYZ = 360^\circ - m\angle XQZ \)
   \[ = 360^\circ - 199^\circ = 161^\circ \]
   \[ = \frac{1}{2} m\angle XYZ = \frac{1}{2} (161^\circ) = 80.5^\circ \]

3. a. \( 360^\circ \div 3 = 120^\circ \)
   b. \( x = 361 - 2(131) = 361 - 262 = 99 \)
   The distance between the lowest point reached by the blades and the ground is 99 feet.

   e. The wheel itself is 150 - 10 = 140 feet tall. The diameter is the total height, or 140 feet. The radius is half the height, or 70 feet.

4. a. \( 360^\circ \div 40 = 9^\circ \)
   b. \( 72^\circ \div 9^\circ = 8; \) 8 angles and 9 total spokes. There are 7 spokes between the two spokes.

   e. The wheel itself is 150 - 10 = 140 feet tall. The diameter is the total height, or 140 feet. The radius is half the height, or 70 feet.

5. Answers will vary.

6. a. \( x^5 = \frac{1}{2} (m\angle NM + m\angle LK) \)
   \[ = \frac{1}{2} (35^\circ + 93^\circ) = 64^\circ \]

   b. \( m\angle LDK = m\angle MDN = 64^\circ \)
   \( m\angle LDM = 180^\circ - m\angle MDN = 180^\circ - 64^\circ = 116^\circ \)
   \( m\angle KDN = m\angle LDM = 116^\circ \)

7. \( x^5 = \frac{1}{2} (m\angle BC - m\angle AD) \)
   \[ 2x^5 = m\angle BC - m\angle AD \]
   \[ m\angle BC = 2x^5 + m\angle AD \]
   \[ y^5 = \frac{1}{2} (m\angle BC + m\angle AD) \]
   \[ 2y^5 = m\angle BC + m\angle AD \]
   \[ m\angle BC = 2y^5 - m\angle AD \]
   \[ 2m\angle AD = 2y^5 - 2x^5 \]
   \[ m\angle AD = y^5 - x^5 \]

Lesson 10.6

Investigating Geometry Activity 10.6 (p. 688)

1. The products \( AE \cdot CE \) and \( BE \cdot DE \) are the same.

2. The products \( AE \cdot CE \) and \( BE \cdot DE \) are the same.

3. \( AE \cdot CE = BE \cdot DE \)

4. \( PT \cdot QT = RT \cdot ST \)
   \[ 9 \cdot 5 = 15 \cdot ST \]
   \[ 45 = 15ST \]
   \[ 3 = ST \]

10.6 Guided Practice (pp. 690–692)

1. \( 6 \cdot (6 + 9) = 5 \cdot (5 + x) \)
   \[ 90 = 25 + 5x \]
   \[ 3x = 24 \]
   \[ 65 = 5x \]
   \[ x = 8 \]

2. \( 3 \cdot x = 6 \cdot 4 \)
   \[ 90 = 25 + 5x \]
   \[ 3x = 24 \]
   \[ 65 = 5x \]
   \[ x = 8 \]

3. \( 3 \cdot [3 + (x + 2)] = (x + 1) \cdot [(x + 1) + (x - 1)] \)
   \[ 3(x + 5) = (x + 1)(2x) \]
   \[ 3x + 15 = 2x^2 + 2x \]
   \[ 2x^2 - x - 15 = 0 \]
   \[ (2x + 5)(x - 3) = 0 \]
   \[ x = 3 \quad \text{or} \quad x = -\frac{5}{2} \text{ is extraneous.} \]

4. \( x^5 = \frac{1}{2} (3 + 1) \)
   \[ x^5 = 4 \]
   \[ x = 2 \quad \text{or} \quad x = -2 \text{ is extraneous.} \]

5. \( 7^5 = 5 \cdot (5 + x) \)
   \[ 49 = 25 + 5x \]
   \[ 24 = 5x \]
   \[ 5 = x \]

6. \( 12^2 = x \cdot (x + 10) \)
   \[ 144 = x^2 + 10x \]
   \[ x^2 + 10x - 144 = 0 \]
   \[ (x - 8)(x + 18) = 0 \]
   \[ x = 8 \quad \text{or} \quad x = -18 \text{ is extraneous.} \]
Chapter 10, continued

7. Use Theorem 10.16.
   \[ 15^2 = x \cdot (x + 14) \]
   \[ 225 = x^2 + 14x \]
   \[ x^2 + 14x - 225 = 0 \]
   \[ x = \frac{-14 \pm \sqrt{196 - 4(1)(-225)}}{2(1)} \]
   \[ x = \frac{-14 \pm 2\sqrt{274}}{2} \]
   \[ x = -7 \pm \sqrt{274} \]
   \[ x = -7 + \sqrt{274} \]
   \( (x = -7 - \sqrt{274} \text{ is extraneous}) \)

   \[ x \cdot 18 = 9 \cdot 16 \]
   \[ 18x = 144 \]
   \[ x = 8 \]

9. Use Theorem 10.15.
   \[ 18 \cdot (18 + 22) = x \cdot (x + 29) \]
   \[ 720 = x^2 + 29x \]
   \[ x^2 + 29x - 720 = 0 \]
   \[ (x - 16)(x + 45) = 0 \]
   \[ x = 16 \] \( (x = -45 \text{ is extraneous}) \)

10. \( EC \) cannot be equal to \( EA \) because that would make \( EC \) tangent to the circle. \( EC \) must be less than \( EA \).

11. It is appropriate to use the approximation symbol in the solution to Example 4 because it is stated in the problem that each moon has a nearly circular orbit.

10.6 Exercises (pp. 692–695)

Skill Practice

1. The part of the secant segment that is outside the circle is called an external segment.

2. A tangent segment intersects the circle in only one point while the secant segment intersects the circle in two points.

3. 12 \( \times \) \( x = 10 \cdot 6 \)
   \[ 12x = 60 \]
   \[ x = 5 \]

4. 9 \( \times \) (x - 3) = 10 \( \cdot \) 18
   \[ 9x - 27 = 180 \]
   \[ 9x = 207 \]
   \[ x = 23 \]

5. \( x \cdot (x + 8) = 6 \cdot 8 \)
   \[ x^2 + 8x = 48 \]
   \[ x^2 + 8x - 48 = 0 \]
   \[ (x - 4)(x + 12) = 0 \]
   \[ x = 4 \] \( (x = -12 \text{ is extraneous}) \)

6. 8 \( \times \) (x + 3) = 6 \( \cdot \) (6 + 10)
   \[ 64 + 8x = 96 \]
   \[ 8x = 32 \]
   \[ x = 4 \]

7. \( x \cdot (x + 4) = 5 \cdot (5 + 7) \)
   \[ x^2 + 4x = 60 \]
   \[ x^2 + 4x - 60 = 0 \]
   \[ (x - 6)(x + 10) = 0 \]
   \[ x = 6 \] \( (x = -10 \text{ is extraneous}) \)

8. \( (x - 2) \cdot [(x - 2) + (x + 4)] = 4 \cdot (4 + 5) \)
   \[ (x - 2)(2x + 2) = 36 \]
   \[ 2x^2 - 2x - 4 = 36 \]
   \[ 2x^2 - 2x - 40 = 0 \]
   \[ x^2 - x - 20 = 0 \]
   \[ (x - 5)(x + 4) = 0 \]
   \[ x = 5 \] \( (x = -4 \text{ is extraneous}) \)

9. \( x^2 = 9 \cdot (9 + 7) \)
   \[ x^2 = 144 \]
   \[ x = 12 \] \( (x = -12 \text{ is extraneous}) \)

10. \( 24^2 = 12 \cdot (12 + x) \)
    \[ 576 = 144 + 12x \]
    \[ 432 = 12x \]
    \[ 36 = x \]

11. \( (x + 4)^2 = x \cdot (x + 12) \)
    \[ x^2 + 8x + 16 = x^2 + 12x \]
    \[ 16 = 4x \]
    \[ 4 = x \]

12. The error is that the wrong segment lengths are being multiplied. Use Theorem 10.15.
    \[ FD \cdot FC = FA \cdot FB \]
    \[ 4 \cdot (4 + CD) = 3 \cdot (3 + 5) \]
    \[ 16 + 4CD = 24 \]
    \[ 4CD = 8 \]
    \[ CD = 2 \]

13. 15 \( \cdot \) (x + 3) = 2x \( \cdot \) 12
    14. \( x \cdot 45 = 27.50 \)
    \[ 15x + 45 = 24x \]
    \[ 45x = 1350 \]
    \[ 45 = 9x \]
    \[ x = 30 \]
    \[ 5 = x \]

15. \( (\sqrt{3})^2 = x \cdot (2 + x) \)
    \[ 3 = 2x + x^2 \]
    \[ x^2 + 2x - 3 = 0 \]
    \[ (x - 1)(x + 3) = 0 \]
    \[ x = 1 \] \( (x = -3 \text{ is extraneous}) \)

16. D:
    \[ x \cdot x = 2 \cdot (2x + 6) \]
    \[ x^2 = 4x + 12 \]
    \[ x^2 - 4x - 12 = 0 \]
    \[ (x - 6)(x + 2) = 0 \]
    \[ x = 6 \] \( (x = -2 \text{ is extraneous}) \)

17. \( MN^2 = PN \cdot (PN + PQ) \)
    \[ 12^2 = 6 \cdot (6 + PQ) \]
    \[ 144 = 36 + 6PQ \]
    \[ 108 = 6PQ \]
    \[ 18 = PQ \]
Chapter 10, continued

18. \(RQ^2 = RS \cdot (RS + SP)\)
   \(RQ^2 = 14 \cdot (14 + 12)\)
   \(RQ^2 = 364\)
   \(PQ^2 + RQ^2 = RP^2\)
   \(PQ^2 + 364 = 26^2\)
   \(PQ^2 = 312\)
   \(PQ = 17.7\)

19. \(AB^2 = BC \cdot (BC + CD + DE)\)
   \(12^2 = 8 \cdot (8 + CD + 6)\)
   \(144 = 112 + 8CD\)
   \(4 = CD\)

\(CD \cdot DE = (r + PD) \cdot (r - PD)\)
\(4 \cdot 6 = (r + 4) \cdot (r - 4)\)
\(24 = r^2 - 16\)
\(40 = r^2\)
\(2\sqrt{10} = r\)

Problem Solving

20. \(x \cdot x = 62 \cdot (250 - 62)\)
    \(x^2 = 11,656\)
    \(x = 107.96\)

The distance from the end of the passage to either side of
the mound is about 108 feet.

21. Given: \(AB\) and \(CD\) are chords
derectangle at \(E\).

Prove: \(EA \cdot EB = EC \cdot ED\)

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<tr>
<td>1. (AB) and (CD) are chords that intersect at (E).</td>
<td>1. Given</td>
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<tr>
<td>2. Draw (BD) and (AC).</td>
<td>2. Two points determine a line.</td>
</tr>
<tr>
<td>3. (\angle C \cong \angle B)</td>
<td>3. (\angle C) and (\angle B) intercept the same arc.</td>
</tr>
<tr>
<td>4. (\angle A \cong \angle D)</td>
<td>4. (\angle A) and (\angle D) intercept the same arc.</td>
</tr>
<tr>
<td>5. (\triangle AEC \sim \triangle DEB)</td>
<td>5. AA Similarity Postulate</td>
</tr>
<tr>
<td>6. (\frac{EA}{ED} = \frac{EC}{EB})</td>
<td>6. Corresponding sides are proportional.</td>
</tr>
<tr>
<td>7. (EA \cdot EB = EC \cdot ED)</td>
<td>7. Cross Product Property</td>
</tr>
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</table>

22. \(AD \cdot (AD + DE) = AB \cdot (AB + BC)\)
    \((AD)^2 + (AD)(DE) = (AB)^2 + (AB)(BC)\)
    \((AD)^2 + (AD)(DE) - (AB)^2 = (AB)(BC)\)
    \[(AD)^2 + (AD)(DE) - (AB)^2\]
    \(AB\)

23. Given: \(EA\) is a tangent segment and \(ED\) is
    a secant segment passing through the center.

Prove: \((EA)^2 = EC \cdot ED\)

Proof: Draw radius \(AT\). By Theorem 10.1, \(EA \perp AT\). By
the Pythagorean Theorem, \((EA)^2 + AT^2 = ET^2\). By the
Segment Addition Postulate, \(ET = EC + CT\). So,
\((EA)^2 + AT^2 = (EC + CT)^2\) by substitution. Simplify the
equation.

\((EA)^2 + AT^2 = (EC + CT)^2\)
\((EA)^2 + AT^2 = EC^2 + (EC)(CT) + CT^2\)
\(AT\) and \(CT\) are both radii, so they are equal. Substitute
\(CT\) for \(AT\).

\((EA)^2 + CT^2 = EC^2 + 2(EC)(CT) + CT^2\)
\((EA)^2 = EC(EC + 2CT)\)
\((EA)^2 = EC(EC + CD)\)
\((EA)^2 = EC \cdot ED\)

24. \(4 \cdot CN = 6 \cdot 8\)
    \(CN = 12\)

The length \(CN\) is 12 centimeters. Sparkles travel from
\(C\) to \(D\) in \(6\) cm \(= 3\) seconds. So, sparkles must travel
from \(C\) to \(N\) in 3 seconds. The sparkles must travel at a
rate of \(12\) cm \(= 4\) cm/sec.

25. Given: \(EB\) and \(ED\) are
    secant segments.

Prove: \(EA \cdot EB = EC \cdot ED\)

Paragraph proof: Draw \(\overline{AD}\) and \(\overline{BC}\). \(\angle B\) and \(\angle D\)
intercept the same arc, so \(\angle B \cong \angle D\). \(\angle E \cong \angle E\) by the
Reflexive Property of Congruence, \(\triangle BCE \sim \triangle DAE\)
by the AA Similarity Theorem. Then, since lengths of
corresponding sides of similar triangles are proportional,
\(\frac{EA}{ED} = \frac{EC}{EB}\). By the Cross Product Property,
\(EA \cdot EB = EC \cdot ED\).
Chapter 10, continued

26. Given: $\overline{EA}$ is a tangent segment. $\overline{ED}$ is a secant segment.
    Prove: $EA^2 = EC \cdot ED$

Paragraph proof: Draw $\overline{AC}$ and $\overline{AD}$. $\angle D = \frac{1}{2} \angle AC$ and $\angle EAC = \frac{1}{2} \angle AC$. So, $\angle D \cong \angle EAC$. $\angle E \cong \angle E$ by the Reflexive Property of Congruence, so $\triangle DAE \cong \triangle ACE$ by the AA Similarity Theorem. Then, since lengths of corresponding sides of similar triangles are proportional, $\frac{EA}{EC} = \frac{ED}{EA}$. By the Cross Product Property, $(EA)^2 = EC \cdot ED$.

27. a. $m \angle CAB = \frac{1}{2} m \angle BD = \frac{1}{2} (120^\circ) = 60^\circ$
    b. $m \angle CAB = m \angle EFD = 60^\circ$, so $\angle CAB \cong \angle EFD$. Also, $\angle ACB \cong \angle FCE$ by the Vertical Angles Theorem. So, $\triangle ABC \cong \triangle FEC$ by the AA Similarity Theorem.
    c. $\overline{EF} = \overline{FC}$
    $\overline{RA} = \overline{AC}$
    $\frac{y}{3} = \frac{x + 10}{6}$
    $y = \frac{3(x + 10)}{6}$
    $y = \frac{x + 10}{2}$
    d. Use Theorem 10.16.
    $EF^2 = FD \cdot FA$
    $y^2 = x \cdot (x + 10 + 6)$
    $y^2 = x(x + 16)$
    e. $y^2 = x(x + 16)$
    $\left(\frac{x + 10}{2}\right)^2 = x(x + 16)$
    $x^2 + 50x + 100 = 4x^2 + 64x$
    $3x^2 + 44x - 100 = 0$
    $3x + 50 = 0$
    $x = 2$ or $3x + 50 = 0$
    $x = -\frac{50}{3}$
    Length cannot be negative, so $x = 2$.
    $y = \frac{x + 10}{2} = \frac{2 + 10}{2} = 6$
    f. $\frac{EF}{AB} = \frac{6}{3} = \frac{2}{1}$, so the ratio of $\triangle FEC$ to $\triangle ABC$ is 2 to 1.
    So, $\frac{CF}{CB} = \frac{2}{1}$.
    Let $CE = 2x$ and $CB = x$.
    By Theorem 10.14,

$CE \cdot CB = AC \cdot CD$

$2x \cdot x = 6 \cdot 10$
$2x^2 = 60$
$x^2 = 30$
$x = \pm \sqrt{30}$

Length cannot be negative, so $x = \sqrt{30}$.
$CE = 2x = 2\sqrt{30}$

28. The distance from the origin to $P'$ is 5 (Pythagorean Theorem). $ON$ is the diameter, so its length is 2.

$(NP')^2 = (ON)^2 + (OP')^2 = 2^2 + 5^2 = 29$
$NP' = \sqrt{29}$. By Theorem 10.16,

$(OP')^2 = PP' \cdot NP'$

$\frac{25}{\sqrt{29}} = PP' = \frac{25\sqrt{29}}{29}$

The distance $d$ is $\frac{25\sqrt{29}}{29}$ units.

Mixed Review

29. $\sqrt{(-10)^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6$
30. $\sqrt{-5 + (-4) + (6 - 1)^2} = \sqrt{-9 + 5^2}$
    $= \sqrt{-9 + 25}$
    $= \sqrt{16} = 4$
31. $\sqrt{[(-2 - (-6))^2 + (3 - 6)^2]} = \sqrt{4^2 + (-3)^2}$
    $= \sqrt{16 + 9} = \sqrt{25} = 5$
32. $\cos 40^\circ = \frac{8}{QR} \rightarrow QR = \frac{8}{\cos 40^\circ} \approx 10.4$
    $\tan 50^\circ = \frac{8}{PR} \rightarrow PR = \frac{8}{\tan 50^\circ} \approx 6.7$
33. $FC^2 = EC^2 + EF^2 = 5^2 + 8^2 = 89$
    $FC = \sqrt{89}$
34. $\overline{mAB} = m \angle AFB = 135^\circ$
35. $\overline{mCD} = m \angle CFD = 60^\circ$
36. $\overline{mBCA} = 360^\circ - \overline{mAB} = 360^\circ - 135^\circ = 225^\circ$
37. $\overline{mCBD} = 360^\circ - \overline{mCD} = 360^\circ - 60^\circ = 300^\circ$
38. $\overline{CD}$ is a semicircle, so $\overline{mCD} = 180^\circ$.
39. $\overline{BAE}$ is a semicircle, so $\overline{mBAE} = 180^\circ$.
40. $\overline{AB}$ does not bisect chord $\overline{RS}$, so $\overline{AB}$ is not a diameter.
Chapter 10, continued

41. \(10^2 = 8^2 + x^2\)
   \[100 = 64 + x^2\]
   \[36 = x^2\]
   \[x = 6\]
   \(AB\) bisects \(CD\) and is perpendicular to \(CD\), so \(AB\) is a diameter.

42. \(AB\) does not bisect chord \(CD\), so \(AB\) is not a diameter.

Problem Solving Workshop 10.6 (p. 696)

1. \(RS = \frac{RQ}{RT}\)
   \[4 = \frac{RQ}{4 + 9}\]
   \[4 = \frac{RQ}{13}\]
   \[RQ = 52\]
   \[RQ = \sqrt{52}\]
   \[RQ = 2\sqrt{13}\]

2. a. \(m\angle BDE = \frac{1}{2} m\angle BCD\)
   \(m\angle AEB = 180^\circ - m\angle BDE = 180^\circ - \frac{1}{2} m\angle BCD\)
   \(m\angle ACD = \frac{1}{2} m\angle BDE\)
   \[= \frac{1}{2} (360^\circ - m\angle BCD) = 180^\circ - \frac{1}{2} m\angle BCD\]
   So, \(m\angle AEB = m\angle ACD\) or \(\angle AEB \equiv \angle ACD\). Also \(\angle A \equiv \angle A\) by the Reflexive Property of Congruence. So, \(\triangle AEB \sim \triangle ACD\) by the AA Similarity Theorem.

b. \(\frac{AC}{AE} = \frac{AD}{AB}\)
   \[15 + 5 = \frac{12 + DE}{15}\]
   \[20 = \frac{12 + DE}{15}\]
   \[300 = 144 + 12DE\]
   \[156 = 12DE\]
   \[13 = DE\]

3. \(7^2 = 5(5 + x)\)
   \[49 = 25 + 5x\]
   \[24 = 5x\]
   \[\frac{24}{5} = x\]

4. \(x(x + w) = y(y + z)\)
   \[x + w = \frac{y(y + z)}{x}\]
   \[w = \frac{y(y + z)}{x} - x\]

10.6 Extension (pp. 697–698)

1.

2. \(1\text{ in.}\)

3. \(1\text{ in.}\)

4. \(1\text{ in.}\)

5. The locus of points consists of two points on \(l\) each 3 centimeters away from \(P\).

6. The locus of points consists of four points on a circle with center at \(Q\) and a radius of 5 centimeters. The four points are the intersections of the circle and two lines parallel to and 3 centimeters away from \(m\).

7. The locus of points consists of a semi-circle centered at \(R\) with radius 10 centimeters. The diameter bordering the semi-circle is 10 centimeters from \(k\) and parallel to \(k\).

8. The portions of line \(l\) and \(m\) that are no more than 8 centimeters from point \(P\) and the portion of the circle, including its interior, with center \(P\) and radius 8 centimeters that is between lines \(l\) and \(m\).
Chapter 10, continued

9. The center is (4, 2) and the radius is 3 ft.

Lesson 10.7

10.7 Guided Practice (pp. 700–701)

1. \(x^2 + y^2 = r^2\)
   \(x^2 + y^2 = 2.5^2\)
   \(x^2 + y^2 = 6.25\)

2. \((x - h)^2 + (y - k)^2 = r^2\)
   \((x - (-2))^2 + (y - 3)^2 = 7^2\)
   \((x + 2)^2 + (y - 5)^2 = 49\)

3. \(r = \sqrt{(3 - 1)^2 + (4 - 4)^2}\)
   \(= \sqrt{2^2 + 0^2} = 2\)
   \((x - h)^2 + (y - k)^2 = r^2\)
   \((x - 1)^2 + (y - 4)^2 = 2^2\)
   \((x - 1)^2 + (y - 4)^2 = 4\)

4. \(r = \sqrt{(-1 - 2)^2 + (2 - 6)^2}\)
   \(= \sqrt{(-3)^2 + (-4)^2} = 5\)
   \((x - h)^2 + (y - k)^2 = r^2\)
   \((x - 2)^2 + (y - 6)^2 = 25\)

5. \((x - 4)^2 + (y + 3)^2 = 16\)
   \((x - 4)^2 + (y - (-3))^2 = 4^2\)
   The center is (4, -3) and the radius is 4.

6. \((x + 8)^2 + (y + 5)^2 = 121\)
   \((x - (-8))^2 + (y - (-5))^2 = 11^2\)
   The center is at (-8, -5) and the radius is 11.

7. Three seismographs are needed to locate an earthquake’s epicenter because two circles intersect in two points. You would not know which point is the epicenter. You need the third circle to determine which point is the epicenter.

10.7 Exercises (pp. 702–705)

Skill Practice

1. The standard equation of a circle can be written for any circle with known center and radius.

2. The location of the center and one point on a circle is enough information to draw the rest of the circle because the distance from the center to the known point is the radius of the circle.

3. The radius is 2.
   \(x^2 + y^2 = 4\)

4. The center is (2, 3). The radius is 2.
   \((x - 2)^2 + (y - 3)^2 = 4\)

5. The radius is 20.
   \(x^2 + y^2 = 20^2\)
   \(x^2 + y^2 = 400\)

6. The center is (5, 0). The radius is 10.
   \((x - 5)^2 + (y - 0)^2 = 10^2\)
   \((x - 5)^2 + y^2 = 100\)

7. The center is (50, 50). The radius is 10.
   \((x - 50)^2 + (y - 50)^2 = 10^2\)
   \((x - 50)^2 + (y - 50)^2 = 100\)

8. The center is (-3, -3). The radius is 9.
   \((x - (-3))^2 + (y - (-3))^2 = 9^2\)
   \((x + 3)^2 + (y + 3)^2 = 81\)

9. \(x^2 + y^2 = 7^2\)
   \(x^2 + y^2 = 49\)

10. \((x - (-4))^2 + (y - 1)^2 = 1^2\)
    \((x + 4)^2 + (y - 1)^2 = 1\)

11. \((x - 7)^2 + (y - (-6))^2 = 8^2\)
    \((x - 7) + (y + 6)^2 = 64\)

12. \((x - 4)^2 + (y - 1)^2 = 5^2\)
    \((x - 4) + (y - 1)^2 = 25\)

13. \((x - 3)^2 + (y - (-5))^2 = 7^2\)
    \((x - 3)^2 + (y + 5)^2 = 49\)

14. \((x - (-3))^2 + (y - 4)^2 = 5^2\)
    \((x + 3)^2 + (y - 4)^2 = 25\)

15. If \((h, k)\) is the center of a circle with radius \(r\), the equation of the circle should be \((x - h)^2 + (y - k)^2 = r^2\) not \((x + h)^2 + (y + k)^2 = r^2\).

16. \(C, r^2 = 16 \text{ so } r = 4.\)
    \(d = 2r = 2(4) = 8\)
Chapter 10, continued

17. \( r = \sqrt{(0 - 0)^2 + (0 - 6)^2} = \sqrt{(-6)^2} = 6 \)
\[ x^2 + y^2 = 36 \]

18. \( r = \sqrt{(1 - 4)^2 + (2 - 2)^2} = \sqrt{(-3)^2} = 3 \)
\[ (x - 1)^2 + (y - 2)^2 = 3^2 \]
\[ (x - 1)^2 + (y - 2)^2 = 9 \]

19. \( r = \sqrt{(-3 - 1)^2 + (5 - 8)^2} \)
\[ = \sqrt{(-4)^2 + (-3)^2} = 5 \]
\[ (x - (-3))^2 + (y - 5)^2 = 5^2 \]
\[ (x + 3)^2 + (y - 5)^2 = 25 \]

20. \( x^2 + y^2 = 49 \)  
The center is (0, 0).  The radius is 7.

21. \( (x - 3)^2 + y^2 = 16 \)  
The center is (3, 0).  The radius is 4.

22. \( x^2 + (y + 2)^2 = 36 \)  
The center is (0, -2).  The radius is 6.

23. \( (x - 4)^2 + (y - 1)^2 = 1 \)  
The center is (4, 1).  The radius is 1.

24. \( (x + 5)^2 + (y - 3)^2 = 9 \)  
The center is (-5, 3).  The radius is 3.

25. \( (x + 2)^2 + (y + 6)^2 = 25 \)  
The center is (-2, -6).  The radius is 5.

26. \( D; (x + 2)^2 + (y - 4)^2 = 25 \)  
\( (0 + 2)^2 + (5 - 4)^2 \neq 25 \)
\[ 2^2 + 1^2 \neq 25 \]
5 \neq 25

27. \( x^2 + y^2 - 6y + 9 = 4 \)  
\[ x^2 + (y^2 - 6y + 9) = 4 \]
\[ x^2 + (y - 3)^2 = 4 \]
The equation is a circle.

28. \( x^2 - 8x + 16 + y^2 + 2y + 4 = 25 \)  
\( (x^2 - 8x + 16) + (y^2 + 2y + 1) = 25 - 3 \)
\[ (x - 4)^2 + (y + 1)^2 = 22 \]
The equation is a circle.

29. \( x^2 + y^2 + 4y + 3 = 16 \)  
\[ x^2 + (y^2 + 4y + 3 + 1) = 16 + 1 \]
\[ x^2 + (y + 2)^2 = 17 \]
The equation is a circle.

30. \( x^2 - 2x + 5 + y^2 = 81 \)  
\[ (x^2 - 2x + 5 - 4) + y^2 = 81 - 4 \]
\[ (x^2 - 2x + 1) + y^2 = 77 \]
\[ (x - 1)^2 + y^2 = 77 \]
The equation is a circle.

31. \( (x - 4)^2 + (y - 3)^2 = 9 \)  
The center is (4, 3).
\[ y = -3x + 1 \]
\[ 3 \neq -3(4) + 6 \]
\[ 3 \neq -6 \]
The line does not contain the center of the circle. Solve for y in both equations and set them equal to find points of intersection.
\[ (x - 4)^2 + (y - 3)^2 = 9 \]
\[ (y - 3)^2 = 9 - (x - 4)^2 \]
\[ y - 3 = \sqrt{9 - (x - 4)^2} \]
\[ y = 3 + \sqrt{9 - (x - 4)^2} \]
\[ -3x + 6 = 3 + \sqrt{9 - (x - 4)^2} \]
\[ -3x + 3 = \sqrt{9 - (x - 4)^2} \]
\[ (-3x + 3)^2 = 9 - (x - 4)^2 \]
\[ 9x^2 - 18x + 9 = 9 - x^2 + 8x - 16 \]
\[ 10x^2 - 26x + 16 = 0 \]
\[ 5x^2 - 13x + 8 = 0 \]
Chapter 10, continued

Use the quadratic equation to solve for \( x \).

\[
\begin{align*}
x & = \frac{-13 \pm \sqrt{(-13)^2 - 4(5)(8)}}{2(5)} \\
x & = \frac{-13 \pm \sqrt{169 - 160}}{10} \\
x & = \frac{-13 \pm 3}{10} \rightarrow x = 1 \text{ or } x = 1.6
\end{align*}
\]

Because \( x \) has two solutions, the line intersects the circle at two points. The line is a secant.

32. \((x + 2)^2 + (y - 2)^2 = 16\)

The center is \((-2, 2)\).

\[
\begin{align*}
y & = 2x - 4 \\
2 & \neq 2(-2) - 4 \Rightarrow 2 \neq -8
\end{align*}
\]

The line does not contain the center of the circle. Solve for \( y \) in both equations and set them equal to find points of intersection.

\[
\begin{align*}
(x + 2)^2 + (y - 2)^2 & = 16 \\
(y - 2)^2 & = 16 - (x + 2)^2 \\
y & = \sqrt{16 - (x + 2)^2} \\
2x - 6 & = \sqrt{16 - (x + 2)^2} \\
4x^2 - 24x + 36 & = 16 - x^2 - 4x - 4 \\
5x^2 - 20x + 24 & = 0
\end{align*}
\]

Use the quadratic equation to solve for \( x \).

\[
\begin{align*}
x & = \frac{20 \pm \sqrt{(-20)^2 - 4(5)(24)}}{2(5)} \\
x & = \frac{20 \pm \sqrt{400 - 480}}{10} \\
x & = \frac{20 \pm \sqrt{-80}}{10}
\end{align*}
\]

The square root of a negative number does not exist. There are no solutions, so the line does not intersect the circle. The line is not a secant or a tangent.

33. \((x - 5)^2 + (y + 1)^2 = 4\)

The center is \((5, -1)\).

\[
\begin{align*}
y & = \frac{1}{5}x - 3 \\
-1 & \neq \frac{1}{5}(5) - 3 \Rightarrow -1 \neq -2
\end{align*}
\]

The line does not contain the center of the circle.

At \( x = 5, \)

\[
\begin{align*}
y & = \frac{1}{5}x - 3 = \frac{1}{5}(5) - 3 = -2
\end{align*}
\]

The point \((5, -2)\) is in the interior of the circle, so the line is a secant.

34. \((x + 3)^2 + (y - 6)^2 = 25\)

The center is \((-3, 6)\).

\[
\begin{align*}
y & = \frac{4}{3}x + 2 \\
6 & \neq \frac{4}{3}(-3) + 2 \\
6 & = 6
\end{align*}
\]

The line contains the center of the circle, so it is a secant that contains a diameter.

35. Let radius \( O \) be \( k \), then center \( O : (15k, 0) \), point: \((63, 16)\).

\[
\begin{align*}
r & = \sqrt{(15k - 63)^2 + (0 - 16)^2} \\
r & = \sqrt{225k^2 - 1890k + 4225} \\
4k & = \sqrt{225k^2 - 1890k + 4225} \\
16k^2 & = 225k^2 - 1890k + 4225 \\
0 & = 209k^2 - 1890k + 4225 \\
k & = \frac{1890 \pm 200}{418} \Rightarrow k = 5 \text{ or } k = 4.04
\end{align*}
\]

\( k = 5 \) because \( k \) is an integer.

\[
\begin{align*}
(x - 15)^2 + y^2 & = 100
\end{align*}
\]

Problem Solving

36. a.

b. \((3, 4)\) is in Zone 2.

\((6, 5)\) is in Zone 3.

\((1, 2)\) is in Zone 1.

\((0, 3)\) is in Zone 1.

\((1, 6)\) is in Zone 2.

37. \( r = \frac{1}{2}d = \frac{1}{2}(4.8) = 2.4 \)

\[
\begin{align*}
x^2 + y^2 & = 2.4^2 \\
x^2 + y^2 & = 5.76 \\
r & = \frac{1}{2}d = \frac{1}{2}(0.6) = 0.3 \\
x^2 + y^2 & = 0.3^2 \\
x^2 + y^2 & = 0.09
\end{align*}
\]

38. center: \((-3, 0)\)

radius: \( AD = 1 \)

\[
\begin{align*}
(x - (-3))^2 + (y - 0)^2 & = 1^2 \\
(x + 3)^2 + y^2 & = 1
\end{align*}
\]
Chapter 10, continued

39. center: (3, 0)
radius: \( BD = 7 \)
\[
(x - 3)^2 + (y - 0)^2 = 7^2
\]
\[
(x - 3)^2 + y^2 = 49
\]

40. Answers will vary.

41. The height (or width) always remains the same as the figure is rolled on its edge.

42. a. Yes, cell towers \( X \) and \( Y \) cover part of the same area, so this area may receive calls from Tower \( X \) or Tower \( Y \).

b. Your home is within range of Tower \( Y \), but your school is not in the range of any tower. So, you can use your phone at home.

c. City \( B \) has better cell phone coverage because it is located entirely within the range of Tower \( Z \). City \( A \) is only partially covered by Tower \( X \) and Tower \( Y \).

43. a. The center \( C \) can be found at the intersection of the lines perpendicular to the tangents.

\[
y = -\frac{4}{3}x + b
\]
\[
5 = -\frac{4}{3}(1) + b
\]
\[
\frac{31}{3} = b
\]
\[
y = -\frac{4}{3}x + \frac{31}{3}
\]
\[
y = \frac{4}{3}x + b
\]
\[
13 = -\frac{4}{3}(4) + b
\]
\[
\frac{23}{3} = b
\]
\[
y = \frac{4}{3}x + \frac{23}{3}
\]
\[
-\frac{4}{3}x + \frac{31}{3} = \frac{4}{3}x + \frac{23}{3}
\]
\[
-4x + 31 = 4x + 23
\]
\[
8 = 8x
\]
\[
1 = x
\]
\[
y = \frac{4}{3}x + \frac{23}{3} - \frac{4}{3}(1) + \frac{23}{3} = 9
\]
The center \( C \) is at \((1, 9)\).

\[
r = \sqrt{(1 - 4)^2 + (9 - 5)^2} = \sqrt{(-3)^2 + 4^2} = 5
\]

44. Given: A circle passing through the points \((-1, 0)\) and \((1, 0)\).

Prove: The equation of the circle is \( x^2 - 2yk + y^2 = 1 \) with center at \((0, k)\).

Construct the perpendicular bisector of chord with the endpoints \((-1, 0)\) and \((1, 0)\). Using Theorem 10.4, this new chord is a diameter of the circle. Since the new chord is a segment of the \( y \)-axis, the center of the circle is located at some point \((0, k)\) which makes the equation of the circle \( x^2 + (y - k)^2 = r^2 \) or \( x^2 + y^2 - 2yk = r^2 - k^2 \). Now consider the right triangle whose vertices are \((0, 0), (0, k), \) and \((1, 0)\) with the distance from \((0, k)\) to \((1, 0)\) being \( r \), the radius of the circle. Using the Pythagorean Theorem, you get \( k^2 + 1^2 = r^2 \) or \( r^2 = k^2 + 1 \).

Substituting you get \( x^2 - 2yk + y^2 = 1 \).

45. a. If \( r = 2 \), there are two possible points of intersection.

Use Theorem 10.1. If the center of the circle is \((8, 8)\), then the point of intersection for \( m \) and \( n \) is \((10, 6)\). If the center of the circle is \((10, 6)\), then the point of intersection is \((8, 8)\). If \( r = 10 \), these are also two possible points of intersection. \( C \) has two possible centers.

To find these centers, set the equation of the circle with radius 10 and center \((8, 8)\) equal to the equation of the circle with radius 10 and center \((10, 8)\).

\[
r_m = 10 \quad \text{center} (8, 6) \quad (x - 8)^2 + (y - 6)^2 = 100
\]
\[
\rightarrow y = 6 + \sqrt{100 - (x - 8)^2}
\]
\[
r_n = 10 \quad \text{center} (10, 8) \quad (x - 10)^2 + (y - 8)^2 = 100
\]
\[
\rightarrow y = 8 + \sqrt{100 - (x - 10)^2}
\]
\[
6 + \sqrt{100 - (x - 8)^2} = 8 + \sqrt{100 - (x - 10)^2}
\]
\[
100 - (x - 8)^2 = (2 + \sqrt{100 - (x - 10)^2})^2
\]
\[
100 - (x - 8)^2 = 4 + 4\sqrt{100 - (x - 10)^2} + 100 - (x - 10)^2
\]
\[
\frac{(x - 10)^2 - (x - 8)^2}{4} = \sqrt{100 - (x - 10)^2}
\]
\[
\frac{-4x + 32}{4} = \sqrt{100 - (x - 10)^2}
\]
\[
(x + 8)^2 = 100 - (x - 10)^2
\]
\[
x^2 - 18x + 32 = 0
\]
\[
(x - 2)(x - 16) = 0
\]
\[
x = 2 \quad \text{or} \quad x = 16
\]
Chapter 10, continued

Possible centers for \( \bigcirc C \) are (2, 14) and (16, 0). If the center of \( \bigcirc C \) is (16, 0), then line \( m \) is \( y = \frac{4}{3} x - \frac{14}{3} \) and line \( n \) is \( y = \frac{3}{4} x + \frac{1}{2} \):

\[
\frac{4}{3} x - \frac{14}{3} = \frac{3}{4} x + \frac{1}{2}
\]

\[
16x - 56 = 9x + 6
\]

\[
x = \frac{6}{7} \rightarrow y = \frac{7}{7} \]

The point of intersection is \( \left( \frac{6}{7}, \frac{7}{7} \right) \).

If the center of \( \bigcirc C \) is (2, 14), then line \( m \) is \( y = \frac{3}{4} x \) and line \( n \) is \( y = \frac{4}{3} x - \frac{16}{3} \):

\[
\frac{3}{4} x = \frac{4}{3} x - \frac{16}{3}
\]

\[
9x = 16x - 64
\]

\[
x = 9 \rightarrow y = \frac{6}{7}
\]

The point of intersection is \( \left( \frac{9}{7}, \frac{6}{7} \right) \).

The points of intersection and the center of \( \bigcirc C \) are collinear.

b. The locus of intersection points of \( m \) and \( n \) for all possible values of \( r \) for \( \bigcirc C \) is the intersection of all circles centered (8, 6) and (10, 8):

\[
(x - 8)^2 + (y - 6)^2 = (x - 10)^2 + (y - 8)^2
\]

\[
x^2 - 16x + 64 + y^2 - 12y + 36
\]

\[
= x^2 - 20x + 100 + y^2 - 16y + 64
\]

\[
4x + 4y - 64 = 0
\]

\[
x + y - 16 = 0
\]

Mixed Review

46. \( P = 2l + 2w = 2(22) + 2(9) = 44 + 18 = 62 \) in.

47. \( P = 4s = 4(18) = 72 \) ft

48. \( x^2 + 40^2 = 57^2 \)

\[
x^2 = 1649
\]

\[
x = 40.6
\]

\[
P = 40.6 + 40 + 57 = 137.6 \) m
\]

49. \( C = 2\pi r = 2(3.14)(17) = 43.96 \) cm

50. \( C = \pi d = (3.14)(160) = 502.4 \) in.

51. \( C = \pi d = (3.14)(48) = 150.72 \) yd

52. \( 15^2 + r^2 = (9 + r)^2 \)

\[
225 + r^2 = 81 + 18r + r^2
\]

\[
144 = 18r
\]

\[
8 = r
\]

53. \( r^2 + 21^2 = (r + 15)^2 \)

\[
r^2 + 441 = r^2 + 30r + 225
\]

\[
216 = 30r
\]

\[
7.2 = r
\]

54. \( r^2 + 28^2 = (r + 20)^2 \)

\[
r^2 + 784 = r^2 + 40r + 400
\]

\[
384 = 40r
\]

\[
9.6 = 4
\]

Quiz 10.6–10.7 (p. 705)

1. \( 6 \cdot x = 8 \cdot 9 \)

\[
6x = 72
\]

\[
x = 12
\]

2. \( 7 \cdot (7 + 5) = 6 \cdot (6 + x) \)

\[
7(12) = 36 + 6x
\]

\[
x = 12
\]

3. \( 16^2 = 12(x + 12) \)

\[
256 = 12x + 144
\]

\[
x = 12
\]

4. center: (1, 4)

radius: 6

\[
112 = 12x
\]

\[
x = 9.3
\]

5. center: (5, 7)

point: (5, 3)

\[
r = \sqrt{(5 - 5)^2 + (7 - 3)^2} = \sqrt{4^2} = 4
\]

\[
(x - 5)^2 + (y - 7)^2 = 4
\]

6. 

center of tire: (–4, 3)

radius of tire: 12.1

\[
(x - (–4))^2 + (y - 3)^2 = 12.1^2
\]

\[
(x + 4)^2 + (y - 3)^2 = 146.21
\]

center of rim: (–4, 3)

radius of rim: 7

\[
(x - (–4))^2 + (y - 3)^2 = 7^2
\]

\[
(x + 4)^2 + (y - 3)^2 = 49
\]

Mixed Review of Problem Solving (p. 706)

1. a. center: (20, 30)

radius: 20

\[
(x - 20)^2 + (y - 30)^2 = 20^2
\]

\[
(x - 20)^2 + (y - 30)^2 = 400
\]


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Geometry

Worked-Out Solution Key

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Chapter 10, continued

b. At \(E(25, 25)\):
\[
\begin{align*}
(x - 20)^2 + (y - 30)^2 &\leq 400 \\
(25 - 20)^2 + (25 - 30)^2 &\leq 400 \\
5^2 + (-5)^2 &\leq 400 \\
50 &\leq 400
\end{align*}
\]
You can receive the signal at \(E\).

At \(F(10, 10)\):
\[
\begin{align*}
(x - 20)^2 + (y - 30)^2 &\leq 400 \\
(10 - 20)^2 + (10 - 30)^2 &\leq 400 \\
(-10)^2 + (-20)^2 &\leq 400 \\
250 &> 400
\end{align*}
\]
You cannot receive the signal at \(F\).

At \(G(20, 16)\):
\[
\begin{align*}
(x - 20)^2 + (y - 30)^2 &\leq 400 \\
(20 - 20)^2 + (16 - 30)^2 &\leq 400 \\
(-14)^2 &\leq 400 \\
16 \leq 400
\end{align*}
\]
You can receive the signal at \(G\).

At \(H(35, 30)\):
\[
\begin{align*}
(x - 20)^2 + (y - 30)^2 &\leq 400 \\
(35 - 20)^2 + (30 - 30)^2 &\leq 400 \\
15^2 &\leq 400 \\
225 &\leq 400
\end{align*}
\]
You can receive the signal at \(H\).

2. a. The areas covered by the cell towers overlap, so there are areas that can transmit calls using more than one tower.

b. Your house is located within range of tower \(X\). You can use your cell phone at your house. You cannot use your cell phone at your friend’s house.

c. City \(B\) has better coverage from the cell phone towers.

3. \(40 \cdot 43 \neq 41 \cdot 42\)

\[
1720 \neq 1722
\]

The track is not circular because the product of the lengths of one chord is not equal to the product of the lengths of the other chord.

4. a. \(17^2 = 6(6 + 2r)\)

\[
\begin{align*}
289 &= 36 + 12r \\
253 &= 12r \\
21.1 &= r
\end{align*}
\]
The radius of the tank is about 21.1 feet.

b. \(x^2 = 4(4 + 12)\)

\[
x^2 = 4(16) \\
x^2 = 64
\]
x = 8

You are 8 feet from a point of tangency.

5. a. The circles intersect at point (2, 3), so the epicenter is at (2, 3).

c. center: (2, 3)

radius: 12

\[
\begin{align*}
(x - 2)^2 + (y - 3)^2 &= 12^2 \\
(x - 2)^2 + (y - 3)^2 &= 144
\end{align*}
\]

At \((14, 16)\):

\[
\begin{align*}
(x - 2)^2 + (y - 3)^2 &= 144 \\
(14 - 2)^2 + (16 - 3)^2 &= 144 \\
12^2 + 13^2 &= 144 \\
313 &> 144
\end{align*}
\]
The point \((14, 16)\) lies outside the range of the earthquake, so you could not feel the earthquake.

6. a. \(y(y + 2x) = 15^2\)

\[
y^2 + 2xy = 225
\]

\[
y^2 + 2xy - 225 = 0
\]

\[
y = \frac{-2x \pm \sqrt{(2y)^2 - 4(1)(-225)}}{2(1)}
\]

\[
y = \frac{-2x \pm \sqrt{4x^2 + 900}}{2}
\]

\[
y = -x + \sqrt{x^2 + 225}
\]

b. \(8 \cdot x = x \cdot x\)

\[
x = 8
\]

c. \(y = -8 \pm \sqrt{8^2 + 225}\)

\[
y = 9
\]

Chapter 10 Review (pp. 708–711)

1. If a chord passes through the center of a circle, then it is called a diameter.

2. An inscribed angle is an angle whose vertex is on the circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc of the angle.
Chapter 10, continued

3. The measure of the central angle and the corresponding minor arc are the same. The measure of the major arc is 360° minus the measure of the minor arc.

4. B: $KZ$ is a tangent segment.

5. C: $LN$ is a secant segment.

6. A: $LM$ is an external segment.

7. \[ XY = XZ \]
\[ 9a^2 - 30 = 3a \]
\[ 3(3a^2 - a - 10) = 0 \]
\[ (3a + 5)(a - 2) = 0 \]
\[ 3a + 5 = 0 \text{ or } a - 2 = 0 \]
\[ a = \frac{5}{3}, a = 2 \]
Length cannot be negative, so $a = 2$.

8. \[ XY = XZ \]
\[ 2c^2 + 9c + 6 = 9c + 14 \]
\[ 2c^2 - 8 = 0 \]
\[ 2(c^2 - 4) = 0 \]
\[ (c - 2)(c + 2) = 0 \]
\[ c = 2 \text{ or } c = -2 \]
Length cannot be negative, so $c = 2$.

9. \[ WZ^2 + XZ^2 = WX^2 \]
\[ r^2 + 9^2 = (r + 3)^2 \]
\[ r^2 + 81 = r^2 + 6r + 9 \]
\[ 72 = 6r \]
\[ 12 = r \]

10. \[ m\angle KLM = m\angle KPL = 100^\circ \]
11. \[ m\angle LM = 180^\circ - m\angle MKN = 180^\circ - 120^\circ = 60^\circ \]
12. \[ m\angle KM = m\angle KLM + m\angle LM = 100^\circ + 60^\circ = 160^\circ \]
13. \[ m\angle KN = 180^\circ - m\angle KLN = 180^\circ - 100^\circ = 80^\circ \]
14. \[ m\angle ED = m\angle ECD = 61^\circ \]
15. \[ m\angle AB = m\angle ED = 61^\circ \]
16. \[ m\angle AB = m\angle ED = 91^\circ \]
17. \[ m\angle YZ = \frac{1}{2} m\angle Y \]
18. \[ m\angle BAC = \frac{1}{2} m\angle BAC \]
\[ c^2 = \frac{1}{2} (56^\circ) \]
\[ c = 28 \]
\[ 40^\circ = \frac{1}{2} x^\circ \]
\[ 80 = x \]
19. \[ m\angle E + m\angle G = 180^\circ \]
\[ q^\circ + 80^\circ = 180^\circ \]
\[ q = 100 \]
\[ m\angle F + m\angle D = 180^\circ \]
\[ 100^\circ + 4q^\circ = 180^\circ \]
\[ 4q = 80 \]
\[ 4 = 20 \]

20. \[ x^\circ = \frac{1}{2} (250^\circ - (360^\circ - 250^\circ)) \]
\[ x = \frac{1}{2} (250 - 110) \]
\[ x = \frac{1}{2} (140) \]
\[ x = 70 \]
21. \[ 40^\circ = \frac{1}{2} (96^\circ - x^\circ) \]
22. \[ x^\circ = \frac{1}{2} (152^\circ + 60^\circ) \]
\[ 80 = 96 - x \]
\[ x = \frac{1}{2} (212) \]
\[ x = 16 \]
\[ x = 106 \]
23. \[ 20^2 = 12 \times (12 + 2r) \]
\[ 400 = 144 + 24r \]
\[ 256 = 24r \]
\[ 10 \frac{2}{3} = r \]
The radius of the rink is $10 \frac{2}{3}$ feet.
24. center: $(4, -1)$
radius: 3
\[ (x - 4)^2 + (y - (-1))^2 = 3^2 \]
\[ (x - 4)^2 + (y + 1)^2 = 9 \]
25. center: $(8, 6)$
radius: 6
\[ (x - 8)^2 + (y - 6)^2 = 6^2 \]
\[ (x - 8)^2 + (y - 6)^2 = 36 \]
26. center: $(0, 0)$
radius: 4
\[ x^2 + y^2 = 4^2 \]
\[ x^2 + y^2 = 16 \]
27. \[ x^2 + y^2 = 9^2 \]
\[ x^2 + y^2 = 81 \]
28. \[ (x - (-5))^2 + (y - 2)^2 = 1.3^2 \]
\[ (x + 5) + (y - 2)^2 = 1.69 \]
29. \[ (x - 6)^2 + (y - 21)^2 = 4^2 \]
\[ (x - 6)^2 + (y - 21)^2 = 16 \]
30. \[ (x - (-3))^2 + (y - 2)^2 = 16^2 \]
\[ (x + 3)^2 + (y - 2)^2 = 256 \]
31. \[ (x - 10)^2 + (y - 7)^2 = 3.5^2 \]
\[ (x - 10)^2 + (y - 7)^2 = 12.25 \]
32. \[ x^2 + y^2 = 5.2^2 \]
\[ x^2 + y^2 = 27.04 \]

Chapter 10 Test (p. 712)

1. \[ AB = AD \]
\[ 5x - 4 = 3x + 6 \]
\[ 2x = 10 \]
\[ x = 5 \]
Chapter 10, continued

2. \(AD^2 + DC^2 = AC^2\)
   \[12^2 + r^2 = (r + 6)^2\]
   \[144 + r^2 = r^2 + 12r + 36\]
   \[108 = 12r\]
   \[r = 9\]

3. \(AB = AD\)
   \[2x^2 + 8x - 17 = 8x + 15\]
   \[2x^2 = 32\]
   \[x^2 = 16\]
   \[x = 4\]

4. \(m\angle CED = m\angle CED = 60^\circ = m\angle AB\)
   \(AB \cong CD\)

5. \(m\angle FG = m\angle FHG = 136^\circ\)
   \(m\angle J = 360^\circ - 224^\circ = 136^\circ\)
   \(FG \cong JL\)

6. \(MN\) is not congruent to \(QR\) because they are not part of two congruent circles.

7. \(AB\) is perpendicular to and bisects \(CD\), so \(AB\) is a diameter.

8. \(AB\) is perpendicular to and bisects \(CD\), so \(AB\) is a diameter.

9. \(20^2 + ZY^2 = 25^2\)
   \(ZY^2 = 225\)
   \(ZY = 15\)
   \(AB\) is perpendicular to but does not bisect \(XY\), so \(AB\) is not a diameter.

10. \(m\angle ABC = \frac{1}{2} m\angle AC = \frac{1}{2}(106^\circ) = 53^\circ\)

11. \(m\angle FD = m\angle FED = \frac{1}{2}(182^\circ) = 164^\circ\)

12. \(m\angle GHJ = m\angle HJG = \frac{1}{2}(43^\circ) = 86^\circ\)
   \(m\angle GHJ = 360^\circ - m\angle JG = 360^\circ - 86^\circ = 274^\circ\)

13. \(m\angle 1 = \frac{1}{2}(238^\circ) = 119^\circ\)

14. \(m\angle 2 = \frac{1}{2}(152^\circ + 112^\circ) = \frac{1}{2}(164^\circ) = 82^\circ\)

15. \(m\angle B = \frac{1}{2}(m\angle AD - m\angle AC)\)
   \(42^\circ = \frac{1}{2}(168^\circ - m\angle AC)\)
   \(84^\circ = 168^\circ - m\angle AC\)
   \(m\angle AC = 84^\circ\)

16. \(8 \cdot x = 14 \cdot 4\)
   \[8x = 56\]
   \[x = 7\]

17. \(12^2 = 9(9 + x)\)
   \[144 = 81 + 9x\]
   \[63 = 9x\]
   \[x = 7\]

18. \(x(x + 28) = 20(20 + 32)\)
   \[x^2 + 28x = 1040\]
   \[x^2 + 28x - 1040 = 0\]
   \[x = \frac{-28 \pm \sqrt{28^2 - 4(1)(-1040)}}{2(1)}\]
   \[x = \frac{-28 \pm \sqrt{4944}}{2}\]
   \[x = 21.2\]

19. \((x + 2)^2 + (y - 5)^2 = 169\)
   \((x - (-2))^2 + (y - 5)^2 = 13^2\)
   The center is \((-2, 5)\) and the radius is 13.

Chapter 10 Algebra Review (p. 713)
1. \(6x^2 + 18x^4 = 6x^2(1 + 3x^2)\)
2. \(16a^2 - 24b = 8(2a^2 - 3b)\)
3. \(9y^2 - 15yz = 3y(3y - 5z)\)
4. \(14x^3 + 27x^3 = x^3(14x^2 + 27)\)
5. \(8x^3 + 6y^2 - 10r = 2(4r^3 + 3r - 5)\)
6. \(9c^2 + 3z + 21z^2 = 3(3z^2 + 7 + 7)z\)
7. \(5y^3 - 4y^3 + 2y^3 = y^3(5y^3 - 4y^2 + 2)\)
8. \(30y - 25y^2 - 10y^4 = 5y^4(6y^3 - 5y - 2)\)
9. \(6x^3y + 15x^3y^3 = 3x^3y(2x + 5y^2)\)
10. \(x^2 + 6x + 8 = (x + 4)(x + 2)\)
11. \(y^2 - y - 6 = (y - 3)(y + 2)\)
12. \(a^2 - 64 = (a - 8)(a + 8)\)
13. \(x^2 - 8x + 16 = (x - 4)(x - 4) = (x - 4)^2\)
14. \(3x^2 + 2x - 1 = (3x - 1)(x + 1)\)
15. \(5b^2 - 16b + 3 = (5b - 1)(b - 3)\)
16. \(4x^4 - 49 = (2x^2 - 7)(2x^2 + 7)\)
17. \(25x^2 - 81 = (5x - 9)(5x + 9)\)
18. \(4x^2 + 12x + 9 = (2x + 3)(2x + 3) = (2x + 3)^2\)
19. \(x^3 + 10x + 21 = (x + 3)(x + 7)\)
20. \(x^2 - 121 = (x - 11)(x + 11)\)
21. \(y^2 + y - 6 = (y + 3)(y - 2)\)
22. \(x^2 + 12x + 36 = (x + 6)(x + 6) = (x + 6)^2\)
23. \(x^2 - 49 = (x + 7)(x - 7)\)
24. \(2x^2 - 12x - 14 = 2(x^2 - 6x - 7) = 2(x - 7)(x + 1)\)

Standardized Test Preparation (p. 715)
1. You can eliminate choice A (20°) because \(m\angle NQ\) must be greater than \(m\angle P\), which is 20°.
2. You can eliminate choice A (−17°) because if \(x = -17\), then \(m\angle E = (x + 8)^\circ = (-17 + 8)^\circ = -9^\circ\). \(m\angle E\) cannot be negative.

Standardized Test Practice (pp. 716–717)
1. C; You do not know if \(\overline{MP} \cong \overline{NQ}\).
Chapter 10, continued

2. B: \( SQ \) is a diameter that is perpendicular to \( PR \), so \( SQ \) also bisects \( PR \).
   \[ PV = \frac{1}{2} PR \]
   \[ 5x - 2 = \frac{1}{2}(4x + 14) \]
   \[ 5x - 2 = 2x + 7 \]
   \[ 3x = 9 \]
   \[ x = 3 \]

3. B: \((x + 2)^2 + (y - 4)^2 = 9\)
   \((x - (-2))^2 + (y - 4)^2 = 9\)
   The center is \((-2, 4)\)

4. D; \( m \angle P = 180^\circ - 105^\circ - 27^\circ = 48^\circ \)
   \( m \angle P = \frac{1}{2} m \angle OR \)
   \[ 48^\circ = \frac{1}{2} m \angle OR \]
   \[ 96^\circ = m \angle OR \]

5. B: A central angle in a regular hexagon measures \( 360^\circ + 6 = 60^\circ \).
   \( m \angle KL = 60^\circ \)

6. B; \( \triangle TSV \) is a right triangle.
   \[ ST^2 + SV^2 = TV^2 \]
   \[ 12^2 + 18^2 = TV^2 \]
   \[ 468 = TV^2 \]
   \[ 22 \text{ in.} = TV \]

7. D; \( m \angle TRS = \frac{1}{2} m \angle RT \)
   \[ 62^\circ = \frac{1}{2} m \angle RT \]
   \[ 124^\circ = m \angle RT \]
   \[ m \angle RT = 360^\circ - m \angle RT = 360^\circ - 124^\circ = 236^\circ \]

8. C; 3 common tangents

9. B; \( m \overline{EF} = (146 - x)^\circ \)
   \[ m \overline{HG} = (172 - x)^\circ \]
   \[ m \overline{EF} + m \overline{FG} + m \overline{GH} + m \overline{HE} = 360^\circ \]
   \( (146 - x)^\circ + x + (172 - x)^\circ + 5^\circ = 360^\circ \)
   \[ 2x + 318 = 360 \]
   \[ 2x = 42 \]
   \[ x = 21 \]

10. \( KL = ML \)

11. \( m \overline{AB} = 180^\circ - m \overline{AD} = 180^\circ - 111^\circ = 69^\circ \)

12. \( 20^2 = 2x(2x + 6x) \)
    \[ 400 = 2x(8x) \]
    \[ 400 = 16x^2 \]
    \[ 25 = x^2 \]
    \[ 5 = x \]

13. \( \angle P \) and \( \angle T \) intercept the same arc, \( \widehat{OS} \), so they are congruent. Also, \( \angle R \cong \angle R \) by the Reflexive Property of Congruence. So, \( \triangle PSR \cong \triangle TQR \) by the AA Similarity Theorem.

14. The measure of an inscribed angle is \( \frac{1}{2} \) the measure of its intercepted arc. So, \( x = \frac{1}{2} y \) or \( y = 2x \).

The slope of the line is \( 2 \).

15. By Theorem 10.3, \( \overline{AC} \cong \overline{EG} \).
    By Theorem 10.5, \( \overline{EF} \cong \overline{FG} \cong \overline{CB} \cong \overline{BA} \).
    By Theorem 10.6, \( \overline{AC} \cong \overline{EG} \).
    All radii of the same circle are congruent, so \( \overline{JB} \cong \overline{JF} \).

16. a-b.
Chapter 10, continued

The point of intersection is the center of the plate.

e. The archeologist could measure the radius found in part (b) and double it. The diameter is always twice the radius.

17. a. Point $P(3, -8)$, center $C(-2, 4)$

\[ r = \sqrt{(-2 - 3)^2 + (4 - (-8))^2} \]
\[ = \sqrt{(-5)^2 + 12^2} \]
\[ = \sqrt{169} \]
\[ = 13 \]

\[(x - (-2))^2 + (y - 4)^2 = 13^2\]
\[(x + 2)^2 + (y - 4)^2 = 169\]

b. \[ m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 - (-8)}{2 - 3} = \frac{12}{-5} = -\frac{12}{5} \]
\[ y = mx + b \]
\[ -8 = -\frac{12}{5}(3) + b \]
\[ -\frac{40}{5} = \frac{36}{5} + b \]
\[ b = \frac{4}{5} \]
\[ y = -\frac{12}{5}x + \frac{4}{5} \]

c. \[ m = \frac{5}{12} \]
\[ y = mx + b \]
\[ -8 = \frac{5}{12}(3) + b \]
\[ -\frac{32}{4} = \frac{5}{4} + b \]
\[ b = -\frac{37}{4} \]
\[ y = \frac{5}{12}x - \frac{37}{4} \]