WESTSIDE

Algebra 2 HADV

Summer Enrichment Assignment

IMPORTANT INSTRUCTIONS FOR STUDENTS!!!

We understand that students come to Algebra II with different strengths and needs. For this reason, students have options for completing the packet and getting assistance!

- **Students should try to answer all the question, if possible. YOU MUST SHOW ALL WORK TO RECEIVE CREDIT.**
- **Khan Academy video tutorials may be very helpful to you. HISD aligned Khan Academy videos are available by clicking this link:** http://www.houstonisd.org/cms/lib2/TX01001591/Centricity/Domain/8050/Khan_Acad_Video_Algmt_Alg1.pdf
- **Notes for each problem are provided at the end of packet.**
- **If you need help with the problems, come to tutorials the first week of school**
- **Finally, honor and integrity are at the heart of a Westside Wolf! Smart wolves never cheat. You are only hurting yourself by attempting to copy someone else's work. This packet is to help you get ready for Algebra II and help your teachers know what you can do.**

Name:____________________________ Date:________________________

Algebra 2 Pre-AP Summer Packet

Solving Equations

Exercises: Solve each equation. Then check your solution.

1. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$

2. $3(a + 1) - 5 = 3a - 2$

Solving Equations and Formulas

Exercises: Solve each equation or formula for the variable specified.

3. $7x + 3y = m$ for y

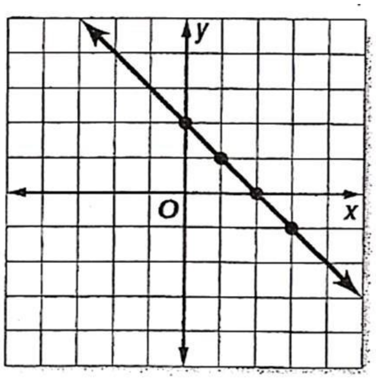
4. $xy + xz = 6 + a$ for x

Describe Number Patterns

5. **Write an equation for the function in functional notation. Then complete the table.**

x	-1	0	1	2	3
y	-2	2	6		

6. **Write an equation in functional notation**.



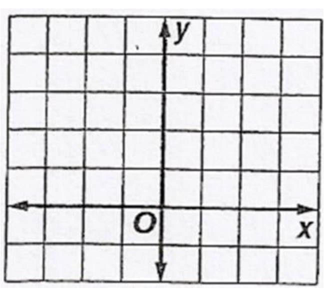
Equations of Linear Functions

Exercise: Write an equation of the line in Standard Form with the given information.

7. Slope: -2, point (5, 3)

Exercise: Graph the function.

8. $3x + y = 2$

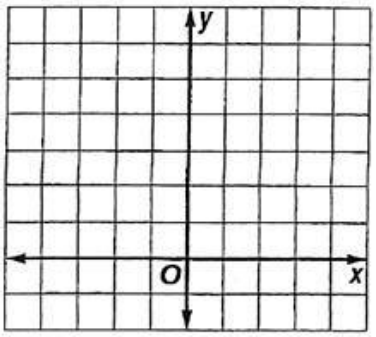


Graphing Systems of Equations

Exercises:

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

9. $2x + y = 6$
$2x - y = -2$



Solving Systems of Equations by Substitution & Elimination

Exercises: Use substitution or elimination to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

10. $y = 4x$
$3x - y = 1$

11. $2m - 3n = 14$
$m + 3n = -11$

Multiplying a Polynomial by a Monomial

12. $-2g\,(g^2 - 2g + 2)$

13. $3x\,(x^4 + x^3 + x^2)$

Factoring Using the Greatest Common Factor

Exercises: Factor each polynomial.

14. $55p^2 - 11p^4 + 44p^5$

15. $14y^3 - 28y^2 + y$

Multiplying Polynomials

Exercises: Find each product.

16. *$(y + 5)(y + 2)$*

17. *$(2x - 1)(x + 5)$*

Factoring Trinomials: $x^2 + bx + c$ & $ax^2 + bx + c$

Exercises: Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

18.. *$x^2 - 4x - 21$*

19.. *$3x^2 + 2x - 8$*

Solving Equations

Addition Property of Equality	*For any numbers a, b, and c, if a = b, then a + c = b +c.*
Subtraction Property of Equality	*For any numbers a, b, and c, if a = b, then a – c = b – c.*
Multiplication Property of Equality	*For any numbers a, b, and c, if a = b, then ac = bc.*
Division Property of Equality	*For any numbers a, b, and c, with c ≠ 0, if a = b, then* $\frac{a}{c} = \frac{b}{c}$.

Example 1

Solve: ***3 ½ p = 1 ½***

$\frac{7}{2}p = \frac{3}{2}$ — *Original equation*

Rewrite each mixed number as an improper fraction

$\frac{2}{7}\left(\frac{7}{2}p\right) = \left(\frac{3}{2}\right)\frac{2}{7}$ — *Multiply each side by the reciprocal of 7/2.*

$p = \frac{3}{7}$ — *Simplify*

Check: *3 ½* $\left(\frac{3}{7}\right)$ *= 1 ½* — *Substitute solution for variable*

$\frac{7}{\cancel{2}}\left(\frac{3}{\cancel{7}}\right) = \frac{3}{\cancel{2}}$ — *Rewrite each mixed number as an improper fraction*

$\frac{3}{7} = \frac{3}{7}$ — *Left Hand Side = Right Hand Side*

LHS = RHS correct

Example 2

Solve: -5n = 60 — *Original equation*

$\frac{-5n}{-5} = \frac{60}{-5}$ — *Divide both sides by -5 or multiply both sides by -1/5*

n = -12 — *Simplify*

Check: *-5 (-12) = 60* — *Substitute solution for variable*

60 = 60 — *Left Hand Side = Right Hand Side*

LHS = RHS correct

To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to get the variable on just one side of the equation. Then use the Multiplication or Division Property of Equality to solve the equation.

Example 1

Solve: 5y – 8 = 3 y + 12

5y – 8 – 3y = 3 y + 12 – 3y

2y – 8 = 12

2y – 8 + 8 = 12 + 8

2y = 20

$\frac{2y}{2} = \frac{20}{2}$

y = 10

Check: *5y – 8 = 3 y + 12*

5 (10) – 8 = 3(10) +12

50 - 8 = 30 + 12

42 = 42

LHS = RHS correct

Example 2

Solve: -11 – 3y = 8y + 1

-11 – 3y + 3y = 8y + 1 + 3y

-11 = 11y + 1

-11 -1 = 11y + 1 -1

-12 = 11y

$\frac{-12}{11} = \frac{11y}{11}$

$-\frac{12}{11} = y$ *or* $-1\frac{1}{11} = y$

Check: *-11 – 3y = 8y + 1*

$-11 - 3\left(\frac{-12}{11}\right) = 8\left(\frac{-12}{11}\right) + 1$

$-11 + \frac{36}{11} = \frac{-96}{11} + 1$

$\frac{-121}{11} + \frac{36}{11} = \frac{-96}{11} + \frac{11}{11}$

$\frac{-85}{11} = \frac{-85}{11}$

LHS = RHS correct

Example 3

Solve: 4 (2a – 1) = -10 (a – 5)

8a – 4 = -10a + 50

8a – 4 + 10a = -10 a + 50 + 10a

18a – 4 = 50

1 8a – 4 + 4 = 50 + 4

18a = 54

$\frac{18a}{18} = \frac{54}{18}$

a = 3

Check: *4 (2a – 1) = -10 (a – 5)*

4 (2(3) – 1) = -10 (3 – 5)

4 (6 – 1) = -10(-2)

4 (5) = 20

20 = 20

LHS = RHS correct

Solving Equations and Formulas

Solve for variables: sometimes you may want to solve an equation such as V = lwh for one of its variables. For example, if you know the values of V, w, and h, then the equation $l = \frac{V}{wh}$ *is more useful for finding the value of l.*

Example 1 Solve 2x – 4 y = 8 for y.	Example 2 Solve 3m – n = km – 8
$2x - 4y = 8$ $2x - 4y - 2x = 8 - 2x$ $-4y = 8 - 2x$ $\frac{-4y}{-4} = \frac{8 - 2x}{-4}$ $y = \frac{8 - 2x}{-4}$ or $\frac{2x - 8}{4}$	$3m - n = km - 8$ $3m - n - km = km - 8 - km$ $3m - n - km = -8$ $3m - n - km + n = -8 + n$ $3m - km = -8 + n$ $m(3 - k) = -8 + n$ $\frac{m(3 - k)}{3 - k} = \frac{-8 + n}{3 - k}$ $m = \frac{-8 + n}{3 - k}$, or $\frac{n - 8}{3 - k}$ *Since division by 0 is undefined,* $3 - k \neq 0$, *or* $k \neq 3$.

Describe Number Patterns

Write Equations: *Sometimes a pattern can lead to a general rule that can be written as an equation.*

Example: Suppose you purchased a number of packages of blank CDs. If each package contains 3 CDs, you could make a chart to show the relationship between the number of packages of compact disks and the number of disks purchased. Use x for the number of packages and y for the number of compact disks.

Make a table of ordered pairs for several points of the graph.

Number of packages	1	2	3	4	5
Number of CDs	3	6	9	12	15

The difference in the x values is 1, and the difference in the y values is 3. This pattern shows that y is always three times x. This suggests the relation y = 3x. Since the relation is also a function, we can write the equation in functional notation as f(x) = 3x.

Equations of Linear Functions

Standard Form	$Ax + By = C$
Slope-Intercept Form	$y = mx + b$, where m is the given slope and b is the y-intercept
Point-Slope Form	$y - y_1 = m(x - x_1)$, where m is the given slope and (x_1, y_1) is the given point

Example 1: Write an equation of a line in standard form whose slope is -4 and whose y-intercept is 3.

$y = mx + b$

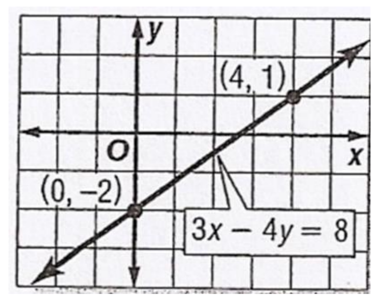
$y = -4x + 3$

$+4x \quad +4x$

$4x + y = 3$

Example 2: Graph 3x – 4y = 8

$3x - 4y = 8$	Original equation
$-4y = -3x + 8$	Subtract 3x from each side
$\dfrac{-4y}{-4} = \dfrac{-3x + 8}{-4}$	Divide each side by -4
$y = \frac{3}{4}x - 2$	Simplify



The y-intercept of $y = 3/4x - 2$ is -2 and the slope is $\frac{3}{4}$. So graph the point (0, -2). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

Graphing Systems of Equations

Solve by Graphing One method of solving a system of equations is to graph the equations on the same coordinate plane.

Example: Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

a. $x + y = 2$

$x - y = 4$

The graphs intersect. Therefore, there is one solution. The point (3, -1) seems to lie on both lines. Check this estimate by replacing x with 3 and y with -1 in each equation.

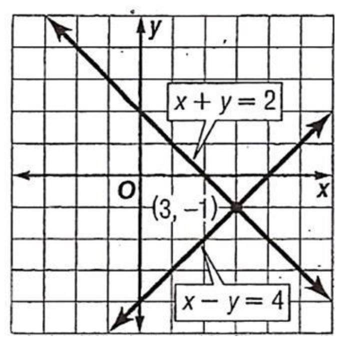
$x + y = 2$

$3 + (-1) = 2$ ✓

$x - y = 4$

$3 - (-1) = 3 + 1$ or 4 ✓

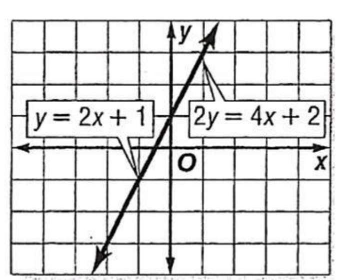
The solution is (3,-1).



b. $y = 2x + 1$

$2y = 4x + 2$

The graphs coincide. Therefore there are infinitely many solutions.



Solving Systems of Equations by Substitution

Example 1: use substitution to solve they system of equations.

$\boldsymbol{y = 2x}$

$\boldsymbol{4x - y = -4}$

Substitute 2x for y in the second equation.

$4x - y = -4$	*second equation*
$4x - 2x = -4$	$y = 2x$
$2x = -4$	*combine like terms*
$x = -2$	*Divide each side by 2 and simplify.*

Use y = 2x to find the value of y.

$y = 2x$	*First equation*
$y = 2(-2)$	$x = -2$
$y = -4$	*simplify*

The solution is (-2,-4).

Example 2: Solve for one variable, then substitute.

$\boldsymbol{x + 3y = 7}$

$\boldsymbol{2x - 4y = -6}$

Solve the first equation for x since the coefficient of x is 1.

$x + 3y = 7$	*First equation*
$x + 3y - 3y = 7 - 3y$	*Subtract 3y from each side*
$x = 7 - 3y$	*Simplify*

Find the value of y by substituting 7 – 3y for x in the second equation.

$2x - 4y = -6$	*Second equation*
$2(7- 3y) - 4y = -6$	$x = 7 - 3y$
$14 - 6y - 4y = -6$	*Distributive Property*
$14 - 10y = -6$	*Combine like terms.*
$14 - 10y - 14 = -6 -14$	*Subtract 14 from each side.*
$-10y = -20$	*Simplify.*
$y = 2$	*Divide each side by -10 and simplify.*

Use y = 2 to find the value of x.

$x = 7- 3y$

$x = 7 - 3(2)$

$x = 1$

The solution is (1, 2).

Elimination Using Addition

Elimination Using Addition: *In systems of equations in which the coefficients of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called elimination.*

Example 1: Use addition to solve the system of equations

$x - 3y = 7$

$3x + 3y = 9$

Write the equations in column form and add to eliminate y.

$$\begin{array}{rl} x - 3y &= 7 \\ \underline{(+)\ 3x + 3y} &\underline{= 9} \\ 4x &= 16 \\ \frac{4x}{4} &= \frac{16}{4} \\ x &= 4 \end{array}$$

Substitute 4 for x either equation and solve for y.

$$\begin{array}{rl} 4x - 3y &= 7 \\ 4 - 3y - 4 &= 7 - 4 \\ \frac{-3y}{-3} &= \frac{3}{-3} \\ y &= -1 \end{array}$$

The solution is (4, -1).

Example 2: The sum of two numbers is 70 and their difference is 24. Find the numbers.

Let x represent one number and y represent the other number.

$$\begin{array}{rl} x + y &= 70 \\ \underline{(+)\ x - y} &\underline{= 24} \\ 2x &= 94 \\ \frac{2x}{2} &= \frac{94}{2} \\ x &= 47 \end{array}$$

Substitute 47 for x in either equation.

$$\begin{array}{rl} 47 + y &= 70 \\ 47 + y - 47 &= 70 - 47 \\ y &= 23 \end{array}$$

The numbers are 47 and 23.

Multiplying a Polynomial by a Monomial

Product of Monomial and Polynomial*: The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.*

Example 1: Find $-3x^2 (4x^2 + 6x - 8)$.

$$\begin{aligned} &-3x^2 (4x^2 + 6x - 8) \\ &= -3x^2 (4x^2) + (-3x^2)(6x) - (-3x^2)(8) \\ &= -12x^4 + (-18x^3) - (-24x^2) \\ &= -12x^4 - 18x^3 + 24x^2 \end{aligned}$$

Example 2: Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$

$$\begin{aligned} &-2(4x^2 + 5x) - x(x^2 + 6x) \\ &= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x) \\ &= -8x^2 + (-10x) + (-x^3) + (-6x^2) \\ &= (-x^3) + [-8x^2 + (-6x^2)] + (-10x) \\ &= -x^3 - 14x^2 - 10x \end{aligned}$$

Factoring Using the Greatest Common Factor

Example 1: Use GCF to factor $12mn + 80m^2$
Find the GCF of $12mn$ and $80m^2$
$12mn = 2 \cdot 2 \cdot 3 \cdot m \cdot n$
$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5\, m \cdot m$
$GCF = 2 \cdot 2 \cdot m$ or $4m$
Write each term as the product of the GCF and its remaining factors.
$12mn + 80m^2 = 4m\,(3 \cdot n) + 4m\,(2 \cdot 2 \cdot 5 \cdot m)$
$= 4m\,(3n) + 4m\,(20m)$
$= 4m\,(3n + 20m)$
$12mn + 80m^2 = 4m\,(3n + 20m)$

Example 2: Factor $6ax + 3ay + 2bx + by$ by grouping.
$6ax + 3ay + 2bx + by$
$= (6ax + 3ay) + (2bx + by)$
$= 3a\,(2x + y) + b\,(2x + y)$
$= (3a + b)(2x + y)$

Check using the FOIL method.
$(3a + b)(2x + y)$
$= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y)$
$= 6ax + 3ay + 2bx + by$

Multiplying Polynomials

Multiply Binomials: To multiply two binomials, you can apply the Distributive Property twice. You can use FOIL (First, Outer, Inner and Last) method.

Example 1: Find $(x + 3)(x - 4)$
$(x + 3)(x - 4)$
$= x\,(x - 4) + 3\,(x - 4)$
$= (x)(x) + x\,(-4) + 3(x) + 3(-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

Example 2: Find $(x - 2)(x + 5)$ using FOIL method.
$(x - 2)(x + 5)$
First Outer Inner Last
$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$
$= x^2 + 5x + (-2x) - 10$
$= x^2 + 3x - 10$

Factoring Trinomials: $x^2 + bx + c$

Factor $x^2 + bx + c$: To factor a trinomial of the form $x^2 + bx + c$, find two integers m and n, whose sum is equal to b and whose product is equal to c.

Example 1: Factor each trinomial.

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1 , 10	11
2 , 5	7

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$.
Notice that $m + n$ is negative and mn is positive, so m and n are both negative.
Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $n = -1$.
$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2: Factor $x^2 + 6x - 16$
In this trinomial, $b = 6$ and $c = -16$. This means $m + n$ is positive and mn is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of -16	Sum of Factors
1, -16	-15
-1 , 16	15
2 , -8	-6
-2, 8	6

Therefore, $m = -2$ and $n = 8$.
$x^2 + 6x - 16 = (x - 2)(x + 8)$

Factoring Trinomials: $ax^2 + bx + c$

Factor $ax^2 + bx + c$: *To factor a trinomial of the form $ax^2 + bx + c$, find two integers m and n, whose sum is equal to b and whose product is equal to ac. If there are no integers that satisfy these requirements, the polynomial is called a* ***prime polynomial****.*

Example 1: Factor $2x^2 + 15x + 18$.

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers whose sum is 15 and whose product is $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors whose sum is 15.

Factors of 36	Sum of Factors
1, 36	37
2 , 18	20
3 , 12	15

Use the pattern $ax^2 + mx + nx + c$ with $a = 2$, $m = 3$, $n = 12$ and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Example 2: Factor $3x^2 - 3x - 18$

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 whose sum is -1.

Factors of -6	Sum of Factors
1, -6	-5
-1, 6	5
-2 , 3	1
2, -3	-1

Now use the pattern $(x + m)(x + n)$ with $m = 2$ and $n = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is

$3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.