Unit 1 - Radian and Degree Measure – Classwork

Definitions to know:

Trigonometry – triangle measurement

Angle: rotating a ray about its endpoint

Initial side, terminal side - starting and ending Position of the ray

Vertex – endpoint of the ray

Standard position – origin if the vertex, initial side is the positive *x*-axis

Positive, negative angles – positive is counterclockwise rotation, negative axis clockwise rotation

Co-terminal angles – angles having the same initial and terminal sides

Measurement of angles:

Degrees: 360 degrees make up one circle.

Radians: one radian is the central angle formed by laying the radius of the circle onto the circumference. There are 2π radians in one circle.

Revolutions: a full rotation of a circle.

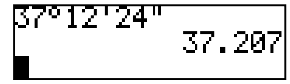
Conversion formula for angles: $\boxed{360^\circ = 2\pi \text{ radians} = 1 \text{ revolution}}$

Example 1) Convert the following angles to the other two measurements

#	Degrees	Radians	Revolutions
a.	180°	$\boxed{\pi}$	$\boxed{\frac{1}{2}}$
b.	30°	$\boxed{\frac{\pi}{6}}$	$\boxed{\frac{1}{12}}$
c.	$\boxed{90^\circ}$	$\frac{\pi}{2}$	$\boxed{\frac{1}{4}}$
d.	$\boxed{135^\circ}$	$\frac{3\pi}{4}$	$\boxed{\frac{3}{8}}$
e.	$\boxed{45^\circ}$	$\boxed{\frac{\pi}{4}}$	$\frac{1}{8}$
f.	$\boxed{240^\circ}$	$\boxed{\frac{4\pi}{3}}$	$\frac{2}{3}$
g.	225°	$\boxed{\frac{5\pi}{4}}$	$\boxed{\frac{5}{8}}$
h.	$\boxed{300^\circ}$	$\frac{5\pi}{3}$	$\boxed{\frac{5}{6}}$
i.	$\boxed{264^\circ}$	$\boxed{\frac{22\pi}{15}}$	$\frac{11}{15}$
j.	$\boxed{\frac{180}{\pi}^\circ}$	1	$\boxed{\frac{1}{2\pi}}$
k.	$\boxed{360\pi^\circ}$	$\boxed{2\pi^2}$	π

Conversion of angles expressed in degrees, minutes, and seconds to decimal degrees:

Example: Express $37°12'24''$ to decimal degrees: $\boxed{37+\dfrac{12}{60}+\dfrac{24}{3600}}$



Example: Express $28.923°$ in degrees, minutes, seconds:

$\boxed{.923(60)=55.38 \quad .38(6)=22.8}$

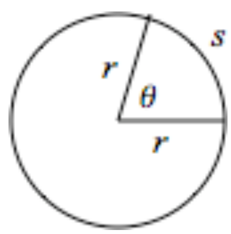
Example 2) Convert from degrees, minutes, and seconds to decimal degrees or vice versa. (3 decimal places)

#	Degrees, Minutes, Seconds	Decimal Degrees
a.	$23°30'$	$\boxed{23.5°}$
b.	$141°25'45''$	$\boxed{14.429°}$
c.	$\boxed{12°15'}$	$12.25°$
d.	$\boxed{39°25'34''}$	$39.426°$
e.	$59°59'59''$	$\boxed{60°}$
f.	$\boxed{127°0'4''}$	$127.001°$

Finding the arc length of a circle:

We know that the circumference of a circle is given by $C = 2\pi r$ where r is the radius of the circle: This formula can also be used to find the length of an arc intercepted by some angle θ.

> Arc length formula: $s = r\theta$ where r is measured in linear units, θ is measured in radians, and s is measured in linear units



Example 3: Find the arc length of the arc with radius = 4 inches and $\theta = 60°$

$$\boxed{s = 4\text{ in}(60°)\left(\frac{2\pi}{360°}\right) = 4.189\text{ in.}}$$

Note: since the angle is measured in radians, it technically has no units so s is measured in linear units.

Example 4: If the arc length is 6 inches and the radius is 2 inches, find the central angle in degrees

$$\boxed{\theta = \frac{s}{r} = \frac{6\text{ in}}{2\text{ in}} = 3\left(\frac{360°}{2\pi}\right) = 171.887°}$$

Example 5: If the arc length is 2 meters and the central angle is 125°, find the radius of the circle.

$$\boxed{r = \frac{s}{\theta} = \frac{2\text{ m}}{125°}\left(\frac{360°}{2\pi}\right) = 0.917\text{ meters}}$$

Example 6: Assuming the earth. is a sphere of radius 4,000 miles. Miami, Florida is at latitude $25°47'9''$N while Erie, Pennsylvania is at $42°7'15''$N and the cities are on the same meridian (one city lies due north of each other). Find the distance between the cities.

$$\boxed{s = 4000 \text{ m}(16.335°)\left(\frac{2\pi}{360°}\right) = 1140.4 \text{ miles}}$$

Imagine an object traveling along a circular arc. The element of time is now added to the equation. In order to do problems in such situations, we need to identify variables that can express certain information.

Important variables for problems in which an object is moving along a circular arc

Variable	Name	Given in	Use in formulas	Sample measures
r	Radius	Linear units	$\frac{\text{linear units}}{\text{radian}}$	4 inches, 1.5 feet
θ (theta)	Angle	Degrees, radians, revs	Radians	$25°, 1.5\pi^R, .75$ revs
s	Arc length	Linear units	Linear units	2.3 ft, 5 cm
t	Time	Time units	Time units	2 sec, 2.5 hrs
v	Linear velocity	$\frac{\text{linear units}}{\text{time}} = \frac{s}{t}$	$\frac{\text{linear units}}{\text{time}}$	$5\frac{\text{ft}}{\text{sec}}, 12mph$
ω (omega)	Angular velocity	$\frac{\text{angle}}{\text{time}} = \frac{\theta}{t}$	$\frac{\text{radians}}{\text{time}}$	$15\frac{\text{degrees}}{\text{sec}}, 8rpm$

Example 7) What variable are you being given (r, θ, s, t, v, ω)

- $\boxed{\theta}$ I make a U-turn with my car.
- $\boxed{t}$ It takes 5 minutes to complete the exam
- $\boxed{r}$ The spoke of a wheel is 58 inches
- $\boxed{s}$ A circular track measures 400 feet
- $\boxed{\omega}$ Around the world in 80 days
- $\boxed{v}$ The space shuttle travels at 3,094 miles per hour

Methods for transforming one variable into another

You may multiply any variable by the fraction "one". Here are some examples:

$$\frac{12 \text{ inches}}{1 \text{ foot}}, \frac{5280 \text{ feet}}{1 \text{ mile}}, \frac{2\pi \text{ radians}}{1 \text{ rev}}, \frac{1 \text{ minute}}{60 \text{ sec}}, \frac{1000 \text{ meters}}{1 \text{ km}}$$

Example 8): Convert the following:

a. 15 miles to feet

$$15 \text{ miles} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} = \boxed{79{,}200 \text{ ft}}$$

b. 1 day to seconds

$$1 \text{ day} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} = \boxed{86{,}400 \text{ sec}}$$

c. 10,000 degrees to revolutions

$$10000° \cdot \frac{1 \text{ rev}}{360°} = \boxed{27.778 \text{ rev}}$$

d. $\frac{20 \text{ feet}}{\text{sec}}$ to miles per hour

$$\frac{20 \text{ ft}}{\text{sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} = \boxed{\frac{13.636 \text{ ft}}{\text{sec}}}$$

e. 55 mph to $\frac{\text{feet}}{\text{sec}}$

$$\frac{55 \text{ mile}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = \boxed{\frac{88.667 \text{ ft}}{\text{sec}}}$$

f. $\frac{1{,}000{,}000{,}000°}{\text{year}}$ to rpm

$$\frac{1{,}000{,}000{,}000°}{\text{year}} \cdot \frac{1 \text{ rev}}{360°} \cdot \frac{1 \text{ year}}{365 \text{ days}} \cdot \frac{1 \text{ day}}{24 \cdot 60 \text{ min}} = \boxed{\frac{5.285 \text{ rev}}{\text{min}}}$$

The Angular velocity – linear velocity formula: When an object is traveling along an arc, it has both an angular velocity and a linear velocity. The formula that ties these two variables together is:

$$v = \omega r \text{ or } \omega = \frac{v}{r} \qquad \omega \text{ is always measured in } \frac{\text{radians}}{\text{time}}$$

Examples 9:

a. A bicycle's wheel has a 30 inch diameter. If the wheel makes 1.5 revolutions per second, find the speed of the bike in mph.

$$v = \omega r = \frac{1.5\text{ rev}}{\text{sec}} \cdot \frac{15\text{ in}}{} \cdot \frac{2\pi}{1\text{ rev}} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{3600\text{ sec}}{1\text{ hr}} = \boxed{8.032\text{ mph}}$$

b. A flight simulator has pilots traveling in a circular path very quickly in order to experience g-forces. If the pilots are traveling at 400 mph and the circular room has a radius of 25 feet, find the number of rotations that simulator makes per second.

$$\omega = \frac{v}{r} = \frac{\frac{400\text{ miles}}{\text{hr}}}{25\text{ ft}} = \frac{400\text{ miles}}{\text{hr}} \cdot \frac{1}{25\text{ ft}} \cdot \frac{5280\text{ ft}}{1\text{ mile}} \cdot \frac{1\text{ rev}}{2\pi} \cdot \frac{1\text{ hr}}{3600\text{ sec}} = \boxed{\frac{3.735\text{ rev}}{\text{sec}}}$$

c. A large clock has its second hand traveling at 2.5 inches per second. Find the length of the second hand.

$$r = \frac{v}{\omega} = \frac{\frac{2.5\text{ in}}{\text{sec}}}{\frac{1\text{ rev}}{1\text{ min}}} = \frac{2.5\text{ in}}{\text{sec}} \cdot \frac{1\text{ min}}{1\text{ rev}} \cdot \frac{1\text{ rev}}{2\pi} \cdot \frac{60\text{ sec}}{1\text{ min}} = \boxed{23.873\text{ in}}$$

d. Two gears are connected by a belt. The large gear has a radius of 6 inches while the small gear has a radius of 3 inches. If a point on the small gear travels at 16 rpm, find the angular velocity of the large gear.

$$v = w_S r_S \qquad v = w_L r_L$$
$$w_S r_{S1} = w_L r_L$$
$$16(3) = 6w_L$$
$$\boxed{w_L = 8\text{ rpm}}$$

Unit 1 - Radian and Degree Measure – Homework

1. Convert the following angles to the other two measurements

	Degrees	Radians	Revolutions
a.	270°	$\frac{3\pi}{2}$	$\frac{3}{4}$
b.	45°	$\frac{\pi}{4}$	$\frac{1}{8}$
c.	240°	$\frac{4\pi}{3}$	$\frac{2}{3}$
d.	330°	$\frac{11\pi}{6}$	$\frac{11}{12}$
e.	135°	$\frac{3\pi}{4}$	$\frac{3}{8}$
f.	300°	$\frac{5\pi}{3}$	$\frac{5}{6}$
g.	315°	$\frac{7\pi}{4}$	$\frac{7}{8}$
h.	30°	$\frac{\pi}{6}$	$\frac{1}{12}$
i.	312°	$\frac{26\pi}{15}$	$\frac{13}{15}$
j.	5°	$\frac{\pi}{36}$	$\frac{1}{72}$
k.	$\left(\frac{900}{\pi}\right)^{\circ}$	5	$\frac{5}{2\pi}$
l.	1800°	10π	5

2. Convert from degrees, minutes, and seconds to decimal degrees or vice versa. (3 decimal places)

	Degrees, Minutes, Seconds	Decimal Degrees
a.	158°15′	158.25°
b.	6°21′35″	6.360°
c.	33°16′4″	33.268°
d.	12°54′	12.9°
e.	24°39′	24.65°
f.	154°30′7″	154.502°
g.	27°8′8″	27.136°
h.	99°59′56″	99.999°

3. Of the three variables r, θ, and s, you will be given two of them. Find the third. Angles should be found in the units specified). Specify units for other variables.

	r	θ	s
a.	4 inches	60°	4.189 in
b.	6.5 ft	135°	16.656 ft
c.	2.3 meters	1.5	3.450 meters
d.	23.65 cm	$14°25'36''$	5.935 cm
e.	16.82 miles	2.504	42.12 miles
f.	125.775 mm	329.856°	724.095 mm
g.	5 inches	$275°1'11''$	2 feet
h.	11.937 in	$\frac{\pi}{3}$	12.5 inches
i.	7.824 ft	72.5°	9.9 ft

4. Convert the given quantity into the specified units. Show your work in the "Convert to" column.

	Given	Convert to
a.	4.25 ft	$\frac{4.25\text{ ft}}{} \cdot \frac{12\text{ in}}{1\text{ft}} = \frac{51\text{ in}}{}$
b.	80 years	$\frac{80\text{ yrs}}{} \cdot \frac{365\text{ days}}{1\text{yr}} \cdot \frac{24\text{ hrs}}{1\text{day}} \cdot \frac{3600\text{ sec}}{1\text{hr}} = \frac{2{,}522{,}880{,}000\text{ sec}}{}$
c.	1,500 revolutions	$\frac{1500\text{ rev}}{} \cdot \frac{360°}{1\text{ rev}} = \frac{540{,}000°}{}$
d.	10 km	$\frac{10\text{ km}}{} \cdot \frac{3280.8\text{ ft}}{1\text{ km}} = \frac{32{,}808\text{ ft}}{}$
e.	2,500π	$\frac{2500\pi}{} \cdot \frac{1\text{ rev}}{2\pi} = \frac{1{,}2500\text{ rev}}{}$
f.	$\frac{25\text{ ft}}{\text{sec}}$	$\frac{25\text{ ft}}{\text{sec}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{3600\text{ sec}}{1\text{ hr}} = \frac{17.045\text{ mph}}{}$
g.	$\frac{12\text{ rev}}{\text{min}}$	$\frac{12\text{ rev}}{\text{min}} \cdot \frac{1\text{ rev}}{2\pi} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{24\text{ hr}}{1\text{ day}} = \frac{2{,}750.197}{\text{day}}$
h.	$\frac{500{,}000°}{\text{week}}$	$\frac{500000°}{\text{week}} \cdot \frac{1\text{ rev}}{360°} \cdot \frac{1\text{ week}}{7\text{ days}} \cdot \frac{1\text{ day}}{24\text{ hours}} \cdot \frac{1\text{ hour}}{60\text{ min}} = .022\text{ rpm}$
i.	60 mph	$\frac{60\text{ miles}}{\text{hour}} \cdot \frac{5280\text{ ft}}{1\text{ miles}} \cdot \frac{12\text{ inch}}{1\text{ ft}} \cdot \frac{1\text{ hour}}{3600\text{ sec}} = \frac{1{,}056\text{ inch}}{\text{sec}}$
j.	$\frac{1\text{ rev}}{80\text{ days}}$	$\frac{1\text{ rev}}{80\text{ days}} \cdot \frac{360°}{1\text{ rev}} \cdot \frac{1\text{ day}}{24\text{ hours}} \cdot \frac{1\text{ hour}}{60\text{ min}} = \frac{0.003°}{\text{min}}$

5. Find the distances between the cities with the given latitude, assuming that the earth is a sphere of radius 4,000 miles and the cities are on the same meridian.

a. Dallas, Texas $32°47'9''$N and Omaha, Nebraska $41°15'42''$N

$$s = r\theta = \frac{4000 \text{ miles}}{} \cdot \frac{8.476°}{} \cdot \frac{2\pi}{360°} = 591 \text{ miles}$$

b. San Francisco, California $37°46'39''$N and Seattle, Washington $47°36'32''$N

$$s = r\theta = \frac{4000 \text{ miles}}{} \cdot \frac{9.831°}{} \cdot \frac{2\pi}{360°} = 686 \text{ miles}$$

c. Copenhagen, Denmark $55°33'18''$N and Rome, Italy $41°49'18''$N

$$s = r\theta = \frac{4000 \text{ miles}}{} \cdot \frac{13.733°}{} \cdot \frac{2\pi}{360°} = 959 \text{ miles}$$

d. Jerusalem, Israel $31°47'0'$N and Johannesburg, South Africa $26°10'$S

$$s = r\theta = \frac{4000 \text{ miles}}{} \cdot \frac{57.95°}{} \cdot \frac{2\pi}{360°} = 4{,}046 \text{ miles}$$

6. What variable are you being given (r, θ, s, t, v, ω) ?

t a. It takes 3 minutes to travel between classes.
ω b. It takes 5 minutes to walk around the school.
θ c. The Space Shuttle made 35 orbits of the earth.
s d. The circumference of the orange is 6.2 inches.
v e. The merry-go-round travels at a constant speed of 4 miles per hour.
θ f. A Ferris-Wheel ride consists of 8 revolutions.
ω g. That Ferris-Wheel completes the 8 revolutions in 6 minutes.
r h. A propeller is 45 inches long.
s i. The park is circular and I walked 2 miles around its circumference.
ω j. An ant walking around a tire lying on the ground can only cover 5 degrees every minute.

7. Complete the chart, finding the missing information in the measurement requested. Show work.

#	ω	r	v	**Units Desired**
a.	80 rpm	2 feet	1,005.31 ft	feet/min
b.	15 rev/sec	2.5 feet	160.65 mph	mph
c	55°/sec	1.1 mile	3801.327 mph	mph
d.	9.549 rpm	1 foot	60 ft/min	rpm
e.	672.27 rpm	15 inches	60 mph	rpm
f.	$\frac{32.554°}{\text{min}}$	2 miles	100 ft/sec	degrees/min
g.	50 rpm	.005 miles	100 mph	miles
h.	100 rev/sec	.08 ft	50 feet/sec	feet
i.	1,000 rev/sec	42.017 in	15,000 mph	inches

8. Applications – For each problem, draw a picture if necessary and show how you got your answer.

a) A clock has a second hand of length 8 inches. How far **in inches** does the tip travel from when it is on the 12 to when it is on the 4.

$$s = r\theta = \frac{8 \text{ in}}{} \cdot \frac{120°}{} \cdot \frac{2\pi}{360°} = 16.755 \text{ inches}$$

b) The pendulum in the Franklin Institute is 40 feet long. It swing through an angle of 11°23'. Find the length of the arc it swings through **in inches.**

$$s = r\theta = \frac{40 \text{ ft}}{} \cdot \frac{11.383°}{} \cdot \frac{2\pi}{360°} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 95.365 \text{ inches}$$

c) When the central angle is small and the distance to an object is large, the arc length formula is a good estimator of the height of the object. The angle of elevation of the Empire State Building from 4 miles away is 4°13'. Use the arc length formula to estimate its height **in feet**.

$$s = r\theta = \frac{4 \text{ miles}}{} \cdot \frac{4.217°}{} \cdot \frac{2\pi}{360°} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} = 1554.32 \text{ ft}$$

d) A car tire with radius 8 inches rotates at 42 rpm. Find the velocity of the car **in mph**.

$$v = \omega r = \frac{42 \text{ rev}}{\text{min}} \cdot \frac{8 \text{ inch}}{} \cdot \frac{2\pi}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ inch}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 1.999 \text{ mph}$$

e) The Spinner is an amusement park ride that straps people to the edge of a circle and spins very fast. If riders are traveling at an actual speed of 25 mph, and the radius of the wheel is 15 feet, find the angular velocity of the wheel in **rpm**.

$$\omega = \frac{v}{r} = \frac{25 \text{ miles}}{\text{hr}} \cdot \frac{1}{15 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ rev}}{2\pi} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 23.343 \text{ rpm}$$