Chapter 2

Prerequisite Skills (p. 70)

1. Sample answer: \( \angle CGA \)
2. Sample answer: \( \angle BGA \) and \( \angle DGE \)
3. Sample answer: \( \angle CGB \) and \( \angle CGE \)
4. Sample answer: \( \angle DGE \) and \( \angle EGF \)
5. Line segment with endpoints \( A \) and \( B \)
6. Line containing points \( C \) and \( D \)
7. The measure of the length of the segment from point \( E \) to \( F \).
8. Ray with endpoint \( G \) containing point \( H \)
9. \( 3x + 5 = 20 \)
   \( 3x = 15 \)
   \( x = 5 \)
10. \( 4(x - 7) = -12 \)
    \( 4x - 28 = -12 \)
    \( 4x = 16 \)
    \( x = 4 \)
11. \( 5(x + 8) = 4x \)
    \( 5x + 40 = 4x \)
    \( x = 40 \)
12. Angle addition postulate
13. Segment addition postulate

Lesson 2.1

2.1 Guided Practice (pp. 72–74)

1.

2. Each number in the pattern is 0.02 more than the previous number: 5.09, 6.01, 6.03
3. Continue the pattern from Example 3. You can connect 6 collinear points \( 10 + 5 \), or 15 different ways and then the 7 collinear points \( 15 + 6 \), or 21 different ways.
4. Find a pattern:
   \( (-1)(-2)(-3) = -6 \)
   \( (-4)(-5)(-6) = -120 \)
   \( (-2)(-5)(-7) = -70 \)
   \( (-4)(-7)(-8) = -224 \)
Conjecture: The sign of the product of any three negative integers is negative.
5. Find a value of \( x \) that is greater than or equal to \( x^2 \).
   \( \frac{1}{2} \geq \frac{1}{4} \)
Because a counterexample exists, the conjecture is false.
6. Sample answer: There will be more girls playing soccer in 2002 than in 2001. This is a reasonable conjecture because the graph shows that the number of girls playing soccer increased each year from 1990-2001.

2.1 Exercises (pp. 75–78)

Skill Practice

1. Sample answer: A conjecture is a statement about an observation that can be true or false.
2. The word counter means contrary or opposite. A counterexample is an example that is contrary to the given conjecture.
3. The shaded portion of the figure is decreasing by one section in each figure. Sketch the fourth figure by shading only the center and the lower left section of the figure.
4. Each figure is created by adding one block to the outermost edges of the previous figure. Sketch the fourth figure by adding one block to the outermost edges of the third figure.
5. C. Each figure is the previous figure rotated 90° counterclockwise. The fourth figure is the third figure rotated 90° counterclockwise.

6. 1, 5, 9, 13, …
   \[ +4 +4 +4 +4 \]

   Each number in the pattern is 4 more than the previous number. The next number is 17.

7. 3, 12, 48, 192, …
   \[ \times 4 \times 4 \times 4 \]

   Each number in the pattern is four times the previous number. The next number is 768.

8. 10, 5, 2.5, 1.25, …
   \[ \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \]

   Each number in the pattern is \( \frac{1}{2} \) times the previous number. The next number is 0.625.

9. 4, 3, 1, \(-2\), …
   \[ -1 -2 -3 -4 \]

   You subtract 1 to get the second number, then you subtract 2 to get the third number, then you subtract 3 to get the fourth number. To find the fifth number, subtract the next consecutive integer, which is 4. The next number is \(-6\).

10. 1, \(-\frac{2}{3}\), \(-\frac{1}{3}\), 0, …
    \[ \frac{1}{3} -\frac{1}{3} -\frac{1}{3} -\frac{1}{3} \]

    Each number in the pattern is \(\frac{1}{3}\) less than the previous number. The next number is \(-\frac{1}{3}\).

11. \(-5\), \(-2\), 4, 13, …
    \[ \frac{1}{3} +6 +9 +\frac{12}{3} \]

    You add 3 to get the second number, then add 6 to get the third number, then add 9 to get the fourth number. To find the fifth number, add the next multiple of 3, which is 12. So, the next number is 25.

12. | Number of points | 3 | 4 | 5 | 6 | 7 |
    |------------------|---|---|---|---|---|
    | Number of corrections | 3 | 6 | 10 | 15 | 21 |
    |------------------|---|---|---|---|---|
    | Number of corrections | 3 | 10 | 15 | 21 | 27 |

   Conjecture: You can connect seven noncollinear points in 21 different ways.

13. Conjecture: The sum of any two odd integers is even.

14. A counterexample is \(-5\)(\(-4\)) = 20. The numbers \(-5\) and \(-4\) are both negative, and their product is positive.

15. A counterexample is \((2 + 5)^2 = 7^2 = 49\).
    \[ 2^2 + 5^2 = 4 + 25 = 29 \]
    \[ 49 \neq 29 \]

16. A counterexample is the number 2 because it is prime and even.

17. A counterexample is 7(4) = 28.
    The number 7 is not even and the product of 7 and 4 is even.

18. The student has made a conclusion about all angles by drawing only one angle. The student could have drawn a right angle or an obtuse angle. The drawing shows only that the conjecture “some angles are acute” is true.

19. For a conjecture to be true, it must be true for all possible cases. A counterexample is a specific case when the conjecture is false. So, if you are able to find one counterexample, then the conjecture is false because it is not true for all possible cases.

20. \(x\) | 1 | 2 | 3 |
    \(y\) | -3 | -2 | -1 |

   The value of \(y\) is four less than the value of \(x\). So, a function rule relating \(x\) and \(y\) is \(y = x - 4\).

21. \(x\) | 1 | 2 | 3 |
    \(y\) | 2 | 4 | 6 |

   The value of \(y\) is twice the value of \(x\). So, a function rule relating \(x\) and \(y\) is \(y = 2x\).

22. B; \(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 81, 243, 729\)
    \[ \times 3 \times 3 \]

   Each number in the pattern is three times the previous number. To find the third number, divide the fourth number by \(3; 81 = 27\). To find the second number, divide the third number by \(3; 27 = 9\). To find the first number, divide the second number by \(3; \frac{9}{3} = 3\).

23. \[ \frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots \]
    \[ +\frac{1}{1} +\frac{1}{1} +\frac{1}{1} \]

    The numerator and denominator of each number in the pattern are one more than the numerator and denominator of the previous number. The next number is \(\frac{6}{5}\).

24. 1, 8, 27, 64, 125, … → \((1)^3, (2)^3, (3)^3, (4)^3, (5)^3, \ldots\)

   The numbers in the pattern are successive perfect cubes starting with 1. The next number is 216.
25. \(0.45, 0.7, 0.95, 1.2, \ldots\)

Each number in the pattern is 0.25 more than the previous number. The next number is 1.45.

26. \(1, 3, 6, 10, 15, \ldots\)

You add 2 to get the second number, then add 3 to get the third number, then add 4 to get the fourth number, then add 5 to get the fifth number. To find the sixth number, add the next consecutive integer, which is 6. The next number is 21.

27. \(2, 20, 100, 500, \ldots\)

You multiply by 10 to get the second number, then you divide by 2 to get the third number, then multiply by 10 to get the fourth number, then divide by 2 to get the fifth number. To find the sixth number, continue the pattern and multiply by 10. The next number is 500.

28. \(0.4(6), 0.4(6)^2, 0.4(6)^3, \ldots\)

The exponent of the 6 in each number in the pattern is one more than the exponent of the 6 in the previous number. The next number is \(0.4(6)^4\).

29. For all values of \(r\) where \(r > 1\), the values of the numbers in the pattern are increasing. For all values of \(r\) where \(0 < r < 1\), the values of the numbers in the pattern are decreasing. Raising numbers greater than 1 by successive natural number powers increases the result, while raising numbers between 0 and 1 by successive natural number powers decreases the result.

30. \(1, 2, 4, \ldots\)

Yes; you can add 1 to get the second number, then add 2 to get the third number. To find the fourth number, add the next consecutive integer, which is 3. The next number is 7.

31. a. \(1, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \ldots\)

You add \(\frac{1}{2}\) to get the second number, then add \(\frac{1}{4}\) to get the third number, then add \(\frac{1}{8}\) to get the fourth number. To find the fifth number, add \(\frac{1}{16}\) times the amount added to the previous number, or \(\frac{1}{2} \times \frac{1}{16} = \frac{1}{32}\). The next number is \(\frac{1}{2} \times \frac{1}{32} = \frac{1}{64}\). The next three numbers are \(\frac{15}{16}, \frac{31}{32}\), and \(\frac{63}{64}\).

b. The values of the numbers are increasing.

c. As the pattern continues, the values of the numbers get closer and closer to 1. This is a reasonable conjecture because the fraction part of the mixed number gets closer and closer to 1, so the value of the number gets closer and closer to 2.

Problem Solving


The pitcher throws one of each pitch first, then throws two of each pitch, then continues to increase the number of throws of each pitch by 1. The next five pitches will be \(C, C, F, F, F\).

33. Conjecture: More person-to-person e-mail messages will be sent in 2004 than in 2003. This is a reasonable conjecture because the graph shows an increase each year from 1996–2003.

34. a.

<table>
<thead>
<tr>
<th>Figure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>Distance</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

b. The distance around each figure is four times the figure number.

c. The distance around the 20th figure is \(4 \times 20 = 80\) units.

35. a.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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</thead>
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<tr>
<td>-3</td>
<td>-5</td>
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<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>15</td>
<td>31</td>
</tr>
</tbody>
</table>

b. The value of \(y\) is one more than twice the value of \(x\). So, an equation relating \(x\) and \(y\) is \(y = 2x + 1\).
Chapter 2, continued

<table>
<thead>
<tr>
<th>Number of tickets sold</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class income</td>
<td>0</td>
<td>0.25</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
<td>1.25</td>
<td>2.5</td>
<td>5.00</td>
</tr>
</tbody>
</table>

There is a linear relationship between the number of tickets sold and the income. The income is 0.25 times the number of tickets sold.

e. Let y be your income and x be the number of tickets sold. An equation for your income is \( y = 0.25x \).

d. When \( y = 14 \):
\[ 14 = 0.25x \]
\[ 56 = x \]
Your class must sell more than 56 tickets to make a profit.

e. To make a profit of $50, you must sell 14 + 50 = $64 worth of tickets. Find the value of x when \( y = 64 \).
\[ 64 = 0.25x \]
\[ 256 = x \]
Your class must sell 256 tickets to make a profit of $50.

37. a. After the first two numbers, each number is the sum of the two previous numbers.

b. 144, 233, 377
c. Sample answer: Spiral patterns on the head of a sunflower

38. a. Sample answer: A counterexample is 15, which is a multiple of 5 but not a multiple of 8. So 15 is a member of set A, but not a member of set B. The conjecture is false because a counterexample exists.

b. Sample answer: A counterexample is 99, which is less than 100 but not a member of set A or set B. The conjecture is false because a counterexample exists.

c. A counterexample is 40, which is in both set A and set B. The conjecture is false because a counterexample exists.

Mixed Review

39. \( 4(x - 5) = 4x - 20 \)
40. \( -2(x - 7) = -2x + 14 \)
41. \( (-2n + 5)4 = -8n + 20 \)
42. \( x(x + 8) = x^2 + 8x \)

43. \( \bar{x} = \frac{1 + 5 + 1 + 0 + 3 + 6 + 4 + 2 + 10 + 1}{10} = \frac{33}{10} = 3.3 \)

44. 0, 1, 1, 2, 3, 4, 5, 6, 10
   The median is \( \frac{2 + 3}{2} = 2.5 \).

45. The mode is 1.

46. The median is most representative. The mean is too high because the data value 10 is much higher than the others; the mode is too low because 1 is near the lowest data value.

47. Let \( l = 7 \) and \( w = 3 \).
   Perimeter = \( 2l + 2w = 2(7) + 2(3) = 20 \)
   Area = \( l \times w = 7 \times 3 = 21 \)
   So, the perimeter is 20 inches and the area is 21 square inches.

48. Let \( s = 4 \).
   Perimeter = \( 4s = 4(4) = 16 \)
   Area = \( s^2 = 4^2 = 16 \)
   So, the perimeter is 16 centimeters and the area is 16 square centimeters.

49. Let \( a = 6 \), \( b = 8 \), and \( c = 10 \).
   Perimeter = \( a + b + c = 6 + 8 + 10 = 24 \)
   Area = \( \frac{1}{2}ab = \frac{1}{2}(6)(8) = 24 \)
   So, the perimeter is 24 feet and the area is 24 square feet.

Lesson 2.2

2.2 Guided Practice (pp. 79–82)

1. If an angle is a 90° angle, then it is a right angle.
2. If \( x = -3 \), then \( 2x + 7 = 1 \).
3. If \( a = 9 \), then \( a^2 = 81 \).
4. If a tourist is at the Alamo, then the tourist is in Texas.
5. Converse: If a dog is large, then it is a Great Dane.
   False, not all large dogs are Great Danes.
   Inverse: If a dog is not a Great Dane, then it is not large.
   False, a dog could be large but not a Great Dane.
   Contrapositive: If a dog is not large, then it is not a Great Dane.
   True, a dog that is not large cannot be a Great Dane.

6. Converse: If a polygon is regular, then the polygon is equilateral.
   True, all regular polygons are equilateral.
   Inverse: If a polygon is not equilateral, then it is not regular.
   True, a polygon that is not equilateral cannot be regular.
   Contrapositive: If a polygon is not regular, then the polygon is not equilateral.
   False, a polygon that is not regular can still be equilateral.

7. True, \( \angle JMF \) and \( \angle FMG \) form a linear pair so they are supplementary.
Chapter 2, continued

8. False. It is not known that $M$ bisects $FH$. So, you cannot state that $M$ is the midpoint for $FH$.

9. True. $\angle JMF$ and $\angle HMG$ are vertical angles because their sides form two pairs of opposite rays.

10. False. It is not shown that $FH$ and $JG$ intersect to form right angles. So, you cannot state that $FH \perp JG$.

11. An angle is a right angle if and only if it measures $90^\circ$.

12. Mary will be in the fall play if and only if she is in theater class.

2.2 Exercises (pp. 82–85)

Skill Practice

1. The converse of a conditional statement is found by switching the hypothesis and the conclusion.

2. Collinear points are points that lie on the same line. Points are collinear if and only if they lie on the same line.

3. If $x = 6$, then $x^2 = 36$.

4. If an angle is a straight angle, then it measures $180^\circ$.

5. If a person is registered to vote, then that person is allowed to vote.

6. The error is in identifying the correct hypothesis and conclusion when writing the if-then form of the statement. The hypothesis is “a student is in high school” and the conclusion is “the student takes four English courses.”

If-then statement: If a student is in high school, then the student takes four English courses.

7. If-then: If two angles are complementary, then they add to $90^\circ$.
Converse: If two angles add to $90^\circ$, then they are complementary.
Inverse: If two angles are not complementary, then they do not add to $90^\circ$.
Contrapositive: If two angles do not add to $90^\circ$, then they are not complementary.

8. If-then: If an animal is an ant, then it is an insect.
Converse: If an animal is an insect, then it is an ant.
Inverse: If an animal is not an ant, then it is not an insect.
Contrapositive: If an animal is not an insect, then it is not an ant.

9. If-then: If $x = 2$, then $3x + 10 = 16$.
Converse: If $3x + 10 = 16$, then $x = 2$.
Inverse: If $x \neq 2$, then $3x + 10 \neq 16$.
Contrapositive: If $3x + 10 \neq 16$, then $x \neq 2$.

10. If-then: If a point is a midpoint, then it bisects a segment.
Converse: If a point bisects a segment, then it is a midpoint.
Inverse: If a point is not a midpoint, then it does not bisect a segment.
Contrapositive: If a point does not bisect a segment, then it is not a midpoint.

11. False; a polygon can have 5 sides without being a regular pentagon.
Counterexample: 

12. True.

13. False; two angles can be supplementary without being a linear pair. Counterexample:

14. True.

15. False; Counterexample: The number 5 is real, but not irrational.

16. True; $\angle ABC$ is a right angle, so $m\angle ABC = 90^\circ$.

17. False; It is not known that $\angle 1$ is a right angle, so you cannot conclude that $\overline{PO} \perp \overline{ST}$.

18. True; $\angle 2$ and $\angle 3$ are adjacent angles whose noncommon sides form opposite rays, so $\angle 2$ and $\angle 3$ are a linear pair. Angles in a linear pair are supplementary, so $m\angle 2 + m\angle 3 = 180^\circ$.

19. An angle is obtuse if and only if its measure is between $90^\circ$ and $180^\circ$.

20. Two angles are a linear pair if and only if they are adjacent angles whose noncommon sides are opposite rays.

21. Points are coplanar if and only if they lie in the same plane.

22. This is not a valid definition. The converse of the statement is not true. Rays can have a common endpoint without being opposite rays.

23. The statement is a valid definition.

24. The statement is not a valid definition. The converse of the statement is false. If the measure of an angle is greater than that of an acute angle, the angle is not necessarily a right angle.

25. A; If you do your homework, then you can go to the movie afterwards. This is the if-then form of the given statement.

26. If $x > 0$, then $x > 4$. A counterexample is $x = 2$. Note that $2 > 0$, but $2 > 4$. Because a counterexample exists, the converse is false.

27. If $-x > -6$, then $x < 6$. The converse is true.

28. If $x \leq 0$, then $x \leq -x$. The converse is true.

29. Sample answer: If $x = 2$, then $x^2 > 0$.

30. If $\angle 1$ and $\angle 2$ are linear pairs, then $m\angle 2$ is $90^\circ$; if $\angle 1$ and $\angle 4$ are linear pairs, then $m\angle 4$ is $90^\circ$; if $\angle 4$ and $\angle 3$ are linear pairs, then $m\angle 3$ is $90^\circ$.
Chapter 2, continued

Problem Solving

31. Statement: If a fragment has a diameter greater than 64 millimeters, then it is called a block or bomb.
Converse: If a fragment is called a block or bomb, then it has a diameter greater than 64 millimeters.
Both the statement and its converse are true. So, the biconditional statement is true.

32. Counterexample: a fragment with a diameter of 1 millimeter
The diameter is less than 64 millimeters, but the fragment is not called a lapilli. Because a counterexample exists, the biconditional statement is false.

33. You can show that the statement is false by finding a counterexample. Some sports do not require helmets, such as swimming or track.

34. a. The statement is true. The mean is the average value of the data, so it will lie between the least and greatest values in the data set.

b. If the mean of your data set is between x and y, then x and y are the least and greatest values in your data set. The converse is false. The mean is between any two numbers in a data set where one of the numbers is less than the mean and the other is greater than the mean. The numbers do not have to be the least and greatest values in the data set.

c. If a data set has a mean, median, and mode, then the mode of the data set will always be one of the measurements. The mode is the data value that occurs most frequently in a data set. So, if the mode exists, then it will always be one of the data values. The median is one of the data values only when there is an odd number of values in the data set. The mean does not have to be a data value.

35. Sample answer: If a student is in the jazz band, then the student is in the band.

36. a. If a rock is formed from the cooling of molten rock, then it is igneous rock.
If a rock is formed from pieces of other rocks, then it is sedimentary rock.
If a rock is formed by changing temperature, pressure, or chemistry, then it is metamorphic rock.

b. If a rock is igneous rock, then it is formed from the cooling of molten rock.
If a rock is sedimentary, then it is formed from pieces of other rocks.
If a rock is metamorphic, then it is formed by changing temperature, pressure, or chemistry.
The converse of each statement is true.
If a rock is classified in one of these ways, it must be formed in the manner described.

e. Sample answer: If it is a rock, then it can be formed in different ways. The converse of the statement is false. If something can be formed in different ways, it doesn't necessarily mean it has to be a rock. It could be soil for example.

37. The statement cannot be written as a true biconditional. The biconditional is false because x = -3 also makes the statement true. A counterexample exists, so the biconditional statement is false.

38. For a statement to be a true biconditional, both the original statement and the converse must be true. If the contrapositive of a statement is true, then you know that the original statement is true. However, you do not know if the converse is true. So, you don’t know if it can be written as a true biconditional.

39. It is Tuesday. Because it is Tuesday, I have art class. Because I have art class, I do not have study hall. Because I do not have study hall, I must have music class.

Mixed Review

40. (-2)(10) = -20
41. (15)(-3) = -45
42. (-12)(-4) = 48
43. (-5)(-4)(10) = 200
44. (-3)(6)(-2) = 36
45. (-4)(-2)(-5) = -40

46.

47.

48.

49.

50. $M\left(\frac{10 + 4}{2}, \frac{5 + 5}{2}\right) = M(7, 5)$
51. $M\left(\frac{4 + (-2)}{2}, \frac{-1 + 3}{2}\right) = M(1, 1)$
52. $M\left(\frac{2 + 1}{2}, \frac{2 + (-2)}{2}\right) = M\left(\frac{3}{2}, 0\right)$
53. The figure is a convex polygon.
54. The figure is not a polygon because part of it is not a segment.
55. The figure is a concave polygon.

Geometry

Worked-Out Solution Key
Chapter 2, continued

Lesson 2.3

Investigating Geometry Activity 2.3 (p. 86)

<table>
<thead>
<tr>
<th></th>
<th>Did not eat beans</th>
<th>Did not eat moonlight</th>
<th>Did not write a math book at age 17</th>
<th>Played piano</th>
<th>Was fluent in Latin</th>
<th>Wrote a math book at age 17</th>
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</table>

1. **If-then form:** If the mathematician is Julio Rey Pastor, then the mathematician wrote a book at age 17.

**Contrapositive:** If the mathematician did not write a book at age 17, then the mathematician is not Julio Rey Pastor.

The contrapositive is a helpful clue because it allows you to eliminate anyone who did not write a book at age 17 as possible choices for Julio Rey Pastor.

2. After clue 6, you know that the person who played the piano was the person who is either responsible for the math for the theory of relativity or used perspective drawing. You know that the person who played the piano was either Maria Agnesi or Emmy Noether. You also know that the person fluent in Latin was either Maria Agnesi or Emmy Noether. The person who is fluent in Latin contributed to differential calculus, so Emmy Noether could not have been fluent in Latin. Emmy Noether had to play the piano.

3. Before Clue 7, you knew that the person who used perspective drawing was either Maria Agnesi, Anaxagoras, or Julio Rey Pastor. Clue 7 stated that the person who used perspective drawing was not Maria Agnesi or Julio Rey Pastor. So, Anaxagoras had to be the one who first used perspective drawing.

4. Because \( x = 4 \) satisfies the hypothesis of a true conditional statement, the Law of Detachment states that the conclusion is also true. So, \( x + 9 > 20 \).

5. Look for a pattern:

- \( 1 + 1 = 2; \ 2 + 2 = 4; \ 3 + 3 = 6 \);
- \( 8 + 8 = 16; \ 10 + 10 = 20; \ 15 + 15 = 30 \)

**Conjecture:** The sum of a number and itself is twice the number.

Let \( n \) be any number. Then \( n + n = 2n \).

So, the sum of a number and itself is 2 times the number.

6. **Sample answer:** The northern elephant seal uses fewer strokes to surface the shallower it dives. The northern elephant seal uses fewer strokes to surface from 130 meters than from 420 meters.

2.3 Exercises (pp. 90–93)

**Skill Practice**

1. If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of Detachment.

2. The man is standing in front of a mirrored ball. You can see the reflections of people standing near him in the mirror.

3. There is a light source to the window side of the pears. You can see shadows cast by the pears opposite the window side.

4. Because \( m \angle A = 90^\circ \) satisfies the hypothesis, the conclusion is also true. So, \( \angle A \) is a right angle.

5. Because \( x = 15 \) satisfies the hypothesis, the conclusion is also true. So, \( -15 < -12 \).

6. Because reading a biography satisfies the hypothesis, the conclusion is also true. So, the book you are reading is nonfiction.

7. If a rectangle has four equal side lengths, then it is a regular polygon.

8. If \( y > 0 \), then \( 2y > 5 \).

9. If you play the clarinet, then you are a musician.

10. If \( \frac{1}{2} a = \frac{1}{2} \), then \( 5a = 15 \).

11. \( 2 + 4 = 6; \ 6 + 10 = 16; \ 4 + 14 = 18; \)

- \( 8 + 12 = 20; \ 10 + 12 = 22; \ 12 + 16 = 28 \)

**Conjecture:** even integer + even integer = even integer

Let \( n \) and \( m \) be any two integers. 

- \( 2n \) and \( 2m \) are even integers because any integer multiplied by 2 is even.
- \( 2n + 2m = 2(n + m) \)
- \( 2(n + m) \) is the product of 2 and an integer \( n + m \). So, \( 2(n + m) \) is an even integer. The sum of an even integer and an even integer is an even integer.

12. Because \( \angle A \) and \( \angle B \) are vertical angles satisfies the hypothesis, the conclusion is also true. So, \( m \angle A = m \angle B \).
Chapter 2, continued

13. In the second statement, the hypothesis and conclusion have been switched, which does not make a true statement.

If two angles are a linear pair, then they are supplementary. Angles C and D are a linear pair, so they are supplementary.

14. a. \( AB = \sqrt{(3 - 1)^2 + (6 - 3)^2} = \sqrt{4 + 9} = \sqrt{13} \)

\( CD = \sqrt{(6 - 4)^2 + (7 - 4)^2} = \sqrt{4 + 9} = \sqrt{13} \)

\( EF = \sqrt{(9 - 7)^2 + (4 - 1)^2} = \sqrt{4 + 9} = \sqrt{13} \)

b. Sample answer: Conjecture: If one endpoint is 2 units to the right and 3 units above the other end point, then the segment is congruent to the given segments.

Let \( M(2, 1) \) and \( N(4, 4) \) be the endpoints of \( MN \).

\( MN = \sqrt{(4 - 2)^2 + (4 - 1)^2} = \sqrt{2^2 + 3^2} = \sqrt{13} \)

Because \( MN = \sqrt{13} \), \( MN = AB \), \( CD = EF \).

c. Let \( S(x, y) \) be one endpoint of the segment. Then \( T(x + 2, y + 3) \) is the other endpoint.

\( ST = \sqrt{[(x + 2) - x]^2 + [(y + 3) - y]^2} \)

\( = \sqrt{2^2 + 3^2} = \sqrt{13} \)

The length of \( ST \) will always be \( \sqrt{13} \), so it will be congruent to \( AB \), \( CD \), and \( EF \).

d. \( MN = \sqrt{(5 - 3)^2 + (2 - 5)^2} = \sqrt{2^2 + 3^2} = \sqrt{13} \)

\( PQ = \sqrt{(4 - 1)^2 + (-3 - (-1))^2} \)

\( = \sqrt{3^2 + 2^2} = \sqrt{13} \)

\( RS = \sqrt{(1 - (-2))^2 + (4 - 2)^2} = \sqrt{3^2 + 2^2} = \sqrt{13} \)

The student is correct. Each segment is congruent to the given segments because each segment has a length of \( \sqrt{13} \).

15. The Law of Syllogism works when used with the contrapositives of a pair of statements. The contrapositive of a true statement is true. So, you can use the Law of Syllogism with the contrapositive of each true statement to write a new true statement.

If a creature is not a marsupial, then it is not a wombat.

If a creature does not have a pouch, then it is not a wombat.

The conclusion of the second true statement is the hypothesis of the first true statement, so you use the Law of Syllogism to write the following new statement.

If a creature does not have a pouch, then it is not a wombat.

Problem Solving

16. Because you saved $1200 does not satisfy the hypothesis, the conclusion is not true. So, you cannot buy a car.

17. Write each statement in if-then form.

If the revenue is greater than the costs, the bakery makes a profit.

If the bakery makes a profit, then you will get a raise.

So, if the revenue is greater than the costs, then you will get a raise.

18. So, Simone may have visited Mesa Verde National Park.

19. So, Billy is with a park ranger.

20. a. Sample answer: If calcite is scratched on gypsum, then a scratch mark is left on the gypsum.

If fluorite is scratched on calcite, then a scratch mark is left on the calcite.

If calcite is scratched on talc, then a scratch mark is left on the talc.

b. You can conclude that Mineral C is talc because it is the only mineral that can be scratched by all 3 other minerals.

Mineral A cannot be fluorite because fluorite cannot be scratched by any of the other minerals. So, Mineral A must be gypsum or calcite.

Mineral B cannot be gypsum because gypsum can only scratch talc, which is Mineral C. So, Mineral B must be calcite or fluorite.

c. Test to see if Mineral D can scratch Mineral B. If Mineral D can scratch Mineral B, then Mineral D is fluorite because it is the only mineral that cannot be scratched. If Mineral D is fluorite, then Mineral B is calcite and Mineral A is gypsum. If Mineral D cannot scratch Mineral B, then Mineral B is fluorite; If Mineral B is fluorite, take Mineral A and Mineral D and do one scratch test to identify them.

21. Deductive reasoning; The conclusion is reached by using laws of logic and the facts about your school rules and what you did that day.

22. Inductive reasoning; The conclusion is reached by using a pattern of past activities to make a conclusion on a future activity.

23. Let \( 2n \) be an even integer and \( 2n + 1 \) be an odd integer.

\( 2n + (2n + 1) = 4n + 1 \)

\( 4n \) is the product of 2 and an integer \( 2n \). So, \( 4n \) is an even integer.

\( 4n + 1 \) is one more than an even integer. So, \( 4n + 1 \) is an odd integer.

The sum of an even integer and an odd integer is an odd integer.

24. Use the Law of Syllogism to write a conditional statement for the first two statements.

For want of a nail the horse is lost.

Use the Law of Syllogism to write a new conditional statement for the statements in the poem.

For want of a nail the rider is lost.

25. The conclusion is true. The game is not sold out, so Arlo went to the game and he bought a hot dog.
Chapter 2, continued

26. The conclusion is true. The game is not sold out, so Arlo and Mia went to the game.
27. The conclusion is false. The statements never mention Mia buying a hot dog. So you cannot make that conclusion.
28. The conclusion is false. The statements never mention Arlo eating popcorn. So, you cannot make that conclusion.
29. a. If Adam is telling the truth, then Bob is lying. So, Bob’s statement about Charlie lying is itself a lie. Charlie must be telling the truth in his statement.
b. Assume Adam is telling the truth. Then Bob is lying and Charlie is telling the truth. Charlie’s statement says that Adam and Bob are both lying, which is a contradiction of the original assumption.
c. From part (b) you know that Adam is lying. Then from Adam’s statement, Bob is telling the truth. Charlie says that Adam and Bob are both lying, which cannot be true because Bob is telling the truth. So, Bob is telling the truth, and Adam and Charlie are both lying.

Mixed Review

30. \( \overline{AF}, \overline{CE} \)
31. \( \overline{DA}, \overline{DE}, \overline{DF}, \overline{DC} \)
32. Sample answer: Points \( C, D, \) and \( E \) are collinear.
33. Points \( B, C, D, \) and \( E \) are coplanar.
34. 

<table>
<thead>
<tr>
<th>( y )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
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<tbody>
<tr>
<td>-3</td>
<td>( y )</td>
<td>( x )</td>
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<tr>
<td>-1</td>
<td>( y )</td>
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<tr>
<td>1</td>
<td>( y )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

\( AB = \sqrt{(5 - 1)^2 + (4 - 4)^2} = \sqrt{4^2} = 4 \)

\( CD = \sqrt{(3 - 3)^2 + (-4 - 0)^2} = \sqrt{(-4)^2} = 4 \)

\( AB \) is equal to \( CD \). So \( AB \) is congruent to \( CD \).

35. 

<table>
<thead>
<tr>
<th>( y )</th>
<th>( A )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( y )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

\( AB = \sqrt{(-1 - (-1))^2 + (5 - 0)^2} = \sqrt{(-5)^2} = 5 \)

\( CD = \sqrt{(1 - (-5))^2 + (2 - 2)^2} = \sqrt{6^2} = 6 \)

\( AB \) is not equal to \( CD \), so \( AB \) is not congruent to \( CD \).

36. If \( x = -2 \), then \( x^2 = 4 \).
37. If an angle is acute, then its measure is less than 90°.
38. If a person is a member, then the person can access the website.

Quiz 2.1–2.3 (p. 93)

1. To find a counterexample, you need to find a product of two positive numbers that is positive.
   Sample answer: \( 6 \cdot 4 = 24 \)
   Both numbers are positive and the product is positive. Because a counterexample exists, the conjecture is false.

2. To find a counterexample, you need to find a sum that is less than the greater number.
   Sample answer: \(-2 + 6 = 4 \)
   \( 4 > 6 \)
   Because a counterexample exists, the conjecture is false.

3. If-then form: If points lie on the same line, then they are called collinear points.
   Contrapositive: If points are not collinear, then they do not lie on the same line.

4. If-then form: If \( x = 5 \), then \( 2x - 8 = 2 \).
   Contrapositive: If \( 2x - 8 \neq 2 \), then \( x \neq 5 \).

5. Because 98° satisfies the hypothesis, the conclusion must be true. So, I will wear shorts.

6. A multiple of 3 is, by definition, divisible by 3. If a number is divisible by a multiple of 3, then the number is a multiple of 3. So, if a number is divisible by a multiple of 3, then it is divisible by 3.

2.3 Extension (p. 95)

1. Use the symbol for negation (\( \neg \)) with the conclusion of the conditional statement, then use the arrow (\( \rightarrow \)) to connect the conclusion and the hypothesis of the new statement, then use the symbol for negation again with the hypothesis of the conditional statement. So, \( \neg q \rightarrow \neg p \).

2. If polygon \( ABCDE \) is equiangular and equilateral, then it is a regular polygon.

3. Polygon \( ABCDE \) is not equiangular and equilateral.

4. If polygon \( ABCDE \) is not a regular polygon, then it is not equiangular and equilateral.

5. Polygon \( ABCDE \) is equiangular and equilateral if and only if it is a regular polygon.

6. If \( x + 5 = 12 \), then \( x = 7 \).
   If \( x = 7 \), then \( 3x = 21 \).

   \( p \rightarrow q \)
   \( q \rightarrow r \)
   \( p \rightarrow r \)

7. The truth value of a statement can be either true (T) or false (F).
   The conditional statement \( p \rightarrow q \) is only false when a true hypothesis produces a false conclusion.

8. a. Hypothesis, \( p \): An animal is a poodle.
   Conclusion, \( q \): The animal is a dog.
Chapter 2, continued

<table>
<thead>
<tr>
<th>b.</th>
<th>p</th>
<th>q</th>
<th>q → p</th>
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<tbody>
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<td>T</td>
</tr>
</tbody>
</table>

The first row means an animal can be a dog and a poodle.
The second row means an animal can be a dog but not a poodle.
The third row means an animal that is not a dog cannot be a poodle.
The fourth row means an animal can be both not a dog and not a poodle.

Lesson 2.4

2.4 Guided Practice (pp. 97–98)

1. Postulate 11: If two planes intersect, then their intersection is a line.
2. Postulate 5: Through points A and B, there exists a line n.
   Postulate 6: Line n contains points A and B.
   Postulate 7: Line n intersects line m at point A.
3. Mark PÆW and QÆW congruent using two tick marks for each segment to make them different from TW and WV.
4. Sample answer: ∠TWP and ∠PWV are supplementary because they form a linear pair.
5. Yes, you can assume plane S intersects plane T at BC, because it is shown in the diagram.
6. Because the diagram shows that AB is perpendicular to plane S, then AB is perpendicular to every line in plane S that intersects AB at point B by the definition of a line perpendicular to a plane. BC is in plane S and intersects AB at point B, so AB ⊥ BC.

2.4 Exercises (pp. 99–102)

Skill Practice

1. A line perpendicular to a plane is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.
2. You cannot assume ∠BMA ≅ ∠CJA because you do not know their measures.
3. Postulate 5: Through any two points there exists exactly one line.
4. Postulate 9: A plane contains at least 3 noncollinear points.
5. a. If-then form: If three points are noncollinear, then there exists exactly one plane through the points.
   b. Converse: If exactly one plane exists through three points, then the three points are noncollinear.
   Inverse: If three points are collinear, then there is not exactly one plane that exists through them.

Contrapositive: If there is not exactly one plane that exists through three points, then they are not noncollinear.
6. Line q contains the three points J, H, and K.
7. Line p intersects line q at point H.
8. Through points G, K, and L, there exists exactly one plane, M.
9. \( W \) does not have to be congruent to \( V \) because \( T \) is not given as the midpoint of \( W \).
10. b. No drawn line connects F, B, and G, so you cannot assume they are collinear.
11. The statement is false. Through any two points there exists exactly one line not through any three points. For example, consider two cars parked on one side of a street and one car parked on the other side of the street.
12. It is true that a point can be in more than one plane.
13. The statement is false. Two planes do not have to intersect each other. For example, consider two panes of glass in a window.
14. True. The diagram shows that planes \( W \) and \( X \) intersect at \( KL \).
15. False. No drawn line connects \( Q \), \( J \), and \( M \), so you cannot say they are collinear.
16. False. The diagram shows that points \( K \), \( L \), \( M \), and \( R \) are not coplanar. Points \( M \) and \( R \) lie in different planes.
17. False. The diagram does not show that \( MN \) and \( RP \) intersect.
18. False. \( R \) lies in plane \( W \), so they cannot be perpendicular.
19. True. The diagram shows that \( JK \) lies in plane \( X \).
20. False. With no right angle marked, you cannot assume that \( ∠PLK \) is a right angle.
21. True. The sides of \( ∠NKJ \) and \( ∠JKM \) form two pairs of opposite rays, so they are vertical angles.
22. True. \( ∠NKJ \) and \( ∠JKM \) are a linear pair, so they are supplementary angles.
23. False. The diagram does not show that \( ∠JKM \) is congruent to \( ∠KLP \), so you cannot assume \( ∠JKM \equiv ∠KLP \).
24. C. The diagram shows \( LN \), \( AB \), and \( DC \) intersecting at point \( M \), \( AB \) bisecting \( LN \), and \( DC \perp LN \).
25. Answers will vary.
26. Sample answer: A line contains at least two points. Three points are sometimes contained in a line.

Geometry

Worked-Out Solution Key
Chapter 2, continued

27. You know that a plane contains at least three noncollinear points by Postulate 9. You also know that there exists exactly one line through any two points by Postulate 5. So, there is at least one line in every plane.

28. You know that plane $M$ contains at least three noncollinear points, one of which is point $X$, by Postulate 9. Let $A$ and $B$ be the other two points. You know that lines $XA$ and $XB$ exist in plane $M$ by Postulate 6.

29. You know that line $m$ contains at least two points by Postulate 6. You also know that through two points on line $m$ and point $C$ not on line $m$ there exists exactly one plane by Postulate 8. So, there is one plane that can be drawn so that line $m$ and point $C$ lie in the plane.

Problem Solving

30. Postulate 5 is suggested by the photo. Through one point, the person’s hand, and the other point, the dog, there exists exactly one line, the leash.

31. Postulate 7 is suggested by the photo. If two lines, the two swords, intersect, then their intersection is exactly one point.

32. Postulate 11 is suggested by the photo. If two planes, two sides of the poster board, intersect, then their intersection is a line.

33. Answers will vary.

34. Sample answer:

35. Sample answer: Through points $Z$ and $U$ there exists $\overline{ZU}$.

36. Sample answer: $\overline{SZ}$ intersects $\overline{ZU}$ at point $Z$.

37. Sample answer: The plane of the pyramid contains the points $S$, $T$, and $Y$.

38. Sample answer: The points $S$ and $Z$ lie in the plane of the pyramid, so $\overline{SZ}$ lies in the plane of the pyramid.

39. a. Streets 1 and 2 intersect at Building A.

b. Streets 1 and 2 intersect at Building A.

c. Building B is directly west of Building A and Building D is directly north of Building A, so the angle formed by streets 1 and 2 is a right angle.

d. Building E is not between Buildings A and B because if it were, then $\angle CAE$ would be an acute angle.

e. Building E is on Street 1.

40. a. See diagram.

b. See diagram.

c. Through points $A$ and $B$, there exists line $\overline{AB}$.

d. Point $C$ would lie on the intersection of plane $X$ and plane $Y$.

e. Plane $X$ contains points $C$, $D$, and $E$.

41. If $P$ and $Q$ are different planes, then points $E$, $F$, and $G$ must be collinear and lie on the line of intersection between planes $P$ and $Q$. If $P$ and $Q$ are the same plane, the points $E$, $F$, and $G$ must be noncollinear.

42. a. See diagram.

b. See diagram.

c. Through points $A$ and $B$, there exists line $\overline{AB}$.

d. Point $C$ would lie on the intersection of plane $X$ and plane $Y$.

e. Plane $X$ contains points $C$, $D$, and $E$.

43. a. See diagram.

b. See diagram.

c. Through points $A$ and $B$, there exists line $\overline{AB}$.

d. Point $C$ would lie on the intersection of plane $X$ and plane $Y$.

e. Plane $X$ contains points $C$, $D$, and $E$.
40

Geometry

Worked-Out Solution Key

44. You know that through any three noncollinear points there exists exactly one plane. When the lengths of all the legs are different there are four different combinations of three of the four leg ends. So, there are four planes determined by the lower ends of the legs. When exactly three of the legs of the second chair have the same length there are two different combinations of leg ends. So, there are two different planes determined by the lower ends of the legs of the second chair.

Mixed Review

46. \( MN + NP = MP \)
    \[ 18 + 9 = 27 \]
    \[ 16 + 16 = 32 \]

47. \( AB + BC = AC \)

48. \( RS + ST = RT \)
    \[ RS = RT - ST \]
    \[ RS = 26 - 8 = 18 \]

49. \( JK = KL \)
    \[ 2x - 3 = x + 10 \]
    \[ x = 13 \]
    \[ JK = 2(13) - 3 = 23 \]

50. \( XY = YZ \)
    \[ 3x - 8 = 2x + 7 \]
    \[ x = 15 \]
    \[ XZ = XY + YZ \]
    \[ XZ = [3(15) - 8] + [2(15) + 7] = 37 + 37 = 74 \]

51. \( AB = BC \)
    \[ x + 6 = 2 - x \]
    \[ 2x = -4 \]
    \[ x = -2 \]
    \[ BC = 2 - (-2) = 4 \]

52. Right angle:

53. Acute angle:

54. Obtuse angle:

55. Straight angle:

56. Let \( x \) be the measure of one angle. Then \( 9x \) is the measure of the other angle.
    \[ x + 9x = 180 \]
    \[ 10x = 180 \]
    \[ x = 18 \]
    \[ 9(18) = 162 \]

One angle measures 18° and the other measures 162°.

Mixed Review of Problem Solving (p. 103)

1. a. The time of sunrise gets earlier each day from Jan. 1 through June 1, then gets later on July 1 and Aug. 1.
   b. Sample answer: 6:00 A.M.

2. a. The statement is true.
   b. The statement is false.

3. \( 1, 2, 5, 10, 17, 26, \ldots \)
    \[ +1 \]
    \[ +3 \]
    \[ +5 \]
    \[ +7 \]
    \[ +9 \]
    \[ +11 \]

You add 1 to get the second number, then add 3 to get the third number, then add 5 to get the fourth number, then add 7 to get the fifth number, then add 9 to get the sixth number. To find the seventh number, add the next consecutive odd integer, which is 11. So, the next number is 37.

4. a. The statement is the result of inductive reasoning because the conclusion is based on an observation in the pattern of the data.
   b. The statement is the result of deductive reasoning because the conclusion compares the values given in the graph.
   c. The statement is the result of inductive reasoning because the conclusion is based on an observation.

5. a. You must have a library card. Without a library card, you could not have checked out the book.
   b. Bob may have visited volcanoes. He has never been to the volcanoes in Hawaii, but he may have visited other volcanoes.

6. \( \angle PNR \) and \( \angle QNS \) are acute angles. You know that \( \angle PNR \) and \( \angle PNS \) are supplementary angles because they are a linear pair. Because \( \angle PNS \) is an obtuse angle, \( \angle PNR \) must be an acute angle. Similarly \( \angle QNS \) and \( \angle PNS \) are supplementary angles, so \( \angle QNS \) is also an acute angle.
**Chapter 2, continued**

**Lesson 2.5**

**Investigating Geometry Activity 2.5 (p. 104)**

Explore: Step 2

Sample answer:

1. a. 16  
   b. 16 • 2  
   c. 32 + 4  
   d. 36 • 5  
   e. 180 + 12  
   f. 192 • 10  
   g. 1920 - 320  
   h. 1600 

2. a. 42  
   b. 42 • 2  
   c. 84 + 4  
   d. 88 • 5  
   e. 440 + 12  
   f. 452 • 10  
   g. 4520 - 320  
   h. 4200 

Your answer is always the number you picked in part (a).

**Draw Conclusions**

1. a. x  
   b. 2x  
   c. 2x + 4  
   d. (2x + 4) • 5  
   e. (10x + 20) + 12  
   f. (10x + 32) • 10  
   g. (100x + 320) - 320  
   h. 100x + 100 

2. x is chosen, 2x doubles x, 2x + 4 is four more than 2x, 5(2x + 4) is five times the previous number, 5(2x + 4) + 12 is 12 more than the previous number, 10[5(2x + 4) + 12] multiples the previous number by 10, 10[5(2x + 4) + 12] - 320 reduces the previous number by 320, crossing out the zeros (dividing by 100) leaves x.

3. Sample answer:
   5  
   5 • 2  
   10 + 18  
   28 ÷ 2  
   14 ÷ 5 

Your answer is 9. Your answer does not depend on the number you choose. Your answer will always be 15 if you change Step 3 to "Add 30 to your answer." You have to add twice the value you want as an answer. Then when you divide your answer in Step 3 by 2 and subtract your original answer, you will always get the value you want as an answer.

4. Answers will vary.

**2.5 Guided Practice (pp. 106-108)**

1. 4x + 9 = -3x + 2  
   4x + 9 + 3x = -3x + 2 + 3x  
   7x + 9 = 2  
   7x + 9 - 9 = 2 - 9  
   7x = -7  
   x = -1 

2. 14x + 3(7 - x) = -1  
   14x + 21 - 3x = -1  
   11x + 21 = -1  
   11x + 21 - 21 = -1 - 21  
   11x = -22  
   x = -2 

3. A = \frac{1}{2}bh  
   2A = bh  
   \frac{2A}{h} = b 

4. Symmetric Property of Equality  
5. Transitive Property of Equality  
6. Reflexive Property of Equality

**2.5 Exercises (pp. 108-111)**

**Skill Practice**

1. Reflexive Property of Equality for angle measure
2. To check your answer, substitute \( a = \frac{r - 154}{0.70} \) into the original equation. Simply to see if the result is a true statement.

3. 3x - 12 = 7x + 8  
   -4x = 8  
   x = -4 

4. 5(x - 1) = 4x + 13  
   5x - 5 = 4x + 13  
   x = 18 

5. D; The statement illustrates the Transitive Property of Equality for segment length.

6. 5x - 10 = -40  
   5x = -30  
   x = -6 

7. 4x + 9 = 16 - 3x  
   7x + 9 = 16  
   x = 1 

8. 5(3x - 20) = -10  
   15x - 100 = -10  
   15x = 90  
   x = 6 

9. 3(2x + 11) = 9  
   6x + 33 = 9  
   6x = -24  
   x = -4 

Geography  
Worked-Out Solution Key
Chapter 2, continued

10. $2(-x - 5) = 12$ Given
   $-2x - 10 = 12$ Distributive Property
   $-2x = 22$ Addition Property of Equality
   $x = -11$ Division Property of Equality

11. $44 - 2(3x + 4) = -18x$ Given
    $44 - 6x - 8 = -18x$ Distributive Property
    $-6x + 36 = -18x$ Simplify
    $36 = -12x$ Addition Property of Equality
    $-3 = x$ Division Property of Equality

12. $4(5x - 9) = -2(x + 7)$ Given
    $20x - 36 = -2x - 14$ Distributive Property
    $22x - 36 = -14$ Addition Property of Equality
    $22x = 22$ Division Property of Equality
    $x = 1$ Division Property of Equality

13. $2x - 15 - x = 21 + 10x$ Given
    $x - 15 = 21 + 10x$ Simplify
    $-9x - 15 = 21$ Subtraction Property of Equality
    $-9x = 36$ Addition Property of Equality
    $x = -4$ Division Property of Equality

14. $3(7x - 9) - 19x = -15$ Given
    $21x - 27 - 19x = -15$ Distributive Property
    $2x - 27 = -15$ Simplify
    $2x = 12$ Addition Property of Equality
    $x = 6$ Division Property of Equality

15. $5x + y = 18$ Given
    $y = -5x + 18$ Subtraction Property of Equality

16. $-4x + 2y = 8$ Given
    $2y = 4x + 8$ Addition Property of Equality
    $y = 2x + 4$ Division Property of Equality

17. $12 - 3y = 30x$ Given
    $-3y = 30x - 12$ Subtraction Property of Equality
    $y = -10x + 4$ Division Property of Equality

18. $3x + 9y = -7$ Given
    $9y = -3x - 7$ Subtraction Property of Equality
    $y = -\frac{1}{3}x - \frac{7}{9}$ Division Property of Equality

19. $2y + 0.5x = 16$ Given
    $2y = -0.5x + 16$ Subtraction Property of Equality
    $y = -0.25x + 8$ Division Property of Equality

20. $\frac{1}{2}x - \frac{3}{4}y = -2$ Given
    $\frac{3}{4}y = \frac{1}{2}x + 2$ Subtraction Property of Equality
    $y = \frac{2}{3}x + \frac{8}{3}$ Division Property of Equality

21. If $AB = 20$, then $AB + CD = 20 + CD$.

22. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.

23. If $AB = CD$, then $AB + EF = CD + EF$.

24. If $5(x + 8) = 2$, then $5x + 40 = 2$.

25. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

26. In step 2, the Subtraction Property of Equality should have been used instead of the Addition Property of Equality.
    $7x = x + 24$ Given
    $6x = 24$ Subtraction Property of Equality
    $x = 4$ Division Property of Equality

27. Answers will vary.

28. Equation (Reason)
    $AD = AB, DC = BC$ (Given)
    $AC = AC$ (Reflexive Property of Equality)
    $AD + DC = AB + DC$ (Addition Property of Equality)
    $AD + DC = AB + BC$ (Substitution)
    $AD + DC + AC = AB + BC + AC$ (Addition Property of Equality)

29. Equation (Reason)
    $AD = CB, DC = BA$ (Given)
    $AC = AC$ (Reflexive Property of Equality)
    $AD + DC = CB + DC$ (Addition Property of Equality)
    $AD + DC = CB + BA$ (Substitution)
    $AD + DC + AC = CB + BA + AC$ (Addition Property of Equality)

30. $ZY + YX = XZ \rightarrow ZY = ZX - YX$
    $YX + XW = YW \rightarrow XW = YW - YX$
    $ZY = (5x + 17) - 3 = 5x + 14$
    $XW = (10 - 2x) - 3 = 7 - 2x$
    Because $\overline{ZY} \cong \overline{XW}$, $ZY = XW$.
    $5x + 14 = 7 - 2x$
    $7x + 14 = 7$
    $7x = -7$
    $x = -1$
    So, $ZY = 5(-1) + 14 = 9$ and $XW = 7 - 2(-1) = 9$.

Problem Solving

31. $P = 2l + 2w$ Given
    $P - 2w = 2l$ Subtraction Property of Equality
    $\frac{P}{2} - w = l$ Division Property of Equality

When $P = 55$ and $w = 11$:
    $l = \frac{55}{2} - 11 = 16.5$

The length is 16.5 meters.

Geometry

42

Worked-Out Solution Key
Chapter 2, continued

32. \( A = \frac{1}{2}bh \)  
Given
\[ 2A = bh \]  
Multiplication Property of Equality
\[ \frac{2A}{b} = h \]  
Division Property of Equality
When \( A = 1768 \) and \( b = 52 \):
\[ h = \frac{2(1768)}{52} = 68 \]
The height is 68 inches.

33. | Equation | Explanation | Reason |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( m\angle 1 = m\angle A ), ( m\angle EHF = 90^\circ ), ( m\angle GHF = 90^\circ )</td>
<td>Marked in diagram.</td>
<td>Given</td>
</tr>
<tr>
<td>( m\angle EHF = \angle GHF )</td>
<td>Substitute ( m\angle GHF ) for ( 90^\circ ).</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>( m\angle EHF = m\angle 1 + m\angle 2 )</td>
<td>Add measures of adjacent angles.</td>
<td>Angle Addition Postulate</td>
</tr>
<tr>
<td>( m\angle GHF = m\angle 3 + m\angle 4 )</td>
<td>Write expressions equal to the angle measures.</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 )</td>
<td>Substitute ( m\angle 1 ) for ( m\angle 4 ).</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>( m\angle 2 = m\angle 3 )</td>
<td>Subtract ( m\angle 1 ) from each side.</td>
<td>Subtraction Property of Equality</td>
</tr>
</tbody>
</table>

34. a. \[ \begin{array}{c}
A \quad B \quad C \quad D
\end{array} \]

b. \( AB + BC = AC \)

\( BC + CD = BD \)

Because \( AC = BD, AB + BC = BC + CD \), so, \( AB \) must equal \( CD \).

c. \( AC = BD \)  
Given
\[ BC = BC \]  
Reflexive Property of Equality
\[ AC = AB + BC \]  
Segment Addition Postulate
\[ BD = BC + CD \]  
Segment Addition Postulate
\[ AB + BC = BC + CD \]  
Substitution Property of Equality
\[ AB = CD \]  
Subtraction Property of Equality

35. \( m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \)
\( m\angle 1 + m\angle 2 + m\angle 1 = 180^\circ \)
\( m\angle 2 = 180^\circ - 2m\angle 1 \)
\( m\angle 1 + m\angle 2 = 148^\circ \)
\( m\angle 2 = 148^\circ - m\angle 1 \)
\( 180^\circ - 2m\angle 1 = 148^\circ - m\angle 1 \)
\( 32^\circ = m\angle 1 \)

So, \( m\angle 2 = 180^\circ - 2(32^\circ) = 116^\circ \).

36. a. \( C = \frac{5}{9}(F - 32) \)  
Given
\[ C = \frac{5}{9}F - \frac{160}{9} \]  
Distributive Property
\[ C + \frac{160}{9} = \frac{5}{9}F \]  
Addition Property of Equality
\[ \frac{9}{5}C + 32 = F \]  
Mult. Property of Equality
\[ \frac{9}{5}C + 32 = F \]  
Distributive Property

b. Temperature (°C) \( \begin{array}{c}
0 \quad 20 \quad 32 \quad 41
\end{array} \)

Temperature (°F) \( \begin{array}{c}
32 \quad 68 \quad 89.6 \quad 105.8
\end{array} \)

c. \[ \begin{array}{c}
\text{Temperature (°C)}
\end{array} \]

This is a linear function.

37. The relationship is symmetric. “Yen worked the same hours as Jim” is the same as “Jim worked the same hours as Yen.”

38. The relationship is transitive. One example of an explanation is that “\( x \) is less than \( 2 \) and \( 2 \) is less than \( 1 \), so \( x \) is less than \( 1 \).”

Mixed Review

39. \( m\angle ADB + m\angle CDB = 124 \)
\[ 4x - 8 + 7x + 22 = 124 \]
\[ 11x + 14 = 124 \]
\[ 11x = 110 \]
\[ x = 10 \]

So, \( m\angle ADB = 4(10) - 8 = 32^\circ \).

40. From Exercise 39, you know that \( x = 10 \).

So, \( m\angle BDC = 7(10) + 22 = 92^\circ \).

41. Sample answer: A counterexample is a rectangle because it is a polygon, but it has four sides. Because a counterexample exists, the conjecture is false.

42. If \( m\angle X = m\angle Y \) and \( m\angle Y = m\angle Z \), then \( m\angle X \) is equal to \( m\angle Z \). This is the Transitive Property of Equality for angle measure.
Chapter 2, continued

Quiz 2.4–2.5 (p. 111)
1. True 2. False 3. True
4. \(x + 20 = 35\) Given
\[x = 15\] Subtraction Property of Equality
5. \(5x - 14 = 16 + 3x\) Given
\[2x - 14 = 16\] Subtraction Property of Equality
\[2x = 30\] Addition Property of Equality
\[x = 15\] Division Property of Equality
6. If \(AB = CD\), then \(AB - EF = CD - EF\).
7. If \(a = b\) and \(b = c\), then \(a = c\).

Lesson 2.6

2.6 Guided Practice (pp. 112–115)
1. 

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (AC = AB + AB)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (AB + BC = AC)</td>
<td>2. Segment Addition Postulate</td>
</tr>
<tr>
<td>3. (AB + AB = AB + BC)</td>
<td>3. Substitution Property</td>
</tr>
<tr>
<td>4. (AB = BC)</td>
<td>4. Subtraction Property of Equality</td>
</tr>
</tbody>
</table>

2. Reflexive Property of Segment Congruence
3. Symmetric Property of Angle Congruence
4. Step 5 would be \(MB + MB = AB\).
   Step 6 would be \(2MB = AB\).
   Step 7 would be \(MB = \frac{1}{2}AB\).
5. It does not matter what the actual distances are in order to prove the relationship between \(AB\) and \(CD\). What matters are the positions of the stores relative to each other.
6. \(A\)

From Example 4, you know that \(AB = CD\). You also know that \(BE = EC\) by the definition of a midpoint. By the definition of congruent segments, \(BE = EC\). So, \(AB + BE = EC + CD\) or \(AE = ED\). The food court and the bookstore are also the same distance from the clothing store.

2.6 Exercises (pp. 116–119)

Skill Practice
1. A theorem is a statement that can be proven. A postulate is a rule that is accepted without proof.
2. Sample answer: You can use postulates such as the Angle Addition Postulate. You can use properties such as the Reflexive Property of Equality. You can also use definitions such as the definition of a right angle.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (RT = 5, RS = 5), (RT \cong TS)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (RS = RT)</td>
<td>2. Transitive Property of Equality</td>
</tr>
<tr>
<td>3. (RT = TS)</td>
<td>3. Definition of congruent segments</td>
</tr>
<tr>
<td>4. (RS = TS)</td>
<td>4. Transitive Property of Equality</td>
</tr>
<tr>
<td>5. (RS \cong TS)</td>
<td>5. Definition of congruent segments</td>
</tr>
</tbody>
</table>

17. If \(QR \cong PQ\) and \(RS \cong PQ\), then \(QR = PQ\) and \(RS = PQ\) by the definition of congruent segments. \(PQ = RS\) by the Symmetric Property of Equality. If \(QR = PQ\) and \(PQ = RS\), then \(QR = RS\) by the Transitive Property of Equality.
\[QR = RS\]
\[2x + 5 = 10 - 3x\] Given
\[5x + 5 = 10\] Addition Property of Equality
\[5x = 5\] Subtraction Property of Equality
\[x = 1\] Division Property of Equality
Chapter 2, continued

18. You know that $6x + (3x - 9)^2 = m\angle ABC$ by the Angle Addition Postulate. You are given that $m\angle ABC = 90^\circ$. So, $6x + (3x - 9) = 90$.

\[
6x + (3x - 9) = 90 \\
9x = 99 \\
x = 11
\]

19. Writing a proof is an example of deductive reasoning because you are using facts, definitions, accepted properties, and laws of logic to reach a desired conclusion.

20. a. You are given that $P$ is the midpoint of $MN$, $AB \cong MP$, and $PN = x$. You know that $MP \cong PN$ by the definition of midpoint. If $AB \cong MP$ and $MP \cong PN$, then $AB \cong PN$ by the Transitive Property of Segment Congruence. $AB = PN$ by the definition of congruent segments. So, $AB = x$.

b. You are given that $P$ is the midpoint of $MN$ and $PN = x$. You know that $AB = PN$ by the Segment Addition Postulate and $MP = PN$ by the definition of midpoint. $MP = PN$ by the definition of congruent segments. $MN = PN + PN$ by the Substitution Property of Equality. So, $MN = x + x = 2x$.

c. You are given that $P$ is the midpoint of $MN$, $Q$ is the midpoint of $MP$, and $PN = x$. You know that $PN = PN$ and $MQ = QP$ by the definition of midpoint. $MP = PN$ and $MQ = QP$ by the definition of congruent segments. $MP = MQ + QP$ by the Segment Addition Postulate. $MP = MQ + QP$, and so $PN = MP + QP$, or $x = 2MQ$, by the Substitution Property of Equality. So, $MQ = \frac{1}{2}x$ by the Division Property of Equality.

d. You know that $MN = MQ + NQ$ by the Segment Addition Postulate. From part (b) you know that $MN = 2x$ and from part (c) you know that $MQ = \frac{1}{2}x$.

Using substitution, $2x = \frac{1}{2}x + NQ$. So, $NQ = \frac{3}{2}x$.

Problem Solving

21. Statements | Reasons
--- | ---
1. $TV$ bisects $\angle UTW$ | 1. Given
2. $\angle 1 \cong \angle 2$ | 2. Definition of angle bisector
3. $\angle 2 \cong \angle 3$ | 3. Given
4. $\angle 1 \cong \angle 3$ | 4. Transitive Property of Angle Congruence

22. Statements | Reasons
--- | ---
1. $QS$ is an angle bisector of $\angle PQR$ | D. Given
2. $\angle PQS \cong \angle SQR$ | A. Definition of angle bisector
3. $m\angle PQS = m\angle SQR$ | F. Definition of congruent angles
4. $m\angle PQS + m\angle SQR = m\angle PQR$ | C. Angle addition Postulate
5. $m\angle PQS + m\angle SQR = m\angle PQR$ | G. Substitution Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$ | B. Distributive Property
7. $m\angle PQS = \frac{1}{2}m\angle PQR$ | E. Division Property of Equality

23. Statements | Reasons
--- | ---
1. $2AB = AC$ | 1. Given
2. $AB + AB = AC$ | 2. Distributive Property
3. $AB + BC = AC$ | 3. Segment Addition Postulate
4. $AB + AB = AB + BC$ | 4. Transitive Property of Equality
5. $AB = BC$ | 5. Subtraction Property of Equality

24. Statements | Reasons
--- | ---
1. $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 1 = 62^\circ$ | 1. Given
2. $m\angle 2 = 180^\circ - m\angle 1$ | 2. Subtraction Property of Equality
3. $m\angle 2 = 180^\circ - 62^\circ$ | 3. Substitution Property of Equality
4. $m\angle 2 = 118^\circ$ | 4. Simplify.

25. Statements | Reasons
--- | ---
1. $A$ is an angle. | 1. Given
2. $m\angle A = m\angle A$ | 2. Reflective Property of Equality
3. $\angle A \cong \angle A$ | 3. Definition of congruent angles
Chapter 2, continued

26. Statements | Reasons
--- | ---
1. \(WX \cong XY\) and \(XY \cong YZ\) | 1. Given
2. \(WX = XY\) and \(XY = YZ\) | 2. Definition of congruent segments
3. \(WX = YZ\) | 3. Transitive Property of Equality
4. \(WX \cong YZ\) | 4. Definition of congruent segments

27. You know that \(\angle 1 \equiv \angle 3\) by the Transitive Property of Congruent Angles, so all the angles are the same. The sculpture is an equiangular triangle.

28. Sample answer: All of the line segments are congruent because when you copy and paste a line segment, the length of each line segment is the same. So, all of the segments are congruent by the definition of congruent segments.

29. 

<table>
<thead>
<tr>
<th>Restaurant</th>
<th>Shoe store</th>
<th>Movie theater</th>
<th>Cafe</th>
<th>Florist</th>
<th>Dry cleaners</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>S</td>
<td>T</td>
<td>U</td>
<td>V</td>
<td>W</td>
</tr>
</tbody>
</table>

b. Given: \(RS = UV\)
\(ST = TU = VW\)
Prove: \(RT = UW\)

e. 

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1. (RS = UV) and (ST = TU = VW)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (RT = RS + ST)</td>
<td>2. Segment Addition Postulate</td>
</tr>
<tr>
<td>3. (UW = UV + VW)</td>
<td>3. Segment Addition Postulate</td>
</tr>
<tr>
<td>4. (RT = UV + VW)</td>
<td>4. Substitution Property of Equality</td>
</tr>
<tr>
<td>5. (RT = UW)</td>
<td>5. Transitive Property of Equality</td>
</tr>
</tbody>
</table>

30. 

<table>
<thead>
<tr>
<th>Moon Valley</th>
<th>Lakewood City</th>
<th>Springfield</th>
<th>Bettsville</th>
<th>Janisburg</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>50 mi</td>
<td>x</td>
<td>S</td>
<td>B</td>
</tr>
</tbody>
</table>

b. Sample answer:

![Diagram of cities connected by lines showing distances and directions.]

e. In the first diagram, all of the cities lie in a straight line, so you know the distance between Moon Valley and Lakewood City, and between Bettsville and Janisburg, is 50 miles. You do not know this distance from the second diagram.

Mixed Review

31. Let \(\angle 2\) be a complement to \(\angle 1\).
\(m\angle 1 + m\angle 2 = 90\degree\)
\(47\degree + m\angle 2 = 90\degree\)
\(m\angle 2 = 43\degree\)
Let \(\angle 3\) be a supplement to \(\angle 1\).
\(m\angle 1 + m\angle 3 = 180\degree\)
\(47\degree + m\angle 3 = 180\degree\)
\(m\angle 3 = 133\degree\)

32. Let \(\angle 2\) be a complement to \(\angle 1\).
\(m\angle 1 + m\angle 2 = 90\degree\)
\(29\degree + m\angle 2 = 90\degree\)
\(m\angle 2 = 61\degree\)
Let \(\angle 3\) be a supplement to \(\angle 1\).
\(m\angle 1 + m\angle 3 = 180\degree\)
\(29\degree + m\angle 3 = 180\degree\)
\(m\angle 3 = 151\degree\)

33. Let \(\angle 2\) be a complement to \(\angle 1\).
\(m\angle 1 + m\angle 2 = 90\degree\)
\(89\degree + m\angle 2 = 90\degree\)
\(m\angle 2 = 1\degree\)
Let \(\angle 3\) be a supplement to \(\angle 1\).
\(m\angle 1 + m\angle 3 = 180\degree\)
\(89\degree + m\angle 3 = 180\degree\)
\(m\angle 3 = 91\degree\)

34. \(5x + 14 = -16\) Given
\(5x = -30\) Subtraction Property of Equality
\(x = -6\) Division Property of Equality

35. \(2x - 9 = 15 - 4x\) Given
\(6x - 9 = 15\) Addition Property of Equality
\(6x = 24\) Addition Property of Equality
\(x = 4\) Division Property of Equality

36. \(x + 28 = -11 - 3x - 17\) Given
\(x + 28 = -3x - 28\) Simplify.
\(4x + 28 = -28\) Addition Property of Equality
\(4x = -56\) Subtraction Property of Equality
\(x = -14\) Division Property of Equality

Problem Solving Workshop 2.6 (p. 121)

1. a. The proofs both use the definition of midpoint and properties of equality to prove the same conclusion. The proof on page 120 uses the Reflexive Property of Equality and the Substitution Property of Equality to prove \(FM = SB\). The proof on page 115 uses the Transitive Property of Congruence to prove \(AB = CD\). The letters chosen to represent the locations are different but the conclusion is the same.
Chapter 2, continued

b. Answers will vary.

2. Substitution Property of Equality, because you could substitute \( m \angle C \) for \( m \angle B \).

3. Food court Music Book store Shoe store Toy store

Given: \( B \) is halfway between \( A \) and \( C \)
\( C \) is halfway between \( B \) and \( D \)
\( D \) is halfway between \( C \) and \( E \)

Prove: \( AB = DE \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( B ) is halfway between ( A ) and ( C ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( C ) is halfway between ( B ) and ( D ).</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( D ) is halfway between ( C ) and ( E ).</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( B ) is the midpoint of ( AC ).</td>
<td>4. Definition of midpoint</td>
</tr>
<tr>
<td>5. ( C ) is the midpoint of ( BD ).</td>
<td>5. Definition of midpoint</td>
</tr>
<tr>
<td>6. ( D ) is the midpoint of ( CE ).</td>
<td>6. Definition of midpoint</td>
</tr>
<tr>
<td>7. ( AB = BC, BC = CD, ) and ( CD = DE )</td>
<td>7. Definition of midpoint</td>
</tr>
<tr>
<td>8. ( AB = CD )</td>
<td>8. Transitive Property of Equality</td>
</tr>
<tr>
<td>10. ( AB = DE )</td>
<td>10. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

4. Given: \( \angle BAC \cong \angle CAD \cong \angle DAE \cong \angle EAF \)

Prove: \( m \angle CAD = \frac{1}{2} m \angle BAF \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle BAC \cong \angle CAD \cong \angle DAE \cong \angle EAF )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m \angle BAC = m \angle CAD = m \angle DAE = m \angle EAF )</td>
<td>2. Definition of congruent angles</td>
</tr>
<tr>
<td>3. ( m \angle CAD = m \angle CAD + m \angle DAE )</td>
<td>3. Angle Addition Postulate</td>
</tr>
<tr>
<td>4. ( m \angle BAF = m \angle BAC + m \angle CAD + m \angle DAE + m \angle EAF )</td>
<td>4. Angle Addition Postulate</td>
</tr>
<tr>
<td>5. ( m \angle BAF = m \angle CAD + m \angle DAE + m \angle CAD + m \angle DAE )</td>
<td>5. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

5. a. Both proofs use the same reasoning to prove the Symmetric Property of Angle Congruence and the Symmetric Property of Segment Congruence. The difference is that one proof deals with angle congruence and the other proof deals with segment congruence.

b. If \( FG \cong DE \) is the second statement, then the reason would have to be the Symmetric Property of Segment Congruence. This is not a valid reason in this proof because the Symmetric Property of Segment Congruence is what is trying to be proven, so it is an unproven theorem.

Lesson 2.7

Investigating Geometry Activity 2.7 (pp. 122–123)

1. \( \angle AEC \) and \( \angle AED \) are supplementary.
2. \( \angle AED \) and \( \angle DEB \) are supplementary.
3. \( m \angle AEC \) is equal to \( m \angle DEB \).

4. When you move \( C \) to a different position it changes the measure of the angles, but it does not change the angle relationships.

Two angles supplementary to the same angle are congruent.

5. Yes, let \( \angle A \) and \( \angle B \) be two angles supplementary to \( \angle C \). Then \( m \angle A + m \angle C = 180^\circ, m \angle B + m \angle C = 180^\circ \to m \angle A + m \angle C = m \angle B + m \angle C \to m \angle A = m \angle B, \) so \( \angle A \cong \angle B \).

6. Yes, the angle measures change, but the angle relationship stays the same.

7. If the non-adjacent sides of \( \angle CEG \) and \( \angle GEB \) are perpendicular, then \( \angle CEG \) and \( \angle GEB \) are complementary angles.

8. If two angles are vertical angles formed by intersecting lines, then the two angles are congruent.

9. The vertical angles are:
\( \angle AEC \) and \( \angle BED \), \( \angle CEG \) and \( \angle FED \), \( \angle GEB \) and \( \angle EAF \), \( \angle CEB \) and \( \angle DEA \), \( \angle GED \) and \( \angle CEF \), \( \angle AEG \) and \( \angle BEF \). The vertical angles in each pair are congruent.

2.7 Guided Practice (pp. 125–127)

1. You save two steps using the Right Angles Congruence Theorem. The following is the proof without using the Right Angle Congruence Theorem.
2. Case 1: Two angles complementary to the same angle are congruent.
Given: \( \angle 4 \) and \( \angle 5 \) are complements.
\( \angle 5 \) and \( \angle 6 \) are complements.
Prove: \( \angle 4 \cong \angle 6 \)

3. Given: \( m\angle 1 = 112^\circ \)
\( m\angle 2 = m\angle 2 = 71^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( 112^\circ + 71^\circ = 183^\circ \)
\( m\angle 2 = 68^\circ \)
\( m\angle 4 = m\angle 2 = 68^\circ \)
\( m\angle 3 = m\angle 1 = 113^\circ \)

4. Given: \( m\angle 2 = 67^\circ \)
\( m\angle 4 = m\angle 2 = 67^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( 180^\circ + 67^\circ = 247^\circ \)
\( m\angle 2 = 68^\circ \)
\( m\angle 3 = m\angle 1 = 113^\circ \)

5. Given: \( m\angle 4 = 71^\circ \)
\( m\angle 2 = m\angle 4 = 71^\circ \)
\( m\angle 3 + m\angle 4 = 180^\circ \)
\( m\angle 3 = 109^\circ \)
\( m\angle 1 = m\angle 3 = 109^\circ \)

6. Congruent Supplements Theorem

7. \( 32 + (3x + 1) = 180 \)
\( 33 + 3x = 180 \)
\( 3x = 147 \)
\( x = 49 \)

8. \( m\angle TPS = m\angle QPR \)
\( m\angle QPR = 3(49) + 1 = 148^\circ \)
\( m\angle TPS = 148^\circ \)

2.7 Exercises (pp. 127–131)

Skill Practice

1. If two lines intersect at a point, then the vertical angles formed by the intersecting lines are congruent.

2. The sum of the measures of complementary angles is \( 90^\circ \). The sum of the measures of supplementary angles is \( 180^\circ \). The measures of vertical angles are equal. The sum of the angle measures of a linear pair is \( 180^\circ \).

3. \( \angle PSM \) and \( \angle PSR \) are both right angles. So, \( \angle PSM \cong \angle PSR \) by the Right Angles Congruence Theorem. \( m\angle MSN = 50^\circ \) and \( m\angle PSQ = 50^\circ \), so \( m\angle MSN \cong m\angle PSQ \) by the definition of congruent angles. \( m\angle PSN \) is the complement of \( m\angle MSN \) and \( m\angle RSQ \) is the complement of \( m\angle PSQ \). So, \( m\angle PSN \cong m\angle RSQ \) by the Congruent Complements Theorem.

4. \( \angle ABC \cong \angle DEF \) and \( \angle CBE \cong \angle FEB \) by the Congruent Supplements Theorem.

5. \( \angle FGH \cong \angle WXZ \); \( \angle WXZ \) is a right angle because \( 58^\circ + 32^\circ = 90^\circ \), so \( \angle FGH \cong \angle WXZ \) by the Right Angles Congruence Theorem.

6. \( \angle KMJ \) and \( \angle KMG \) are both right angles. So, \( \angle KMJ \cong \angle KMG \) by the Right Angles Congruence Theorem. \( \angle GML \) and \( \angle HML \) and \( \angle GMH \) and \( \angle LMJ \) are two pairs of vertical angles. So, \( \angle GML \cong \angle HML \) and \( \angle GMH \cong \angle LMJ \) by the Vertical Angles Congruence Theorem.

7. The four angles are congruent right angles. They are all right angles by the definition of perpendicular lines. All right angles are congruent by the Right Angles Congruence Theorem.

8. Given: \( m\angle 1 = 145^\circ \)
\( m\angle 3 = m\angle 1 = 145^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( 145^\circ + m\angle 2 = 180^\circ \)
\( 145^\circ + m\angle 2 = 25^\circ \)
\( m\angle 2 = 35^\circ \)
\( m\angle 4 = m\angle 2 = 35^\circ \)
\( m\angle 4 = m\angle 2 = 142^\circ \)

9. Given: \( m\angle 3 = 168^\circ \)
\( m\angle 2 = m\angle 3 = 168^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)

10. Given: \( m\angle 4 = 37^\circ \)
\( m\angle 3 + m\angle 4 = 180^\circ \)
\( m\angle 3 + m\angle 4 = 180^\circ \)
\( m\angle 3 + m\angle 4 = 180^\circ \)
\( m\angle 3 + m\angle 4 = 180^\circ \)

11. Given: \( m\angle 2 = 62^\circ \)
\( m\angle 4 = m\angle 2 = 62^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)
\( m\angle 1 + m\angle 2 = 180^\circ \)

12. Using the Vertical Angles Congruence Theorem:
\( 8x + 7 = 9x - 4 \)
\( 11 = x \)
\( 5y = 7y - 34 \)
\( -2y = -34 \)
\( y = 17 \)
Chapter 2, continued

13. Using the Vertical Angles Congruence Theorem:
   \[ 4x = 6x - 26 \]
   \[-2x = -26 \]
   \[ x = 13 \]
   \[ 6y + 8 = 7y - 12 \]
   \[ 20 = y \]

14. Using the Vertical Angles Congruence Theorem:
   \[ 10x - 4 = 6(x + 2) \]
   \[ 10x - 4 = 6x + 12 \]
   \[ 4x = 16 \]
   \[ x = 4 \]
   \[ 18y - 18 = 16y \]
   \[ -18 = -2y \]
   \[ 9 = y \]

15. The error is assuming that \( \angle 1 \) and \( \angle 4 \) and \( \angle 2 \) and \( \angle 3 \) are vertical angle pairs. They are not formed by the intersection of two lines. So, \( \angle 1 \not\equiv \angle 4 \) and \( \angle 2 \not\equiv \angle 3 \).

16. D: \( m\angle A + m\angle D = 90^\circ \)
   \[ 4x^2 + m\angle D = 90^\circ \]
   \[ m\angle D = 90 - 4x^2 \]

17. 30\(^\circ\); If \( m\angle 3 = 30^\circ \), then \( m\angle 6 = 30^\circ \) by the Vertical Angles Congruence Theorem.

18. 25\(^\circ\); If \( m\angle BHF = 115^\circ \), then \( m\angle 2 = 65^\circ \) by the Linear Pair Postulate. Because \( m\angle BHG = 90^\circ \), \( m\angle BHD = 90^\circ \) by the Linear Pair Postulate. \( \angle 3 \) is the complement of \( \angle 2 \) because \( m\angle BHD = 90^\circ \). So, \( m\angle 3 = 25^\circ \).

19. 27\(^\circ\); If \( m\angle 6 = 27^\circ \), then \( m\angle 1 = 27^\circ \) by the Congruent Complements Theorem.

20. 133\(^\circ\); If \( m\angle DHF = 133^\circ \), then \( m\angle CHG = 133^\circ \) by the Vertical Angles Congruence Theorem.

21. 58\(^\circ\); If \( m\angle BHG = 90^\circ \), then \( m\angle BHD = 90^\circ \) by the Linear Pair Postulate. \( \angle 2 \) is the complement of \( \angle 3 \) because \( m\angle BHD = 90^\circ \). So, \( m\angle 2 = 58^\circ \).

22. The statement is false. \( \angle 1 \) and \( \angle 2 \) are not a linear pair and you know the intersecting lines are not perpendicular, so \( \angle 1 \not\equiv \angle 2 \).

23. The statement is true. \( \angle 1 \) and \( \angle 3 \) are vertical angles.

24. The statement is false. \( \angle 1 \) and \( \angle 4 \) are a linear pair and you know that the intersecting lines are not perpendicular, so \( \angle 1 \not\equiv \angle 4 \).

25. The statement is false. \( \angle 2 \) and \( \angle 3 \) are a linear pair and you know that the intersecting lines are perpendicular, so \( \angle 3 \not\equiv \angle 2 \).

26. The statement is true. \( \angle 2 \) and \( \angle 4 \) are vertical angles.

27. The statement is true. \( \angle 3 \) and \( \angle 4 \) are a linear pair, so they are supplementary.

28. Using the Linear Pair Postulate:
   \[ 10y + 3y + 11 = 180 \]
   \[ 13y = 169 \]
   \[ y = 13 \]
   \[ 7x + 4 + 4x - 22 = 180 \]
   \[ 11x - 18 = 180 \]
   \[ 11x = 198 \]
   \[ x = 18 \]

The measure of each angle is:
\[ 3(13) + 11 = 50^\circ \]
\[ 10(13) = 130^\circ \]
\[ 4(18) - 22 = 50^\circ \]
\[ 7(18) + 4 = 130^\circ \]

29. Using the Vertical Angle Congruence Theorem:
   \[ 2(5x - 5) = 6x + 50 \]
   \[ 10x - 10 = 6x + 50 \]
   \[ 4x = 60 \]
   \[ x = 15 \]
   \[ 5y + 5 = 7y - 9 \]
   \[ 14 = 2y \]
   \[ 7 = y \]

The measure of each angle is:
\[ 5(7) + 5 = 40^\circ \]
\[ 2(5 \cdot 15 - 5) = 140^\circ \]
\[ 7(7) - 9 = 40^\circ \]
\[ 6(15) + 50 = 140^\circ \]

30. Sample answer: \( m\angle ABY = 80^\circ \) because \( \overline{XY} \) bisects \( \angle ABC \), \( m\angle CBX = 100^\circ \) because \( \overline{CBY} \) and \( \overline{CBX} \) are supplementary.

31. \( \angle EGH \equiv \angle FGH \) by the definition of angle bisector.

32. \( \angle 1 \equiv \angle 9 \) by the Congruent Supplements Theorem.

33. Sample answer: \( \angle AED \equiv \angle BEC \) by the definition of perpendicular lines and the Vertical Angles Congruence Theorem.

34. \( \angle 5 \equiv \angle 1 \) by the Congruent Complements Theorem.

35. Lines \( l \) and \( m \) bisect supplementary angles. The sum of supplementary angles is 180\(^\circ\); so half the sum of each angle pair is 90\(^\circ\). Line \( l \) is perpendicular to line \( m \) by the definition of perpendicular lines.
Chapter 2, continued

Problem Solving

36. Statements | Reasons
--- | ---
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 4$ are supplements. $\angle 1 \cong \angle 4$ | 1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 3 + m\angle 4 = 180^\circ$ | 2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ | 3. Transitive Property of Equality
4. $m\angle 1 = m\angle 4$ | 4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$ | 5. Substitution Property of Equality
6. $m\angle 2 = m\angle 3$ | 6. Subtraction Property of Equality
7. $\angle 2 \cong \angle 3$ | 7. Definition of congruent angles

37. Statements | Reasons
--- | ---
1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements. | 1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$ | 2. Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$ | 3. Transitive Property of Equality
4. $m\angle 2 = m\angle 3$ | 4. Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$ | 5. Definition of congruent angles

38. Statements | Reasons
--- | ---
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle. | 1. Given
2. $m\angle ABD = 90^\circ$; $m\angle CBE = 90^\circ$ | 2. Definition of right angle
3. $m\angle ABC + m\angle CBD = m\angle ABD$ $m\angle CBD + m\angle DBE = m\angle CBE$ | 3. Angle Addition Postulate
4. $\angle ABC$ and $\angle CBD$ are complements. $\angle CBD$ and $\angle DBE$ are complements. | 4. Definition of complementary angles
5. $\angle ABC \cong \angle DBE$ | 5. Congruent Supplements Theorem

39. Statements | Reasons
--- | ---
1. $\overline{JK} \perp \overline{JM}, \overline{KL} \perp \overline{ML}$ $\angle J \cong \angle M$, $\angle K \cong \angle L$ | 1. Given
2. $\angle J$ is a right angle; $\angle L$ is a right angle. | 2. Definition of perpendicular lines
3. $\angle M$ is a right angle; $\angle K$ is a right angle. | 3. Right Angle Congruence Theorem
4. $\overline{JM} \perp \overline{ML}, \overline{JK} \perp \overline{KL}$ | 4. Definition of perpendicular lines

40. a. Given: $m\angle 1 = x^\circ$ $m\angle 2 = (180 - x)^\circ$ because $\angle 1$ and $\angle 2$ are supplements. $m\angle 3 = x^\circ$ because $\angle 1$ and $\angle 3$ are vertical angles. $m\angle 4 = (180 - x)^\circ$ because $\angle 3$ and $\angle 4$ are supplements.
b. Sample answer: $x = 120$ $m\angle 1 = m\angle 3 = 120^\circ$ $m\angle 2 = m\angle 4 = 180 - 120 = 60^\circ$
c. As $\angle 4$ gets larger, $\angle 2$ gets smaller and $\angle 1$ and $\angle 3$ get larger. $\angle 1$ and $\angle 4$ are supplementary and $\angle 2$ and $\angle 3$ are supplementary. As one angle measure gets smaller, the other must get larger to keep the sum of $180^\circ$.

41. Given: $\angle 4$ and $\angle 5$ are complementary. $\angle 6$ and $\angle 7$ are complementary. $\angle 5 \cong \angle 7$
Prove: $\angle 4 \cong \angle 6$
Chapter 2, continued

42. Statements | Reasons
--- | ---
1. ∠1 ≅ ∠3 | 1. Given
2. ∠1 ≅ ∠2 | 2. Vertical Angles Congruence Theorem
3. ∠3 ≅ ∠4 | 3. Vertical Angles Congruence Theorem
4. ∠1 ≅ ∠4 | 4. Transitive Property of Angle Congruence
5. ∠2 ≅ ∠4 | 5. Transitive Property of Angle Congruence
6. m∠4 = m∠6 | 6. Subtraction Property of Equality
7. ∠4 ≅ ∠6 | 7. Definition of congruent angles

43. Statements | Reasons
--- | ---
1. ∠QRS and ∠PSR are supplementary. | 1. Given
2. ∠QRS and ∠QRL are supplementary. | 2. Linear Pair Postulate
3. ∠QRL ≅ ∠PSR | 3. Congruent Supplements Theorem

44. Statements | Reasons
--- | ---
1. ∠1 is complementary to ∠3. | 1. Given
2. ∠2 is complementary to ∠4. | 2. Vertical Angles Congruence Theorem
3. ∠1 ≅ ∠4 | 3. Congruent Complements Theorem

45. Statements | Reasons
--- | ---
1. Given
2. Definition of complementary angles
3. Transitive Property of Equality
4. Definition of congruent angles
5. Substitution Property of Equality
6. Transitive Property of Equality
7. Definition of congruent angles

46. a. m∠3 = m∠7; They are both right angles.
   b. m∠4 = m∠6; They are vertical angles.
   c. m∠8 + m∠6 < 150°; The sum must be equal to 90° because m∠8 + m∠7 + m∠6 = 180°.
   d. If m∠4 = 30°; then m∠5 > m∠4; ∠4 and ∠5 are a linear pair.

47. Statements | Reasons
--- | ---
1. m∠WYZ = m∠TWZ = 45° | 1. Given
2. m∠WYZ ≅ m∠TWZ | 2. Definition of congruent angles
3. m∠W and m∠XYW are a linear pair. m∠TWZ and m∠SWZ are a linear pair.
4. m∠WYZ and m∠XYW are supplements. m∠TWZ and m∠SWZ are supplements.
5. m∠SWZ ≅ m∠XYW | 5. Congruent Supplements Theorem
48. Statements | Reasons
--- | ---
1. The hexagon is regular. | 1. Given
2. The interior angles are congruent. | 2. Definition of regular polygon
3. The measures of the interior angles are equal. | 3. Definition of congruent angles
4. ∠2 and its adjacent interior angle are a linear pair. | 4. Definition of linear pair
5. ∠2 and its adjacent interior angle are supplements. | 5. Linear Pair Postulate
6. The sum of m∠2 and the measure of its adjacent interior angle is 180°. | 6. Definition of supplementary angles
7. The sum of m∠2 and the measure of any interior angle is 180°. | 7. Substitution Property of Equality
8. ∠1 and the interior angle whose sides form two pairs of opposite rays are vertical angles. | 8. Definition of vertical angles
9. ∠1 and the interior angle whose sides form two pairs of opposite rays are congruent. | 9. Vertical Angles Congruence Theorem
10. m∠1 and the measure of any interior angle are equal. | 10. Definition of congruent angles
11. m∠2 + m∠1 = 180° | 11. Substitution Property of Equality

53. The figure is rotated clockwise in each figure. The next figure is:

Recipe 2.6–2.7 (p. 131)
1. B; Symmetric Property of Congruence
2. C; Transitive Property of Congruence
3. A; Reflexive Property of Congruence

Mixed Review of Problem Solving (p. 132)
1. a. Statements | Reasons
--- | ---
1. BD bisects ∠ABC and BC bisects ∠DBE. | 1. Given
2. ∠ABD ≅ ∠DBC and ∠DBC ≅ ∠CBE | 2. Definition of Angle Bisector
3. ∠ABD ≅ ∠CBE | 3. Transitive Property of Congruence
4. m∠ABD = m∠CBE | 4. Definition of congruent angles

b. m∠ABE = m∠ABD + m∠DBC + m∠CBE
m∠ABD = m∠DBC = m∠CBE
Let x = m∠ABD.
99 = x + x + x
99 = 3x
33 = x
So, m∠DBC = 33°.
2. All of the strips will have the same width. They are cut into congruent pieces each time, so all of the strips will have the same width.
3. m∠1 + m∠2 + m∠3 + m∠4 = 360°
m∠1 + m∠1 + 80° + 80° = 360°
2 • m∠1 = 200°
m∠1 = 100°
Chapter 2, continued

4. The Congruence Supplements Theorem can be used because it states that if two angles are supplementary to the same angle, then they are congruent. The sum of any pair of supplementary angles is 180°. So, the Transitive Property of Equality for Angles can be used to show that angles supplementary to the same angle are congruent.

5. a. \( T = c(1 + s) \) Given
   \[ T = c + sc \] Distributive Property
   \[ T - c = sc \] Subtraction Property of Equality
   \[ \frac{T - c}{c} = s \] Division Property of Equality

b. When \( T = 26.75 \) and \( c = 25 \):
   \[ s = \frac{26.75 - 25}{25} = 0.07 \]
   So, the sales tax rate is 7%.

c. Sample answer: You could have first divided each side by \( c \), by the Division Property of Equality. Then subtract 1 from each side by the Subtraction Property of Equality.

6. Sample answer: Either \( m \angle BAC \) or \( m \angle CAD \); because \( \angle GAD \) is a straight angle, if two of the three angles are known, the third angle can be found.

7. You know that \( \angle 1 \) and \( \angle 2 \) are a pair of vertical angles because \( m \angle 1 = m \angle 2 \), while \( m \angle 3 = 3m \angle 1 \).
   \( m \angle 3 = m \angle 4 \) because \( \angle 3 \) and \( \angle 4 \) are the other pair of vertical angles. \( \angle 1 \) and \( \angle 3 \) are a linear pair, so \( m \angle 1 + m \angle 3 = 180° \). Let \( m \angle 1 = x \), then \( m \angle 3 = 3x \) and \( x + 3x = 180 \), or \( x = 45 \). So, \( m \angle 1 = 45° \), \( m \angle 2 = 45° \), \( m \angle 3 = 3(45) = 135° \), and \( m \angle 4 = 135° \).

8. \( \angle BAC \) and \( \angle CAF \) are a linear pair because \( AB \) and \( AF \) are opposite rays. \( m \angle BAC + m \angle CAF \) = 180° by the Linear Pair Postulate.
   \( m \angle CAF = m \angle CAE + m \angle EAF \) and \( m \angle CAF = m \angle CAD + m \angle DAE \) by the Angle Addition Postulate. You know that \( m \angle CAD + m \angle DAE = 90° \) because \( \angle CAD \) and \( \angle DAE \) are complements. \( m \angle BAC + (m \angle CAE + m \angle EAF) = 180° \) and \( m \angle CAE = 90° \) by the Substitution Property of Equality. So, \( m \angle BAC + 90° + m \angle EAF = 180° \), or \( m \angle BAC + m \angle DAE = 90° \), so \( \angle BAC \) and \( \angle EAF \) are complements.

Chapter 2 Review (pp. 134–137)

1. A statement that can be proven is called a theorem.

2. The inverse negates the hypothesis and conclusion of a conditional statement. The converse exchanges the hypothesis and conclusion of a conditional statement.

3. When \( m \angle A = m \angle B \) and \( m \angle B = m \angle C \), then \( m \angle A = m \angle B \).

4. \(-20, 480, -5120, -1280, -320, \ldots\)
   \[ \div 4 \]
   Each number in the pattern is the previous number divided by 4. The next three numbers are \(-80, -20, -5\).

5. Counterexample: \( \frac{-24}{8} = 3 \)
   Because a counterexample exists, the conjecture is false.

6. If-then: If an angle measures 34°, then it is an acute angle.
   Converse: If an angle is an acute angle then it measures 34°.
   Inverse: If an angle does not measure 34°, then it is not an acute angle.
   Contrapositive: If an angle is not an acute angle, then it does not measure 34°.

7. This is a valid definition because it can be written as a true biconditional statement.

8. All the interior angles of a polygon are congruent if and only if the polygon is an equiangular polygon.

9. Because \( \angle B \) is a right angle it satisfies the hypothesis, so the conclusion is also true. So, \( \angle B \) measures 90°.

10. The conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement if \( 4x = 12 \), then \( 2x = 6 \).

11. Look for a pattern:
    \[ 1 + 3 = 4, 5 + 7 = 12, 9 + 3 = 12 \]
    Conjecture: Odd integer + odd integer = even integer
    Let \( 2n + 1 \) and \( 2m + 1 \) be any two odd integers
    \( (2n + 1) + (2m + 1) = 2n + 2m + 2 = 2(n + m + 1) \)
    \( 2(n + m + 1) \) is the product of 2 and an integer
    \( (n + m + 1) \). So, \( 2(m + n + 1) \) is an even integer.
    The sum of any two odd integers is an even integer.

12. \( x = \)?

13. B: With no right angle marked, you cannot assume \( CD \parallel \text{plane} \ P \).

14. \[ -9x - 21 = -20x - 87 \] Given
   \[ 11x - 21 = -87 \] Addition Property of Equality
   \[ 11x = -66 \] Addition Property of Equality
   \[ x = -6 \] Division Property of Equality

15. \[ 15x + 22 = 7x + 62 \] Given
   \[ 8x + 22 = 62 \] Subtraction Property of Equality
   \[ 8x = 40 \] Subtraction Property of Equality
   \[ x = 5 \] Division Property of Equality

16. \[ 3(2x + 9) = 30 \] Given
   \[ 6x + 27 = 30 \] Distributive Property
   \[ 6x = 3 \] Subtraction Property of Equality
   \[ x = \frac{1}{2} \] Division Property of Equality

Geometry
Worked-Out Solution Key
Chapter 2, continued

17. \(5x + 2(2x - 23) = -154\)  
   \(5x + 4x - 46 = -154\)  
   Distributive Property  
   \(9x - 46 = -154\)  
   Simplify.  
   \(9x = -108\)  
   Addition Property of Equality  
   \(x = -12\)  
   Division Property of Equality

18. Symmetric Property of Congruence
19. Reflexive Property of Congruence
20. Transitive Property of Equality
21. Given: \(\angle 1 \cong \angle 2\) and \(\angle 2 \cong \angle 3\)
Prove: \(\angle 1 \cong \angle 3\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
</table>
| 1. \(\angle 1 \cong \angle 2\)  
   \(\angle 2 \cong \angle 3\) | 1. Given |
| 2. \(m\angle 1 = m\angle 2\)  
   \(m\angle 2 = m\angle 3\) | 2. Definition of congruent angles |
| 3. \(m\angle 1 = m\angle 3\) | 3. Transitive Property of Equality |
| 4. \(\angle 1 \cong \angle 3\) | 4. Definition of congruent angles |

22. Given: \(m\angle 1 = 114^\circ\)  
   \(m\angle 3 = m\angle 1 = 114^\circ\)  
   \(m\angle 2 = m\angle 4 = 57^\circ\)  
   \(m\angle 1 + m\angle 2 = 180^\circ\)  
   \(m\angle 3 + m\angle 2 = 180^\circ\)  
   \(114^\circ + m\angle 2 = 180^\circ\)  
   \(57^\circ + m\angle 3 = 180^\circ\)  
   \(m\angle 2 = 66^\circ\)  
   \(m\angle 3 = 123^\circ\)  
   \(m\angle 4 = m\angle 2 = 66^\circ\)  
   \(m\angle 1 = m\angle 3 = 123^\circ\)

23. Given: \(m\angle 4 = 57^\circ\)

24. Statements  
   1. \(\angle 3\) and \(\angle 2\)  
      are complementary.  
      \(m\angle 1 + m\angle 2 = 90^\circ\)  
   2. \(\angle 1\) and \(\angle 2\)  
      are complementary.  
   3. \(\angle 3 \cong \angle 1\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Given</td>
<td></td>
</tr>
<tr>
<td>2. Definition of complementary angles</td>
<td></td>
</tr>
<tr>
<td>3. Congruent Complements Theorem</td>
<td></td>
</tr>
</tbody>
</table>

Chapter 2 Test (p. 128)

1. The figure is rotated counterclockwise. The next figure is:

2. Two pieces are added to the figure alternating between unshaded and shaded pattern. The next figure is:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. (-6, -1, 4, 9, \ldots)</td>
<td></td>
</tr>
<tr>
<td>(\frac{+5}{+5} \frac{+5}{+5} \frac{+5}{+5})</td>
<td></td>
</tr>
<tr>
<td>Each number in the pattern is five more than the previous number. The next number is 14.</td>
<td></td>
</tr>
<tr>
<td>4. (100, -50, 25, -12.5, \ldots)</td>
<td></td>
</tr>
<tr>
<td>(\times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>Each number in the pattern is (-\frac{1}{2}) times the previous number. The next number is 6.25.</td>
<td></td>
</tr>
<tr>
<td>5. If-then form: If two angles are right angles, then they are congruent.</td>
<td></td>
</tr>
<tr>
<td>Converse: If two angles are congruent, then they are right angles.</td>
<td></td>
</tr>
<tr>
<td>Inverse: If two angles are not right angles, then they are not congruent.</td>
<td></td>
</tr>
<tr>
<td>Contrapositive: If two angles are not congruent, then they are not right angles.</td>
<td></td>
</tr>
<tr>
<td>6. If-then form: If an animal is a frog, then it is an amphibian.</td>
<td></td>
</tr>
<tr>
<td>Converse: If an animal is an amphibian, then it is a frog.</td>
<td></td>
</tr>
<tr>
<td>Inverse: If an animal is not a frog, then it is not an amphibian.</td>
<td></td>
</tr>
<tr>
<td>Contrapositive: If an animal is not an amphibian, then it is not a frog.</td>
<td></td>
</tr>
<tr>
<td>7. If-then form: If (x = -2), then (5x + 4 = -6).</td>
<td></td>
</tr>
<tr>
<td>Converse: If (5x + 4 = -6), then (x = -2).</td>
<td></td>
</tr>
<tr>
<td>Inverse: If (x \neq -2), then (5x + 4 \neq -6).</td>
<td></td>
</tr>
<tr>
<td>Contrapositive: If (5x + 4 \neq -6), then (x \neq -2).</td>
<td></td>
</tr>
<tr>
<td>9. Because you are going to the football game satisfies the hypothesis, the conclusion is true. So, you will miss band practice.</td>
<td></td>
</tr>
<tr>
<td>10. The conclusion of the first statement is the hypothesis of the second statement, so you write the following new statement.</td>
<td></td>
</tr>
<tr>
<td>If Margot goes to college, then she will need to buy a lab manual.</td>
<td></td>
</tr>
<tr>
<td>11. Sample answers: Line (\overline{TO}) contains points (N, Q,) and (T).</td>
<td></td>
</tr>
<tr>
<td>Plane (Y) contains points (Q, R,) and (S).</td>
<td></td>
</tr>
<tr>
<td>12. Sample answer: Plane (Y) contains points (Q, R,) and (S).</td>
<td></td>
</tr>
<tr>
<td>Planes (X) and (Y) intersect at (\overline{TO}).</td>
<td></td>
</tr>
<tr>
<td>14. (9x + 31 = -23)</td>
<td></td>
</tr>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>(9x = -54)</td>
<td></td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td></td>
</tr>
<tr>
<td>(x = -6)</td>
<td></td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td></td>
</tr>
<tr>
<td>15. (-7(-x + 2) = 42)</td>
<td></td>
</tr>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>(7x - 14 = 42)</td>
<td></td>
</tr>
<tr>
<td>Distributive Property</td>
<td></td>
</tr>
<tr>
<td>(7x = 56)</td>
<td></td>
</tr>
<tr>
<td>Addition Property of Equality</td>
<td></td>
</tr>
<tr>
<td>(x = 8)</td>
<td></td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 2, continued

16. \(26 + 2(3x + 11) = -18x\)
   Given
   \(26 + 6x + 22 = -18x\)
   Distributive Property
   \(6x + 48 = -18x\)
   Simplify.
   \(48 = -24x\)
   Subtraction Property of Equality
   \(-2 = x\)
   Division Property of Equality

17. B; Symmetric Property of Congruence
18. A; Reflexive Property of Congruence
19. C; Transitive Property of Congruence

20. \(7y = 5y + 36\)
   \(2y = 36\)
   \(y = 18\)
   \(3x - 21 = 2x + 4\)
   \(x = 25\)

   The measures of the angles are:
   \(7(18) = 2(25)\) = 126°
   \(2(25) + 4 = 54°\)
   \(5(18) + 36 = 126°\)
   \(3(25) - 21 = 54°\)

21. Statements | Reasons
   1. \(AX \equiv DX, X X = XC\)
      \(AX = DX, X X = XC\)
   2. Definition of congruent segments
   3. \(AX + XC = AC, AC = BD\)
      \(DX + XB = BD\)
   4. Substitution Property of Equality
   5. \(AC = BD\)
   6. Transitive Property of Equality
   7. \(\overline{AC} \equiv BD\)
      \(\overline{BC} \equiv CD\)

Chapter 2 Algebra Review (p. 139)

1. \(\frac{5x^2}{3} \cdot \frac{2x^4}{x} \cdot \frac{x^3}{y} = \frac{x^5}{4}\)
2. \(-\frac{12ab}{9a^3b} = -4 \cdot \frac{y^3}{3} \cdot \frac{xy}{a} \cdot \frac{b \cdot b}{b} = -\frac{4b^2}{3a}\)
3. \(\frac{5m + 35}{5} = \frac{7(m + 7)}{3} = m + 7\)
4. \(\frac{36m - 48m}{6m} = \frac{-12m}{6m} = -2\)
5. \(\frac{k + 3}{-2k + 3}; \text{cannot be simplified}\)
6. \(\frac{4}{m^2 + 4m} = \frac{m + 4}{m(m^4 + 4)} = \frac{1}{m}\)
7. \(\frac{12x + 16}{8 + 6x} = \frac{2(6x + 8)}{4(x + 3)} = 2\)
8. \(\frac{3x^3}{5x + 8x^2} = \frac{3x^3}{x(5 + 8x)} = \frac{3x^2}{5 + 8x}\)

9. \(\frac{3x^2 - 6x}{4x^2 - 3x} \cdot \frac{x - 2}{x - 1} = \frac{x - 2}{x - 1}\)
10. \(\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}\)
11. \(-\sqrt{180} = -\sqrt{36} \cdot \sqrt{5} = -6\sqrt{5}\)
12. \(\pm \sqrt{128} = \pm \sqrt{64 \cdot \sqrt{2}} = \pm 8\sqrt{2}\)
13. \(\sqrt{2} - \sqrt{18} + \sqrt{6} = \sqrt{2} - 3\sqrt{2} + \sqrt{6} = \sqrt{6} - 2\sqrt{2}\)
14. \(\sqrt{28} - \sqrt{63} - \sqrt{35} = 2\sqrt{7} - 3\sqrt{7} - \sqrt{35} = -\sqrt{7} - \sqrt{35}\)
15. \(4\sqrt{8} + 3\sqrt{32} = 8\sqrt{2} + 12\sqrt{2} = 20\sqrt{2}\)
16. \((6\sqrt{5})^2 = 12\sqrt{10}\)
17. \((-4\sqrt{10})(-5\sqrt{5}) = 20\cdot 50 = 20 \cdot 5\sqrt{2} = 100\sqrt{2}\)
18. \((2\sqrt{6})^2 = (2)^2 \cdot (\sqrt{6})^2 = 4 \cdot 6 = 24\)
19. \(\sqrt{25^2} = 25\)
20. \(\sqrt{x^2} = x\)
21. \(\sqrt{-a^2}; \text{cannot be simplified}\)
22. \(\sqrt{(3y)^2} = \sqrt{9y^2} = 3y\)
23. \(v^2 + 2^2 = v^2 + 4 = \sqrt{13}\)
24. \(\sqrt{h^2 + k^2}; \text{cannot be simplified}\)

Standardized Test Preparation (p. 141)

1. The solution is given full credit. Each part is answered correctly and parts (b) and (c) are explained clearly.

Standardized Test Practice (pp. 142–143)

1. a. \(H = \frac{4}{3}(200 - A)\)
   \(H = 160 - \frac{4}{3}A\)
   Distributive Property

2. a. The statement does not follow the data because the data show 200 students use cars, 400 use public transit, 200 use the school bus and 400 walk. The data adds up to a total of 200 + 400 + 200 + 400 = 1200 students, not the 1500 given in the statement.

b. When \(H = 12:\)
   \(A = \frac{5}{4}(12) + 200 = 185\)
   A bowler’s average score is 185.

c. No; If a bowler’s average score is above 200, then the handicap would be negative, which does not make sense in the context of the problem.

2. a. The statement does not follow the data because the data show 200 students use cars, 400 use public transit, 200 use the school bus and 400 walk. The data adds up to a total of 200 + 400 + 200 + 400 = 1200 students, not the 1500 given in the statement.

b. The data show 400 students use public transportation which is \(\frac{1}{3}(1200),\) or 400, students. So, the statement follows from the data.
Chapter 2, continued

e. Sample answer: The pattern of the data show that most of the students either use public transportation or walk to school, so Porter High School must be in a city or city suburb. Because the conjecture is based on the pattern of the data, John used inductive reasoning.
d. The data show that 400 students walk to school and 200 students use a car to go to school. So, twice as many students walk to go to school than use cars to go to school. Because Betty was comparing values given on the graph, she used deductive reasoning.

3. a. Sample answer:

<table>
<thead>
<tr>
<th>Officer Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior President</td>
</tr>
<tr>
<td>Sophomore President</td>
</tr>
<tr>
<td>Senior Secretary</td>
</tr>
<tr>
<td>Junior Treasurer</td>
</tr>
<tr>
<td>Junior Vice President</td>
</tr>
<tr>
<td>Sophomore Vice President</td>
</tr>
<tr>
<td>Senior Treasurer</td>
</tr>
<tr>
<td>Principal</td>
</tr>
</tbody>
</table>

b. Officers have to be seated in seats 6–9 based on the given conditions, so none of those seats are available. The senior class president has to sit to the left of the principal and next to the junior class president.
The junior class treasurer has to sit next to the senior class treasurer. So, the only seat left available for the senior class secretary is the seat between the junior class president and the junior class treasurer.
c. No; the sophomore class president must sit across from the senior class president and the two sophomores must sit next to each other. So, the sophomore class vice president must sit across from the junior class president.

4. C; An odd integer times an odd integer is an odd integer. An odd integer plus 1 is an even integer. So, 3d + 1 is an even integer.

5. D; The four digit number is repeated 75 times to reach the 300th place. So, 4 is in the 300th place.

6. Using the Vertical Angles Congruence Theorem:

\[3x + 31 = 15x - 5\]

\[36 = 12x\]

\[3 = x\]

So, the value of \(x\) is 3.

7. Using the Vertical Angles Congruence Theorem and the Angle Addition Postulate:

\[x + 20 + y = 180\]

\[x + y = 160\]

8. \[\overrightarrow{PR} = \overrightarrow{RQ}\]

\[PS + SR = RT + TQ\]

\[SR + ST = RT + RT\]

\[2SR = 2RT\]

\[SR = RT\]

\[ST = SR + RT = RT + RT = 2RT\]

When \(ST = 20\):

\[20 = 2RT \Rightarrow RT = 10\]

\[PT = PS + SR + RT = 10 + 10 + 10 = 30\]

So, if \(ST = 20\), then \(PT = 30\).

9. This is not a correct conclusion. The conclusion of the conditional statement is true, but this does not imply that the hypothesis is true.

10. \[192, -48, 12, -3, \ldots\]

\[
\times -\frac{1}{4} \times -\frac{1}{4} \times -\frac{1}{4} \times -\frac{1}{4}
\]

Each number in the pattern is \(-\frac{1}{4}\) times the previous number. The next number is \(\frac{3}{4}\).

11. Points \(A, B,\) and \(F\) are collinear, so they lie on the same line. \(AB \perp CD\) and \(CD \perp EF\), so \(E\) and \(F\) must lie on the same line as \(A\) and \(B\), because \(F\) is collinear with \(A\) and \(B\). So points \(A, B, E,\) and \(F\) lie on the same line.

12. The three tutors who can work any Wednesday are Lou, Mike, and Nina. Lou can work on Tuesdays and Wednesdays. Mike’s only restriction is that he cannot work Fridays, so he can work any Wednesday. Nina’s only restriction is that she cannot work Tuesdays, so she can work any Wednesday.