NAME:______________________________ DATE:__________

Multivariable Calculus 3 Honors Summer Review Packet
For Students whose most recent course was BC Calculus

DUE FIRST DAY OF CLASS FALL!!!

Welcome to the Calculus 3 class. You should recognize this set of problems since it is one of the practice packets you got last year as you prepared for the AP Calculus BC Exam. It is important for you to rework this packet (or perhaps, work for the 1st time) so that you are refreshed on the Calculus 1 (AB) and Calculus 2 (BC) material that is imperative for your success in Calculus 3.

- Solutions to the problems contained in this packet are on the last page.
- Since you will also be CalPALS (Calculus Peer-Assisted Learners) for the AB and BC students for the upcoming school year, it will be necessary for you to remember the curriculum you needed to know for that material.

- **There will be a Quiz over this material the second day of class!!**
- Finally, honor and integrity is at the heart of a Westside Wolf! Smart wolves never cheat. You are only hurting yourself by attempting to copy someone else's work. This packet is to help you be ready for Multivariable Calculus 3 Honors and to help me know what you can do.

EXAM VI
CALCULUS BC
SECTION I PART A
Time–60 minutes
Number of questions–30

BC

MASTER

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. Which of the following is an antiderivative of $\sqrt{4-2x}$?

(A) $-\frac{1}{3}(4-2x)^{3/2}$ (B) $\frac{2}{3}(4-2x)^{3/2}$ (C) $-\frac{1}{6}(4-2x)^{3}$

(D) $\frac{1}{2}(4-2x)^{2}$

Ans

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2. A particle moves along a straight line with its position at any time $t \geq 0$ given by $s(t) = \int_0^t (\sqrt{x} - x + 1)\, dx$, where s is measured in meters and t in seconds. What is the velocity of the particle when its acceleration is zero?

(A) $-\frac{1}{4}$ m/s (B) $\frac{1}{4}$ m/s (C) $\frac{1}{2}$ m/s (D) $\frac{5}{4}$ m/s

Ans

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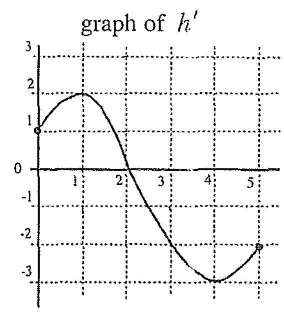
3. If $\dfrac{dy}{dx} = \dfrac{\cos x}{e^y}$ and $y(0) = 0$, then $y\left(\dfrac{\pi}{2}\right) =$

(A) 0
(B) $\ln 2$
(C) 1
(D) $\dfrac{1}{2}$

Ans

4. The function h is defined on the interval $0 \le x \le 5$ and a graph of its derivative function h' is shown in the figure. Which of the following are true?



I. The function h is decreasing on the interval $(1, 2)$.
II. The function h has a local maximum at the point where $x = 2$.
III. Given $h(1) = -1$, an equation of the tangent line to the graph of h at the point $(1, -1)$ is $y = 2x - 3$.

(A) I only (B) II only (C) III only (D) II and III only

Ans

5. $\displaystyle\int_1^{\infty} \frac{x}{1+x^2}\,dx$ is

(A) $\dfrac{\pi}{4}$ (B) 1 (C) $\dfrac{\pi}{2}$ (D) divergent

Ans

6. If $x = \sin t$ and $y = \cos^2 t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $-\frac{1}{4}$ (D) -2

Ans

7. In an effort to enhance fishing, 100 trout were initially put in a small lake. Fishery Department biologists predict that the rate of growth of the trout population is modeled by the logistic differential equation $\frac{dP}{dt} = 0.12P\left(1 - \frac{P}{600}\right)$, where time t is measured in months. Which of the following is true?

I. The growth rate of the fish population is greatest at $P = 600$.

II. If $P > 600$, the population of fish is decreasing.

III. $\lim_{t \to \infty} P(t) = 600$

(A) I only

(B) II only

(C) II and III only

(D) I, II and III

Ans

8. If f and g are both continuous and differentiable functions for all real numbers, which of the following definite integrals is equal to $f(g(5)) - f(g(3))$?

(A) $\int_3^5 f(g'(x)) \cdot g(x)\, dx$

(B) $\int_3^5 f'(g(x)) \cdot g(x)\, dx$

(C) $\int_3^5 f'(g(x)) \cdot g'(x)\, dx$

(D) $\int_3^5 f(g(x)) \cdot f'(x)\, dx$

Ans

9. $\lim\limits_{x \to +\infty} \dfrac{x - \frac{1}{2x}}{2x - \frac{1}{6x}} =$

(A) -3 (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{2}$

Ans

10. An equation of the line tangent to the graph of $y = \dfrac{3x+4}{4x-3}$ at the point where $x = 1$ is

(A) $y + 25x = 32$ (B) $y - 31x = -24$ (C) $y + 5x = 12$
(D) $y - 25x = -18$

Ans

11. If $\dfrac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0, 4]$ is

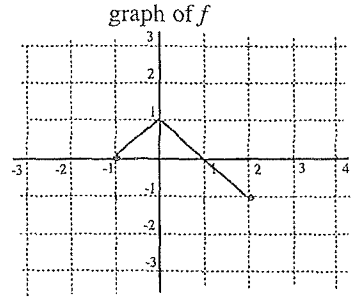
(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$

Ans

12. A graph of the function f, consisting of two line segments, is shown in the figure. If $g(x) = \int_1^x f(t)\, dt$, then the maximum value of g on the closed interval $[-1,2]$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) 1

graph of f



Ans

13. The total area of the region enclosed by the polar graph of $r = 1 + \cos\theta$ is given by which of the following expressions?

(A) $\frac{1}{2}\int_0^{\pi} (1 + \cos\theta)^2\, d\theta$

(B) $\int_0^{\pi} (1 + \cos\theta)^2\, d\theta$

(C) $\frac{1}{2}\int_0^{2\pi} (1 + \cos\theta)\, d\theta$

(D) $2\int_0^{2\pi} (1 + \cos\theta)^2\, d\theta$

Ans

14. A particle moves along the x-axis with acceleration given by $a(t) = \cos t$ ft/sec^2 for $t \geq 0$. At time $t = 0$ seconds the velocity of the particle is 2 ft/sec. The total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 1 ft (B) $\frac{\pi}{2}$ ft (C) π ft (D) $\pi+1$ ft

Ans

15. A curve is parametrically defined by the equations $x = 2\cos t$ and $y = 2\sin t$. The length of the arc from $t = 0$ to $t = 2$ is

(A) 2 (B) 4 (C) 6 (D) 8

Ans

16. $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{2k} =$

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) 1 (D) ∞

Ans

17. The shortest distance from the curve $xy = 4$ to the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Ans

18. If the function f is defined so that $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

(A) f is not continuous at $x = 0$

(B) $f(a) = 0$

(C) $f'(a) = 0$

(D) $f(a)$ does not exist

Ans

19. A particle moves in the xy-plane so that its velocity vector at time $t \geq 0$ is $v(t) = (2t, \sin t)$ and the particle's position vector at time $t = 0$ is (0, 1). The position vector of the particle when $t = \pi$ is

(A) $(\pi^2 + 1, 3)$ (B) $(\pi^2, 3)$ (C) $(\pi^2, 2)$ (D) $(-\pi^2, 3)$

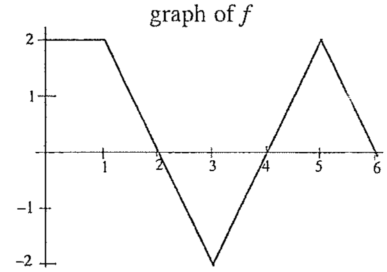
Ans

20. The density of cars (in cars per mile) along a 20-mile stretch of the Jersey Turnpike starting at a toll plaza is given by $f(x) = 500 + 100\sin(\pi x)$ where x is the distance in miles from the toll plaza and $0 \le x \le 20$. The total number of cars along the 20-mile stretch is

(A) 8500 (B) 9000 (C) 9500 (D) 10,000

Ans

21. The function G is defined on the interval [0, 6] by $G(x) = \int_0^x f(t)\,dt$ where f is the function graphed in the figure. A linear approximation of G near $x = 3$ is



(A) $6 - x$ (B) $8 - x$ (C) $5 - 2x$ (D) $8 - 2x$

Ans

22. What is the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{3^n (x+2)^n}{n+1}$?

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1

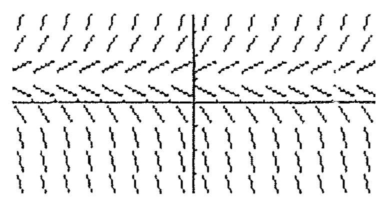
Ans

23. The infinite first quadrant region bounded above by the curve $y = e^{-2x}$ is rotated about the x-axis to generate a solid of revolution. The volume of the solid is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) ∞

Ans

24. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. Which of the following statements are true?



I. A solution curve that contains the point (0, 2) also contains the point (–2, 0).

II. As y approaches 1 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of y.

(A) I only (B) II only (C) I and II only (D) II and III only

Ans

25. If the definite integral $\int_1^7 \ln x \, dx$ is approximated by 3 circumscribed rectangles of equal width on the x-axis, then the approximation is

(A) $\frac{1}{2}(\ln 3 + \ln 5 + \ln 7)$

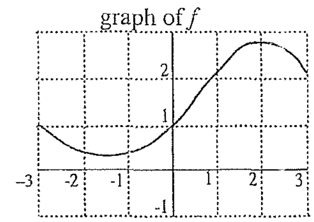
(B) $\frac{1}{2}(\ln 1 + \ln 3 + \ln 5)$

(C) $2(\ln 3 + \ln 5 + \ln 7)$

(D) $2(\ln 3 + \ln 5)$

Ans

26. The function f is defined on the interval $-3 \le x \le 3$ and its graph is shown in the figure. If the graph of f has a horizontal tangent at $x = 2$, which of the following statements are true?



I. $f(2) > f'(2)$

II. $\int_0^1 f(x)\,dx > f''(2)$

III. $1 - x + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{15} - \cdots$ is a Maclaurin series representation of the function f

(A) I only (B) II only (C) I and II only (D) I, II and III

Ans

27. Given the differential equation $\frac{dy}{dx} = \frac{10x}{x+y}$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans

28. The coefficient of $(x-1)^5$ in the Taylor series for $x \ln x$ about $x = 1$ is

(A) $-\frac{1}{20}$ (B) $\frac{1}{20}$ (C) $-\frac{1}{24}$ (D) $\frac{1}{24}$

Ans

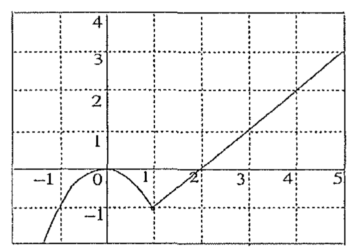
29. If the substitution $\sqrt{x} = u - 1$ is made in $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}\,dx$, the resulting integral is

(A) $\int_1^4 \frac{2}{u}\,du$ (B) $\int_1^4 \frac{1}{u(u-1)}\,du$ (C) $\int_1^4 \frac{1}{u}\,du$ (D) $\int_2^3 \frac{2}{u}\,du$

Ans

30. If the graph of f shown in the figure has a horizontal tangent at $x = 0$, which of the following statements are true.

graph of f



I. $\lim\limits_{x\to 2} \frac{f(x)}{\sin(x-2)} = 1$

II. $\lim\limits_{x\to 1} \frac{f(x-1)}{f(x+1)} = 1$

III. $\lim\limits_{x\to 2} \frac{\left[f(x)\right]^2}{(x-2)^2} = 1$

(A) I only (B) II only (C) I and III only (D) I, II, III

Ans

EXAM VI
CALCULUS BC
SECTION I PART B
Time–45 minutes
Number of questions–15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t+1)(t-4)^3$. For what value of t, $2 \leq t \leq 4$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [2, 4]?

   (A) 2.644 (B) 2.744 (C) 2.844 (D) 2.944

Ans

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2. If the function f defined by $f(x) = x^4 + ax^2 + 8x - 5$ has a horizontal tangent line and a point of inflection at the same value of x, then a is

   (A) –12 (B) –6 (C) 0 (D) 1

Ans

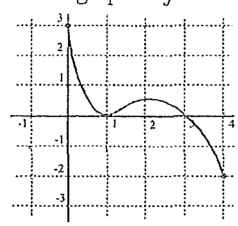
3. A graph of the second derivative of a function f is shown in the figure. Use the graph to determine which of the following are true.

   I. The f- graph is concave down on the interval (1, 3).

   II. The f-graph has points of inflection at $x = 1$ and $x = 3$.

   III. If $f'(2) = 0$, f is increasing at $x = 3$.

graph of f''



(A) I only (B) II only (C) III only (D) I, II, III

Ans

4. The area of the first quadrant region bounded above by the graph of $y = 1 + 2\sin x$ and below by the graph of $y = e^{x/2}$ is

(A) 2.312(B) 1.398 C) 1.343 (D) None of these.

Ans

5. The base of a solid is the region bounded below by the curve $y = x^2$ and above by the line $y = d$, where d is a positive constant. Every cross-section of the solid perpendicular to the y-axis is a square. If the volume of the solid is 72, what is the value of d?

(A) 4 (B) 6 (C) 8 (D) 10

Ans

6. The function g is defined by $g(x) = \int_{\pi/2}^{x} \cos t \, dt$. The maximum value of g on the closed interval $[-\pi, \pi]$ is

(A) –2 (B) –1 (C) 0 (D) 1

Ans

7. The Cartesian equation for the polar curve $r = \sin\theta$ is

(A) $x^2 + y^2 = x$

(B) $x^2 + y^2 = y$

(C) $x^2 + y^2 = x + y$

(D) $(x + y)^2 = y$

Ans

8. The function f is defined for all real numbers by $f(x) = \begin{cases} e^{-x} + 3, & \text{for } x > 0, \\ ax + b & \text{for } x \le 0. \end{cases}$ If f is differentiable at $x = 0$, then $a + b =$

(A) 0 (B) 1 (C) 2 (D) 3

Ans

9. If the derivative of the function f is given by $f'(x) = \cos\left(\frac{x}{2}\right) \cdot \ln x$ for $0 < x < 3\pi$, then the graph of f is increasing and concave down on the interval

(A) (0, 1.967) (B) (1.967, 3.141) (C) (3.141, 6.601)
(D) (6.601, 9.424)

Ans

10. The least integer value of a for which the the series $\sum_{n=1}^{\infty} \frac{1}{n^{a-27}}$ converges is

(A) 26 (B) 27 (C) 28 (D) 29

Ans

11. The graph of the function represented by the Maclaurin series $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \frac{x^6}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ intersects the graph of $y = 1 + x^2$ at the point where $x =$

(A) 0.718 (B) 0.738 (C) 0.758 (D) 0.778

Ans

12. A particle is traveling along the circle $x^2 + y^2 = 4$. When it is at the point $(1, \sqrt{3})$, $\frac{dx}{dt} = 2$. Find $\frac{dy}{dt}$ at this instant.

(A) $-\frac{2}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{2}{\sqrt{3}}$

Ans

13. Suppose interest on money in a bank account accumulates at an annual rate of 4% per year compounded continuously. If the balance $B = B(t)$ in the account satisfies the equation $\frac{dB}{dt} = .04B$, then approximately how much money should be invested today so that 5 years from now it would be worth $4000?

(A) $3600

(B) $3300

(C) $3000

(D) $2700

Ans

14. A graph of the function *f*, is shown in the figure. Which of the following are true?

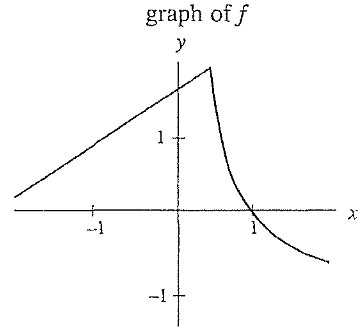
(A) $f''(1) < f'(1) < f(1)$

(B) $f'(1) < f(1) < f''(1)$

(C) $f(1) < f'(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

graph of *f*



Ans

15. If $\int f(x) \cdot \sec^2 x \, dx = f(x) \cdot \tan x - \int 9x^2 \cdot \tan x \, dx$, then $f(x)$ could be

(A) $x^3 \cdot \sec^2 x$ (B) $x^3 \cdot \tan x$ (C) $3x^3$ (D) $3x^2$

Ans

EXAM VI
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

1. At an electrical substation readings of the rate at which power is being used were recorded at 3 hour intervals over a 24-hour period and listed in the following table. The rate of power usage is in kilowatts per hour and is given by a differentiable function P at time t.

t	0	3	6	9	12	15	18	21	24
$P(t)$	1245	1268	1321	1316	1393	1369	1369	1451	1428

(a) Using a midpoint approximation with four equally spaced intervals, approximate $\int_0^{24} P(t)\,dt$.

Using correct units, explain the meaning of your answer in terms of power usage.

(b) Estimate how fast the rate of change of power usage is increasing at time $t = 12$. Show the computation that leads to your answer. Indicate units of measure.

(c) Assume that the function f, given by $f(t) = 1245 + 10te^{0.25\cos t}$, is an accurate model of the rate of power usage at time t, where t is measured in hours and $f(t)$ is in kilowatts per hour. Use $f(t)$ to approximate the average rate of power usage during the 24-hour time period. Undicate units of measure.

2. Let g be the function given by $g(x) = \int_0^x \cos(e^{t/2})\,dt$ for $-1 \le x \le 4$.

a) Find $g(1/2)$.

b) Find the instantaneous rate of change of g, with respect to x, at $x = 0$.

c) Find all values of x on the interval $(-1, 4)$ at which g has a relative maximum. Justify your answer.

d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-1, 4)$. Justify your answer.

e) Find the absolute maximum of g on the closed interval $[0, 4]$. Justify your answer.

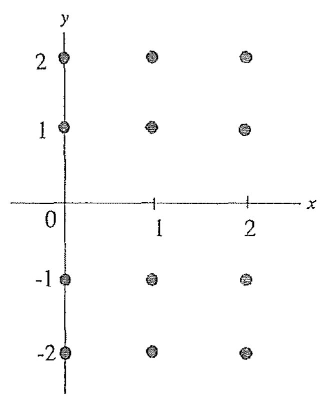
Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems

3. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = \frac{x-1}{y}$. with initial condition $f(0) = -2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Sketch the solution curve that passes through the point (0, –2).

(c) Find $f''(0)$.

(d) Find the particular solution to the differential equation that satisfies the initial condition $f(0) = -2$.

4. A particle with coordinates $(x(t), y(t))$ moves along a curve in the first quadrant so that

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t+1}} \quad \text{and} \quad \frac{dy}{dt} = \frac{1}{(t+1)^2} \qquad \text{for } t \geq 0.$$

(a) Find the coordinates of the particle in terms of t if, when $t = 0$, $x = 1$ and $y = 0$.

(b) Write an equation expressing y in terms of x.

(c) Find the average rate of change of y with respect to x as t varies from $t = 3$ to $t = 15$.

(d) Find the instantaneous rate of change of y with respect to x when $t = 8$.

5. The part of the curve of $y^2 - y + e^x = \cos x$ that is near the point (0, 1) defines y as a function of x implicitly.

(a) Find $\frac{dy}{dx}$.

(b) Find an equation of the line tangent to curve at the point (0, 1).

(c) Find $\frac{d^2y}{dx^2}$ at the point (0, 1).

(d) Use the tangent line approximation in part (b) to estimate the value of y for the point on the curve near (0, 1) where $x = 0.5$.

(e) Does the tangent line approximation in part (d) yield a larger or smaller value than the actual y-value. Explain.

6. Let f be the function defined by $f(x) = \frac{1}{4+x^2}$.

(a) Write the first four terms and general term for the power series expansion of f about $x = 0$.

(b) Determine the radius of convergence of the series found in part (a).

(c) Given the function g defined by $g(x) = \int_0^x \frac{1}{4+t^2}\,dt$, find the Taylor expansion of g about $x = 0$.

(d) Using your answer in part (c), show that $\frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \cdots = \frac{\pi}{8}$

EXAM V

Part A				Part B	
1.	C	18.	B	1.	C
2.	C	19.	B	2.	B
3.	C	20.	B	3.	B
4.	C	21.	D	4.	D
5.	B	22.	D	5.	B
6.	C	23.	B	6.	B
7.	D	24.	D	7.	C
8.	D	25.	B	8.	D
9.	A	26.	A	9.	A
10.	C	27.	D	10.	A
11.	D	28.	D	11.	D
12.	A	29.	C	12.	D
13.	C	30.	D	13.	B
14.	D			14.	D
15.	C			15.	A
16.	B				
17.	A				

EXAM VI

Part A				Part B	
1.	A	18.	C	1.	D
2.	D	19.	B	2.	B
3.	B	20.	D	3.	C
4.	D	21.	D	4.	B
5.	D	22.	C	5.	B
6.	D	23.	B	6.	C
7.	C	24.	D	7.	B
8.	C	25.	C	8.	D
9.	D	26.	C	9.	B
10.	A	27.	C	10.	D
11.	C	28.	A	11.	B
12.	C	29.	D	12.	A
13.	B	30.	C	13.	B
14.	D			14.	B
15.	B			15.	C
16.	A				
17.	D				

EXAM V Section II Part A

1. (a) $y - e^a = e^a(x-a)$ $R = (a-1, 0)$ (b) $A = \frac{1}{2}e^a|a-1|$

(c) Area max when $a = 0$. $A'(0) = 0$ and $A''(0) < 0$

$A(-1) = 0.368$, $A(0) = 0.5$, $A(1) = 0$

2. (a) $R_n = \sum_{k=1}^{n}\left(1\cdot\Delta x_k\cdot\frac{1}{1+x_k}\right)(x_k)^2$ When $n = 4$, $\Delta x = 1$, $x_1 = 0$

$R_4 = (1\cdot1\cdot1)(0)^2 + (1\cdot1\cdot\frac{1}{2})(1)^2 + (1\cdot1\cdot\frac{1}{3})(2)^2 + (1\cdot1\cdot\frac{1}{4})(3)^2 = \frac{49}{12}$

(b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1\cdot\Delta x_k\cdot\frac{1}{1+x_k}\right)(x_k)^2 = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{(x_k)^2}{1+x_k}\Delta x = \int_0^4\frac{x^2}{1+x}dx$

(c) $\int_0^4\frac{x^2}{1+x}\,dx = 5.609$ grams

EXAM VI Section II Part A

1. (a) $6(1268+1316+1369+1451) = 32{,}424$ K

(b) $\left(\frac{1369-1316}{6}\right) = 8.833$ K/hr^2

(c) $\frac{1}{24}\int_0^{24}(1245+10te^{.25\cos t})\,dt = 1364.478$ K/hr

2. (a) $g(1/2) = 0.210$ (b) $g'(0) = 0.540$

(c) $x = 0.903$ (d) $x = 2.290$ or $x = 3.676$

(e) 0.269

EXAM V Section II Part B

3. (a) $T_3(x) = \frac{1}{3}(x-2) + \frac{1}{18}(x-2)^2 + \frac{1}{81}(x-1)^3$

(b) Interval of convergence is all x for which $-1 \le x < 5$.

4. (a) $\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}$ (b) $y - 0 = -\frac{4}{5}(x-2)$

(c) vertical $x = \pm1$, or $\pm\sqrt{\frac{283}{27}}$ horizontal $x = 0$

5. (a) $x = 10$ (b) $5 < x < 12.5$

(c) critical points $(10, 60)$ and $(15, 47.5)$

inflection points $(5, 35)$ and $(12.5, 53.75)$

6. (b) $|y| = 2e^{x^2/4}$ (c) $y = 2e$

(d) $y = 3$; exact: $y(2) = 5.437$

EXAM VI Section II Part B

3. c) $-\frac{3}{8}$ d) $y = -\sqrt{x^2 - 2x + 4}$.

4. (a) $x(t) = \sqrt{t+1}$; $y(t) = 1 - \frac{1}{t+1}$ (b) $y = 1 - \frac{1}{x^2}$

(c) $\frac{3}{32}$ (d) $\frac{2}{27}$

5. (a) $\frac{dy}{dx} = \frac{-e^x - \sin x}{2y-1}$ (b) $y = 1 - x$

(c) At $(0,1)$, $\frac{d^2y}{dx^2} = -4$ (d) $y(0.5) \approx 0.5$ (e) larger

6. (a) $\frac{1}{4} - \frac{x^2}{16} + \frac{x^4}{64} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2^{2n+2}}$ (b) R = 2

(c) $\frac{1}{4}x - \frac{x^3}{48} + \frac{x^5}{320} - \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)2^{2n+2}}$

(d) Let $x = 2$