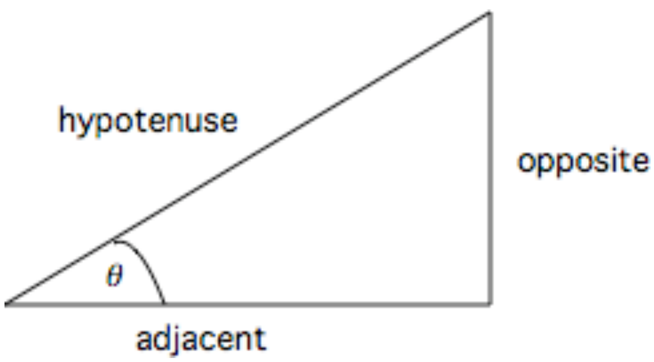
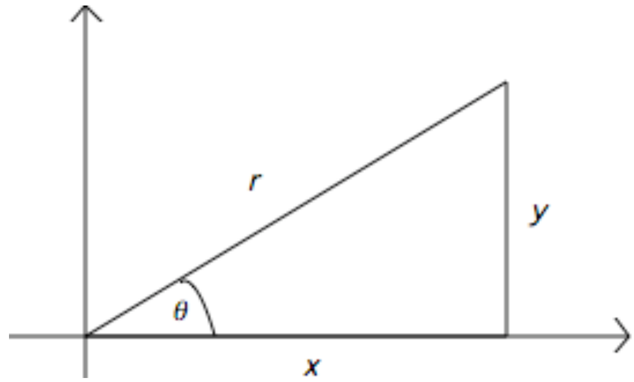
<u>Unit 2 - The Trigonometric Functions - Classwork</u>





Given a right triangle with one of the angles named θ, and the sides of the triangle relative to θ named opposite, adjacent, and hypotenuse (picture on the left) , we define the 6 trig functions to be:

The Basic Trig Definitions

the sine function : $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	the cosecant function : $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
the cosine function : $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	the secant function : $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
the tangent function : $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	the cotangent function : $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Given a right triangle with one of the angles named θ with θ in standard position, and the sides of the triangle relative to θ named *x*, *y*, and *r*. (picture on the right) , we define the 6 trig functions to be:

the sine function : $\sin x = \dfrac{y}{r}$	the cosecant function : $\csc\theta = \dfrac{r}{y}$
the cosine function : $\cos\theta = \dfrac{x}{r}$	the secant function : $\sec\theta = \dfrac{r}{x}$
the tangent function : $\tan\theta = \dfrac{y}{x}$	the cotangent function : $\cot\theta = \dfrac{x}{y}$
The Pythagorean theorem ties these variable together : $x^2 + y^2 = z^2$	

You **MUST, MUST, MUST** know the above thoroughly, inside and out, backwards and forward, and can never forget it. It must be part of you. Expect quizzes every day for the immediate future to test whether you know these definitions. You will find that if you learn them now, this section will be incredibly easy. If you learn them and immediately forget them, you will struggle throughout this course.

A good way to remember the basic definitions is to remember the terms SOH-CAH-TOA. Sine = Opposite, Hypotenuse, … Cosine = Adjacent, Hypotenuse…. Tangent – Opposite, Adjacent. For the other trig functions (called the co-functions), Sine goes with Cosecant (S goes with C), Cosine goes with Secant (C goes with S), and the other functions both use the words tangent.

Finally, remember that there is no such thing as sine. Sine doesn't exist by itself. It is $\sin\theta$ or $\sin\alpha$ or $\sin x$. Every trig function is a function of an angle. **The angle must be present**.

Example 1) Let P be a point on the terminal side of θ. Draw a picture and find the 6 trig functions of θ.

a) $P(3,4)$

$\sin\theta = \dfrac{4}{5}$	$\csc\theta = \dfrac{5}{4}$
$\cos\theta = \dfrac{3}{5}$	$\sec\theta = \dfrac{5}{3}$
$\tan\theta = \dfrac{4}{3}$	$\cot\theta = \dfrac{3}{4}$

b) $P(15,8)$

$\sin\theta = \dfrac{8}{17}$	$\csc\theta = \dfrac{17}{8}$
$\cos\theta = \dfrac{15}{17}$	$\sec\theta = \dfrac{17}{15}$
$\tan\theta = \dfrac{8}{15}$	$\cot\theta = \dfrac{15}{8}$

c) $P(5,2)$

$\sin\theta = \dfrac{2}{\sqrt{29}}$	$\csc\theta = \dfrac{\sqrt{29}}{2}$
$\cos\theta = \dfrac{5}{\sqrt{29}}$	$\sec\theta = \dfrac{\sqrt{29}}{5}$
$\tan\theta = \dfrac{2}{5}$	$\cot\theta = \dfrac{5}{2}$

d) $P(1,7)$

$\sin\theta = \dfrac{7}{5\sqrt{2}}$	$\csc\theta = \dfrac{5\sqrt{2}}{7}$
$\cos\theta = \dfrac{1}{5\sqrt{2}}$	$\sec\theta = 5\sqrt{2}$
$\tan\theta = 7$	$\cot\theta = \dfrac{1}{7}$

e) $P(1,1)$

$\sin\theta = \dfrac{\sqrt{2}}{2}$	$\csc\theta = \sqrt{2}$
$\cos\theta = \dfrac{\sqrt{2}}{2}$	$\sec\theta = \sqrt{2}$
$\tan\theta = 1$	$\cot\theta = 1$

e) $P\left(\sqrt{2},\sqrt{7}\right)$

$\sin\theta = \dfrac{\sqrt{7}}{3}$	$\csc\theta = \dfrac{3}{\sqrt{7}}$
$\cos\theta = \dfrac{\sqrt{2}}{3}$	$\sec\theta = \dfrac{3}{\sqrt{2}}$
$\tan\theta = \sqrt{\dfrac{7}{2}}$	$\cot\theta = \sqrt{\dfrac{2}{7}}$

Quadrant Angles:

Let's examine the trig functions if point P is not in the first quadrant. Let's make a chart of the signs of x, y, and r in all of the quadrants and thus, the signs of the trig functions in those quadrants. (r is always positive)

II		I	
$\sin\theta = \dfrac{+}{+}$	$\csc\theta = \dfrac{+}{+}$	$\sin\theta = \dfrac{+}{+}$	$\csc\theta = \dfrac{+}{+}$
$\cos\theta = \dfrac{-}{+}$	$\sec\theta = \dfrac{+}{-}$	$\cos\theta = \dfrac{+}{+}$	$\sec\theta = \dfrac{+}{+}$
$\tan\theta = \dfrac{+}{-}$	$\cot\theta = \dfrac{+}{-}$	$\tan\theta = \dfrac{+}{+}$	$\cot\theta = \dfrac{+}{+}$
$\sin\theta = \dfrac{-}{+}$	$\csc\theta = \dfrac{+}{-}$	$\sin\theta = \dfrac{-}{+}$	$\csc\theta = \dfrac{+}{-}$
$\cos\theta = \dfrac{-}{+}$	$\sec\theta = \dfrac{+}{-}$	$\cos\theta = \dfrac{+}{+}$	$\sec\theta = \dfrac{+}{+}$
$\tan\theta = \dfrac{-}{-}$	$\cot\theta = \dfrac{-}{-}$	$\tan\theta = \dfrac{-}{+}$	$\cot\theta = \dfrac{+}{-}$
III		IV	

A good way to remember this is the term: A-S-T-C. It says the quadrants in which the 3 basic trig functions are positive: (All – Sine – Cosine – Tangent)

When we draw pictures of trig functions in quadrants other than quadrant I, the triangle is always drawn to the x-axis. The angle inside the triangle will be called the reference angle. It is defined at the acute angle formed by the terminal side of θ and the horizontal axis.

Example 2) Let P be a point on the terminal side of θ. Draw a picture showing the reference angle and find the 6 trig functions of θ.

a) $P(-8,6)$

$\sin\theta = \frac{3}{5}$	$\csc\theta = \frac{5}{3}$
$\cos\theta = -\frac{4}{5}$	$\sec\theta = -\frac{5}{4}$
$\tan\theta = -\frac{3}{4}$	$\cot\theta = -\frac{4}{3}$

b) $P(7,-24)$

$\sin\theta = -\frac{24}{25}$	$\csc\theta = -\frac{25}{24}$
$\cos\theta = \frac{7}{25}$	$\sec\theta = \frac{25}{7}$
$\tan\theta = -\frac{24}{7}$	$\cot\theta = -\frac{7}{24}$

c) $P(-2,-2)$

$\sin\theta = -\frac{\sqrt{2}}{2}$	$\csc\theta = -\sqrt{2}$
$\cos\theta = -\frac{\sqrt{2}}{2}$	$\sec\theta = -\sqrt{2}$
$\tan\theta = 1$	$\cot\theta = 1$

d) $P(-1,\sqrt{3})$

$\sin\theta = \frac{\sqrt{3}}{2}$	$\csc\theta = \frac{2\sqrt{3}}{3}$
$\cos\theta = -\frac{1}{2}$	$\sec\theta = -2$
$\tan\theta = -\sqrt{3}$	$\cot\theta = -\frac{\sqrt{3}}{3}$

Example 3) We can be given information about one trig function and ask about the others. Draw a picture.

a) If $\sin\theta = \frac{12}{13}, \theta$ in quadrant I, find $\cos\theta$ and $\tan\theta$.

$$\cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}$$

b) If $\cos\theta = \frac{2}{3}, \theta$ in quadrant IV, find $\sin\theta$ and $\tan\theta$.

$$\sin\theta = -\frac{\sqrt{5}}{3}, \tan\theta = -\frac{\sqrt{5}}{2}$$

c) If $\tan\theta = 3, \theta$ in quadrant III, find $\cos\theta$ and $\csc\theta$.

$$\cos\theta = -\frac{1}{\sqrt{10}}, \csc\theta = -\frac{\sqrt{10}}{3}$$

d) If $\csc\theta = \frac{6}{5}$, find $\cos\theta$ and $\cot\theta$.

$$\cos\theta = \pm\frac{\sqrt{11}}{6}, \cot\theta = \pm\frac{\sqrt{11}}{6}$$

Example 4) In what quadrant(s) is

a) $\sin\theta > 0$ and $\cos\theta < 0$

II

b) $\sec\theta < 0$ and $\cot\theta < 0$

II

c) $\csc\theta < 0$ and $\cos\theta > 0$

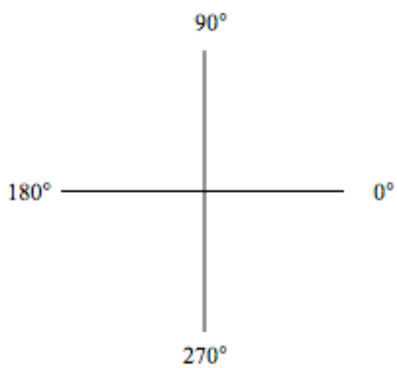
IV

d) all trig functions are negative?

None

Trig functions of quadrant angles:

The picture below shows quadrant angles: Choose a point for each quadrant angle, determine *x*, *y*, and *r*, and determine all six trig functions for those angles: Note that angles can be in degrees or in radians.



$0°$ (or 0) Point $(1,0\_)$		$90°\left(\text{or } \frac{\pi}{2}\right)$ Point $(0,1)$	
$x=1$ $y=0$ $r=1$		$x=0$ $y=1$ $r=1$	
$\sin 0° = 0$	$\csc 0° = DNE$	$\sin 90° = 1$	$\csc 90° = 1$
$\cos 0° = 1$	$\sec 0° = 1$	$\cos 90° = 0$	$\sec 90° = DNE$
$\tan 0° = 0$	$\cot 0° = DNE$	$\tan 90° = DNE$	$\cot 90° = 0$
$180°$ (or π) Point $(-1,0)$		$270°\left(\text{or } \frac{3\pi}{2}\right)$ Point $(0,-1)$	
$x=-1$ $y=0$ $r=1$		$x=0$ $y=-1$ $r=1$	
$\sin 180° = 0$	$\csc 180° = DNE$	$\sin 270° = -1$	$\csc 270° = -1$
$\cos 180° = -1$	$\sec 180° = -1$	$\cos 270° = 0$	$\sec 270° = DNE$
$\tan 180° = 0$	$\cot 180° = DNE$	$\tan 270° = DNE$	$\cot 270° = 0$

Example 5) Calculate the following without looking at the chart above:

a) $5\sin 90° - 12\cos 180°$

$5(1) - 12(-1)$
17

b) $\dfrac{6\tan 180° + 3\csc 270°}{-2\sec 0°}$

$\dfrac{6(0) + 3(-1)}{-2(1)} = \dfrac{3}{2}$

c) $\left(4\sin 90° - 2\cos 270° - 5\right)^2$

$\left[4(1) - 2(0) - 5\right]^2 = 1$

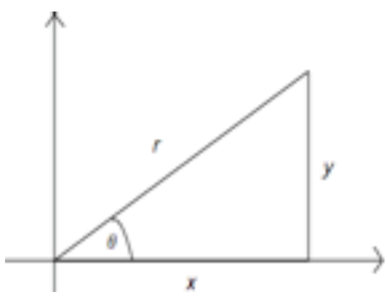
d) $\dfrac{-6\sin\frac{\pi}{2} - 5\cot 90°}{3\cos^2 0 - (3\cos 0)^2}$

$\dfrac{-6(1) - 5(0)}{3(1)^2 - \left[3(1)\right]^2} = \dfrac{-6}{3-9} = 1$

Domain and Range of trig functions:

Domain: We can take the sin and cosine of any angle . But since $\tan\theta = \frac{y}{x}$ and $\cot\theta = \frac{x}{y}$, we have to worry about angles where y = o or $x = 0$. $x = 0$ along the y-axis so we cannot take the tangent of 90° or 270°. $y = 0$ along the x-axis so we cannot take the cotangent of 0° or 180°. For the csc function we have to be concerned about angles where $y = 0$ (0° or 180°.) and for the sec function, we have to be concerned about angles where x = 0 (90° or 270°).

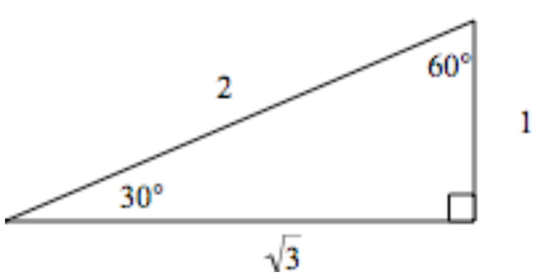
Range: Since we know that trig functions are based on the picture below and that in any right triangle, $r > x$ and $r > y$, r must always be the larger side. So since $\sin\theta = \frac{y}{r} = \frac{\text{smaller}}{\text{larger}}$, we find that the range of the sine (and cosine) functions must be less than (or equal to) 1. And since $\csc\theta = \frac{y}{r} = \frac{\text{larger}}{\text{smaller}}$, the range of the cosecant (and secant) functions must be greater than (or equal to) 1. Since $\tan\theta = \frac{y}{x}$, we find that there is no restriction on the values of the tangent function and cotangent functions. This can be summarized by the table on the right:

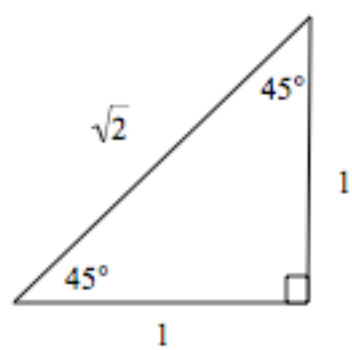


Domain :	Range :
$\sin\theta$: all real numbers	$-1 \le y \le 1$ or $[-1,1]$
$\cos\theta$: all real numbers	$-1 \le y \le 1$ or $[-1,1]$
$\tan\theta : \theta \ne 90°,: \theta \ne 270°$	all real numbers or $(-\infty,\infty)$
$\csc\theta : \theta \ne 0°,: \theta \ne 180°$	$y \le -1$ or $y \ge 1$ or $(-\infty,-1] \cup [1,\infty)$
$\sec\theta : \theta \ne 90°,: \theta \ne 270°$	$y \le -1$ or $y \ge 1$ or $(-\infty,-1] \cup [1,\infty)$
$\cot\theta : \theta \ne 0°,: \theta \ne 180°$	all real numbers or $(-\infty,-1] \cup [1,\infty)$

Special Triangles:

You must know the relationship of sides in both $30° - 60° - 90°$ and $45° - 45° - 90°$ triangles.





In a $30° - 60° - 90°$, the ratio of sides is $1 - \sqrt{3} - 2$

In a $45° - 45° - 90°$, the ratio of sides is $1 - 1 - \sqrt{2}$

So, complete the chart:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30° \left(\text{or } \frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45° \left(\text{or } \frac{\pi}{4}\right)$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60° \left(\text{or } \frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

The Special (Friendly) Angles

Any multiple of $30°, 45°$ or $60°$ is considered a special angle (or a quadrant angle) and we can compute trig functions of these angles. A) Draw it. B) Establish the quadrant and fill in the signs of the sides remembering ASTC. C) Find the reference angle (which will be $30°, 45°$ or $60°$) D) It is one of the special angles above. Determine the lengths of the sides (the signs are waiting for you). E) Find the trig functions of these angles.

θ	Radians	Drawing	Reference angle	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
120°	$\frac{2\pi}{3}$		60°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
135°	$\frac{3\pi}{4}$		45°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$\sqrt{2}$ -	-1
150°	$\frac{5\pi}{6}$		30°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
210°	$\frac{7\pi}{6}$		30°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
225°	$\frac{5\pi}{4}$		45°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$\frac{4\pi}{3}$		60°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\frac{\sqrt{3}}{3}$
300°	$\frac{5\pi}{3}$		60°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\frac{\sqrt{3}}{3}$
315°	$\frac{7\pi}{4}$		45°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330°	$\frac{11\pi}{6}$		30°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$

Example 6) Calculate each of the following expressions. Do not look at the chart on the previous page as you will not have it in an exam. As you did, draw a picture which will help you to calculate the values of trig expressions. Label the picture in case you have to use it again.

a) $8\sin 30° - 6\cos 60°$

$$8\left(\frac{1}{2}\right) - 6\left(\frac{1}{2}\right) = 1$$

b) $-2\tan^2 150° + 4\cot^2 300°$

$$-2\left(\frac{-\sqrt{3}}{3}\right)^2 + 4\left(\frac{-\sqrt{3}}{3}\right)^2 = \frac{-2}{3} + \frac{4}{3} = \frac{2}{3}$$

c) $\sin^2 315° + \cos^2 315°$

$$\left(\frac{-\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

d) $(4\sin 150° + \cos 240°)^2$

$$\left[4\left(\frac{1}{2}\right) - \frac{1}{2}\right]^2 = \frac{9}{4}$$

e) $\dfrac{-5\sin 90° - 2\cos 120°}{-5\sin 90° + 2\cos 120°}$

$$\frac{-5(1) - 2\left(\frac{-1}{2}\right)}{-5(1) + 2\left(\frac{-1}{2}\right)} = \frac{-5+1}{-5-1} = \frac{2}{3}$$

f) $\left(2\cot\dfrac{5\pi}{4} - \sin^2\dfrac{2\pi}{3}\right)^2$

$$\left[2(1) - \left(\frac{\sqrt{3}}{2}\right)^2\right]^2 = \left(2 - \frac{3}{4}\right)^2 = \frac{25}{16}$$

Co-terminal Angles:

So far, our angles have all been between $0°$ and $360°$. What about angles outside that range? We will find that since $360°$ represents one full rotation, that when we take a trig function of an angle greater than $360°$, the reference angle is the same as the angle created when subtracted $360°$ from the original angle. So we can make this claim. We may add or subtract any multiple of $360°$ (2π) to any angle and the trig functions of that angle remain the same. Not that we are not saying the angle remains the same; $100°$ and $460°$ are clearly different angles, but $\sin 100° = \sin 460°$

Example 7) For each angle given, find the angle between $0°$ and $360°$ which is co-terminal and then find the **signs** of the trig functions of that angle.

θ	Co-terminal angle (between $0°$ and $360°$)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$400°$	$40°$	+	+	+	+	+	+
$850°$	$130°$	+	-	-	+	-	-
$1275°$	$195°$	-	-	+	-	-	+
$-231°$	$129°$	+	-	-	+	-	-
$359°$	$1°$	-	+	-	-	+	-
$\dfrac{17\pi}{3}$	$\dfrac{5\pi}{3}$	-	+	-	-	+	-

Unit 2 - The Trigonometric Functions - Homework

1. Let P be a point on the terminal side of θ. Draw a picture showing the reference angle and find the 6 trig functions of θ.

a) $P(12,9)$

$\sin\theta = \frac{3}{5}$ $\quad\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\quad\sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\quad\cot\theta = \frac{4}{3}$

b) $P(30,16)$

$\sin\theta = \frac{8}{17}$ $\quad\csc\theta = \frac{17}{8}$

$\cos\theta = \frac{15}{17}$ $\quad\sec\theta = \frac{17}{15}$

$\tan\theta = \frac{8}{15}$ $\quad\cot\theta = \frac{15}{8}$

c) $P(1,2)$

$\sin\theta = \frac{2}{\sqrt{5}}$ $\quad\csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}}$ $\quad\sec\theta = \frac{\sqrt{5}}{1}$

$\tan\theta = \frac{2}{1}$ $\quad\cot\theta = \frac{1}{2}$

d) $P(3,\sqrt{7})$

$\sin\theta = \frac{\sqrt{7}}{4}$ $\quad\csc\theta = \frac{4}{\sqrt{7}}$

$\cos\theta = \frac{3}{4}$ $\quad\sec\theta = \frac{4}{3}$

$\tan\theta = \frac{\sqrt{7}}{3}$ $\quad\cot\theta = \frac{3}{\sqrt{7}}$

e) $P(-8,-6)$

$\sin\theta = -\frac{3}{5}$ $\quad\csc\theta = -\frac{5}{3}$

$\cos\theta = -\frac{4}{5}$ $\quad\sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\quad\cot\theta = \frac{4}{3}$

f) $P(1,-3)$

$\sin\theta = -\frac{3}{\sqrt{10}}$ $\quad\csc\theta = -\frac{\sqrt{10}}{3}$

$\cos\theta = \frac{1}{\sqrt{10}}$ $\quad\sec\theta = \frac{\sqrt{10}}{1}$

$\tan\theta = \frac{-3}{1}$ $\quad\cot\theta = \frac{-1}{3}$

g) $P(6,-\sqrt{13})$

$\sin\theta = -\frac{\sqrt{13}}{7}$ $\quad\csc\theta = -\frac{7}{\sqrt{13}}$

$\cos\theta = \frac{6}{7}$ $\quad\sec\theta = \frac{7}{6}$

$\tan\theta = \frac{-\sqrt{13}}{6}$ $\quad\cot\theta = \frac{-6}{\sqrt{13}}$

h) $P(-\sqrt{2},-\sqrt{2})$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\quad\csc\theta = -\frac{1}{\sqrt{2}}$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\quad\sec\theta = -\frac{1}{\sqrt{2}}$

$\tan\theta = 1$ $\quad\cot\theta = 1$

2. Given information about one trig function, find other trig functions:

a) If $\tan\theta = \frac{4}{3}, \theta$ in quadrant I, find $\cos\theta$ and $\sin\theta$.

$\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$

b) If $\cos\theta = \frac{\sqrt{3}}{2}, \theta$ in quadrant IV, find $\sin\theta$ and $\tan\theta$.

$\sin\theta = \frac{-1}{2}, \tan\theta = -\frac{\sqrt{3}}{3}$

c) If $\sin\theta = \frac{5}{8}, \theta$ in quadrant II, find $\sec\theta$ and $\cot\theta$. d) If $\sec\theta = -\frac{5}{2}, \theta$ in quadrant III, find $\sin\theta$ and $\tan\theta$.

$\boxed{\sec\theta = \frac{-8}{\sqrt{39}}, \cot\theta = \frac{-\sqrt{39}}{5}}$

$\boxed{\sin\theta = \frac{-\sqrt{21}}{5}, \tan\theta = \frac{\sqrt{21}}{2}}$

e) If $\tan\theta = -5, \theta$ in quadrant IV, find $\sin\theta$ and $\sec\theta$. f) If $\cos\theta = \frac{\sqrt{2}}{3}$ and $\sin\theta < 0$, find $\sin\theta$ and $\tan\theta$.

$\boxed{\sin\theta = \frac{-5}{\sqrt{26}}, \sec\theta = \sqrt{26}}$

$\boxed{\sin\theta = \frac{-\sqrt{7}}{3}, \tan\theta = -\sqrt{\frac{7}{2}}}$

g) If $\sec\theta = \frac{6}{5}$, find $\sin\theta$ and $\tan\theta$.

$\boxed{\sin\theta = \frac{\pm\sqrt{11}}{6}, \tan\theta = \frac{\pm\sqrt{11}}{5}}$

h) If $\tan\theta = \frac{4\sqrt{5}}{5}$, find $\sin\theta$ and $\cos\theta$.

$\boxed{\sin\theta = \frac{\pm 4}{\sqrt{21}}, \cos\theta = \frac{\pm 5}{\sqrt{105}} \text{ or } \pm\sqrt{\frac{5}{21}}}$

3. In what quadrant is

a) $\sin\theta > 0$ and $\cos\theta < 0$

$\boxed{\text{II}}$

b) $\csc\theta > 0$ and $\cot\theta < 0$

$\boxed{\text{II}}$

c) $\sec\theta < 0$ and $\tan\theta < 0$

$\boxed{\text{II}}$

d) $\csc\theta < 0$ and $\cos\theta < 0$

$\boxed{\text{III}}$

4. Find the value of the following (do not look at the chart – make a small picture and calculate the values)

a) $\boxed{\begin{array}{l} 5\sin 90° - 7\cos 180° \\ 5(1) - 7(-1) = 12 \end{array}}$

b) $\boxed{\begin{array}{l} 4\sec 0° + 7\csc 270° \\ 4(1) + 7(-1) = -3 \end{array}}$

c) $\boxed{\begin{array}{l} \sin^2 180° + \cos^2 180° \\ (0)^2 + (-1)^2 = 1 \end{array}}$

d) $\boxed{\begin{array}{l} \left(6\cot\frac{3\pi}{2} + 3\sec\pi\right)^3 \\ \left(6\cot 0 + 3\sec(-1)\right)^3 = -27 \end{array}}$

e) $\boxed{\begin{array}{l} \cos 0° \sin 270° - \cos 270° \sin 0° \\ 1(-1) - (0)(0) = -1 \end{array}}$

f) $\boxed{\begin{array}{l} (\sin 270° - \sec 0°)(\sin 270° + \sec 0°) \\ (-1-1)(-1+1) = 0 \end{array}}$

5. For each statement, determine whether or not it is Possible (P) or Impossible (I).

a) $\sin\theta = -5$ **Impossible**

b) $\tan\theta + 1 = 3.79$ **Possible**

c) $2\cos\theta + 5.5 = 4$ **Possible**

d) $\sin\alpha + \cot\beta = 8$ **Possible**

e) $\csc\alpha + \sin\beta = .5$ **Possible**

f) $\sin\alpha + \cos\beta = 2$ **Possible**

6. Find the value of the following (do not look at the chart – make a small picture and calculate the values)

a) $6\sin 30° - 4\cos 150°$

$$6\left(\frac{1}{2}\right) - 4\left(\frac{-\sqrt{3}}{2}\right) = 3 + 2\sqrt{3}$$

b) $8\sin 60° - 4\sin 300°$

$$8\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{-\sqrt{3}}{2}\right) = 6\sqrt{3}$$

c) $(4\tan 120°)(8\cos 225°)$

$$4\left(-\sqrt{3}\right)(8)\left(-\frac{\sqrt{2}}{2}\right) = 16\sqrt{6}$$

d) $6\sin 315° + 8\tan 135°$

$$6\left(-\frac{\sqrt{2}}{2}\right) + 8(-1) = -3\sqrt{2} - 8$$

e) $\dfrac{8\csc 30°}{\cot 330°}$

$$\frac{8(2)}{-\sqrt{3}} = \frac{-16}{\sqrt{3}} \text{ or } \frac{-16\sqrt{3}}{3}$$

f) $-2\cos 225° - 4\cot 315° + 3$

$$-2\left(\frac{-\sqrt{2}}{2}\right) - 4(-1) + 3 = 7 + \sqrt{2}$$

g) $\sin^2 225° - \cos^2 225°$

$$\left(-\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{2}}{2}\right)^2 = 0$$

h) $\cos^3 630° - \csc^3(-30°)$

$$0 - (-2)^3 = 8$$

i) $\left(\sin\dfrac{\pi}{6} - 4\cos\dfrac{2\pi}{3}\right)^2$

$$\left[\frac{1}{2} - 4\left(\frac{-1}{2}\right)\right]^2 = \frac{25}{4}$$

j) $\left(\cos^2\dfrac{3\pi}{4} - \csc^2\dfrac{7\pi}{6}\right)^4$

$$\left[\left(-\frac{\sqrt{2}}{2}\right)^2 - (-2)^2\right]^4 = \left(\frac{-7}{2}\right)^4 = \frac{2401}{16} = 150.063$$

7. For each value of θ, determine the co-terminal angle and the signs of the trig functions of that angle.

θ	Co-terminal angle (between 0° and 360°)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
700°	340°	-	+	-	-	+	-
1525°	85°	+	+	+	+	+	+
–485°	235°	-	-	+	-	-	+
2.5π	$\frac{\pi}{2}$	1	0	∞	1	∞	0
$\frac{-20\pi}{7}$	$\frac{8\pi}{7}$	-	-	+	-	-	+