NAME:______________________________ DATE:___________

Multivariable Calculus 3 Honors Summer Review Packet
For Students who took BC Calculus in 2020 – 2021

DUE FIRST DAY OF CLASS FALL 2021!!!

Welcome to the Calculus 3 class. You should recognize this set of problems since it is one of the practice packets you got last year as you prepared for the AP Calculus BC Exam. It is important for you to rework this packet (or perhaps, work for the 1st time) so that you are refreshed on the Calculus 1 (AB) and Calculus 2 (BC) material that is imperative for your success in Calculus 3.

- Solutions to the problems contained in this packet are on the last page.
- Since you will also be CalPALS (Calculus Peer-Assisted Learners) for the AB and BC students for the 2021-2022 school year, it will be necessary for you to remember the curriculum you needed to know for that material.

- **There will be a Quiz over this material the second day of class!!**
- Finally, honor and integrity is at the heart of a Westside Wolf! Smart wolves never cheat. You are only hurting yourself by attempting to copy someone else's work. This packet is to help you be ready for Multivariable Calculus 3 Honors and to help me know what you can do.

BC

EXAM IV
CALCULUS BC
SECTION I PART A
Time–60 minutes
Number of questions–30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. The value of $\int_0^{\infty} e^{-x}dx$ is

(A) -1 (B) 0 (C) $\frac{1}{e}$ (D) 1

Ans

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2. The graph of $f(x) = x\sin x$ defined on $0 < x < \pi$ has an inflection point whenever

(A) $\tan x = -\frac{2}{x}$

(B) $\tan x = \frac{2}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

Ans

3. The area of the region in the first quadrant bounded by the curve $y = \sqrt{2x+1}$ and the line $x = 4$ is equal to

(A) 2

(B) $\frac{16}{3}$

(C) $\frac{26}{3}$

(D) $\frac{35}{3}$

Ans

4. $\lim_{x\to 3}\left[\frac{\ln\left(\frac{x-1}{2}\right)}{3-x}\right]$

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

Ans

5. The interval of convergence for the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ is

(A) $-\infty < x < \infty$

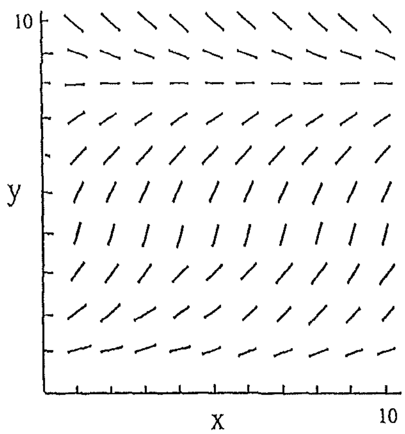
(B)) $-1 < x < 1$

(C) $-1 \le x < 1$

(D) $-1 < x \le 1$

Ans

6. A slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure at the right. Which of the following statements are true?



I. The value of $\frac{dy}{dx}$ at the point (3, 3) is approximately 1.

II. As y approaches 8 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of x.

(A) I only (B) II only (C) I and II only (D) I, II, III

Ans

7. $\lim_{h \to 0}\left(\frac{\tan(x+h) - \tan x}{h}\right) =$

(A) $\sec x$ (B) $-\sec x$ (C) $\sec^2 x$ (D) $-\sec^2 x$

Ans

8. $\int \frac{x^2 + 2x + 9}{x^2 + 9}\, dx =$

(A) $x + \frac{1}{8}\text{Arctan}\frac{x}{3} + C$

(B) $x + \frac{1}{4}\text{Arctan}\frac{x}{3} + C$

(C) $x + \frac{1}{2}\ln(x^2 + 9) + C$

(D) $x + \ln(x^2 + 9) + C$

Ans

9. If $y = \ln(\cos x)$ and $0 < x < \frac{\pi}{2}$, what is $\frac{d^2y}{dx^2}$ in terms of x ?

(A) $\tan x$

(B) $-\tan x$

(C) $\sec^2 x$

(D) $-\sec^2 x$

Ans

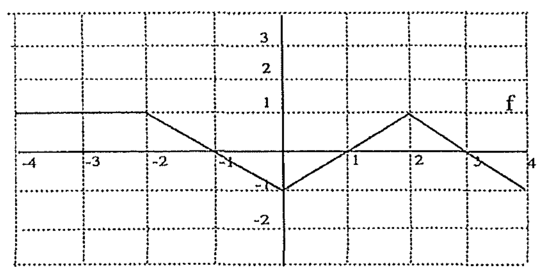
10. The graph of f is shown at the right. Which of the following statements must be true?

I. $f'(3) > f'(1)$

II. $\int_0^2 f(x)\,dx > f'(3.5)$

III. $\int_1^0 f(x)\,dx = \int_2^3 f(x)\,dx$

graph of f



A) I only B) II only (C) II and III only (D) I, II and III

Ans

11. The base of a solid is the region in the first quadrant bounded by the curve $y = \sqrt{\sin x}$ for $0 \le x \le \pi$. If each cross section of the solid perpendicular to the x-axis is a square the volume of the solid is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$

Ans

12. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)(t-6)^3$. The particle is moving to the left for

(A) $t > 3$

(B) $2 < t < 6$

(C) $3 < t < 6$

(D) $0 \leq t < 3$

Ans

13. $\int x^3 \ln x \, dx =$

(A) $\frac{x^4}{4}(4\ln x - 1) + C$

B) $\frac{x^4}{16}(4\ln x - 1) + C$

(C) $\frac{x^2}{4}(\ln x - 1) + C$

(D) $3x^2(\ln x - \frac{1}{2}) + C$

Ans

14. The slope of the tangent line to the curve $y(\cos x) + e^y = 5$ at the point where $x = \frac{\pi}{2}$ is

(A) 0 (B) 5 (C) $\frac{\ln 5}{5}$ (D) none of these

Ans

15. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{\cos y}$ with initial condition $y(1) = 0$?

(A) $y = \sin^{-1}(\frac{x^2-1}{2})$

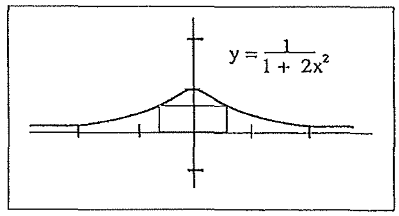
(B) $y = \sin^{-1}(\frac{x^2}{2})$

(C) $y = \ln[\cos(x-1)]$

(D) $y = \ln(\sin x)$

Ans

16. The height of the rectangle with the largest area that can be inscribed under the graph of $y = \frac{1}{1 + 2x^2}$ is



(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) none of these

Ans

17. If $f(x) = \frac{x^2+1}{e^x}$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, 3)$ (D) $(3, 4)$

Ans

18. The function f is defined by $f(x) = x^3 + 1$. If f^{-1} is the inverse function of f and $h(x) = f^{-1}(x)$, then $h'(2)$ is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 1

Ans

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19. If $f(x) = \begin{cases} x^2 & \text{for } 0 \le x \le 2 \\ 6 - x & \text{for } x > 2 \end{cases}$, then $\int_0^4 f(x)\,dx$ is

(A) $21\frac{1}{3}$

(B) $18\frac{2}{3}$

(C) 16

(D) $8\frac{2}{3}$

Ans

20. An equation of the line normal to the graph of $f(x)=\dfrac{x}{x-2}$ at $(1,-1)$ is

(A) $2x+y+1=0$

(B) $x-2y+3=0$

(C) $x+2y+1=0$

(D) $x-2y-3=0$

Ans

21. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1+t,\ t^3\rangle$. If the position vector at $t=0$ is $\langle 5, 0\rangle$, then the position of the particle at $t=2$ is

(A) $\langle 1, 12\rangle$
(B) $\langle 4, 4\rangle$
(C) $\langle 5, 9\rangle$
(D) $\langle 9, 4\rangle$

Ans

22. Let f be defined by $f(x)=x^{2/3}(2x-5)$. f is decreasing on the interval

(A) $0<x<1$ (B) $0<x<\frac{5}{8}$ C) $x>1$ (D) $-\frac{5}{2}<x<0$

Ans

23. The average value of $f(x) = e^{2x} + 1$ on the interval $0 \le x \le \frac{1}{2}$ is

(A) e B) $\frac{e}{2}$ (C) $\frac{e}{4}$ (D) $2e - 1$

Ans

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24. If f is continuous at $x = 2$, and if $f(x) = \begin{cases} \dfrac{\sqrt{x+2} - \sqrt{2x}}{x-2} & \text{for } x \ne 2 \\ k \text{ for } x = 2 \end{cases}$

then $k =$

(A) $-\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{4}$

Ans

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25. The approximate value of $y = \sqrt{x^2 + 3}$ at $x = 1.04$, obtained from the tangent to the graph at $x = 1$, is

(A) 2.01

(B) 2.02

(C) 2.03

(D) 2.04

Ans

26. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 3 (B) $\frac{7}{2}$ (C) $\frac{15}{4}$ (D) $\frac{19}{4}$

Ans

27. Which of the following are true?

I. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} b_n$ diverges.

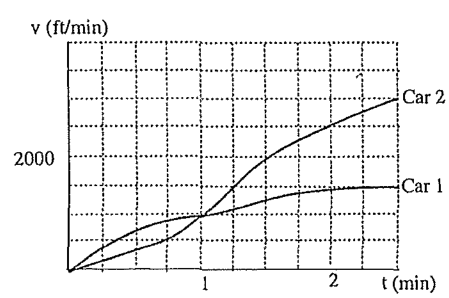
II. The sum of the geometric series $\frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \ldots$ is 2.

III. If $\sum a_k$ diverges then $\lim_{k\to\infty} a_k \neq 0$.

(A) I only (B) II only (C) III only (D) I and II only

Ans

28. Two cars start from rest at a traffic light and accelerate for several minutes. The figure at the right shows their velocities as a function of time. Which of the following statements are true?



I. Car 1 is ahead at one minute.

II. Car 2 is ahead at two minutes.

III. Car 1 and Car 2 are accelerating at the same rate at $t = 1$.

(A) I only (B) I and II only (C) II and III only (D) I, II, III

Ans

29. For which of the following series does the Ratio Test fail to give a conclusive answer.

I $\sum_{n=1}^{\infty} \frac{e^n}{n!}$ II, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ III. $\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n}}$

(A) I only (B) II only (C) II and III only (D) I, II, III

Ans

30. The average rate of change of the differentiable function f from $(3, f(3))$ to $(x, f(x))$ is given by $\frac{x^2 - x - 6}{x - 3}$. The value of $f'(3)$ is

(A) 0

(B) 1

(C) 3

(D) 5

Ans

EXAM IV
CALCULUS BC
SECTION I PART B
Time–45 minutes
Number of questions–15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distanced traveled in the first 10 seconds is

(A) 150 ft (B) 250 ft (C) 350 ft (D) 450 ft

Ans

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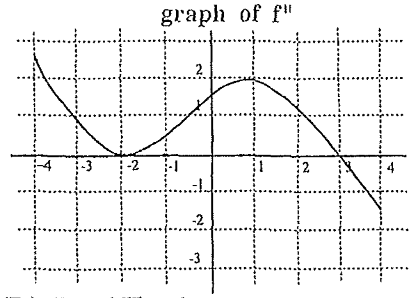
2. If $f(x) = \dfrac{\ln x^2 - x\ln x}{x-2}$ when $x \neq 2$, and f is continuous for all positive real numbers, then $f(2)$ is

(A) -1 (B) -2 (C) $-\frac{e}{4}$ (D) $-\ln 2$

Ans

3. The graph of the **second derivative** of a function f is shown below. Which of the following are true about the original function f ?

graph of f''



I. The graph of f has an inflection point at $x = -2$.

II. The graph of f is concave down on the interval $(0, 4)$.

III. If $f'(0) = 0$, then f is increasing at $x = 2$.

(A) I only (B) II only (C) III only (D) I and II only

Ans

4. For $0 \le t \le 21$, the rate of change of the number of black flies on a coastal island at time t days is modeled by $R(t) = 3\sqrt{t}\cos\left(\frac{t}{3}\right)$ flys per day. There are 500 flies on the island at time $t = 0$. To the nearest whole number, what is the maximum number of flies for $0 \le t \le 21$.

(A) 500 (B) 510 (C) 520 (D) 530

Ans

5. A function f is defined for all real numbers and has the following property: $f(a+b) - f(a) = 3a^2b + 2b^2$. $f'(x)$ is

(A) 0

(B) 1

(C) $3x^2$

(D) $3x^2 + b$

Ans

6. The position of a particle moving in the xy-plane is given by

$$x = t^2 + 2t, \qquad y = 2t^2 - 6t.$$

What is the speed of the particle when $t = 2$?

(A) $2\sqrt{10}$

(B) $4\sqrt{10}$

(C) $6\sqrt{10}$

(D) $8\sqrt{10}$

Ans

7. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\frac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinate $(1, 2)$ is equal to

(A) 1.565 (B) 2.236 (C) 3.354 (D) 6.708

Ans

8. If $F(x) = \int_0^x \frac{\sin t}{1 + \cos t}\, dt$, then $F''\left(\frac{\pi}{3}\right)$ is

(A) $-\frac{1}{2}$

(B) $4 - 2\sqrt{3}$

(C) $2 - \sqrt{3}$

(D) $\frac{2}{3}$

Ans

9. If the substitution $u = \sqrt{x+1}$ is made in $\int_0^3 \frac{1}{x\sqrt{x+1}}\,dx$, the resulting integral is

(A) $\int_1^2 \frac{1}{u^2-1}\,du$ (B) $\int_1^2 \frac{2}{u^2-1}\,du$ (C) $\int_0^3 \frac{1}{(u-1)(u+1)}\,du$

(D) $2\int_1^2 \frac{1}{u(u^2-1)}\,du$

Ans

10. The functions f and g are defined on the closed interval $[0, b]$ by $f(x) = \cos(2x)$ and $g(x) = e^x - 1$. They will have the same average value if b is

(A) 0.848 (B) 0.852 (C) 0.854 (D) 0.858

Ans

11. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$ is

(A) ∞ (B) $\frac{1}{3}$ (C) 1 (D) 3

Ans

12. Which of the following functions has a derivative at $x = 0$?

I. $y = \arcsin(x^2 - 1) - x$

II. $y = x \cdot |x|$

III. $y = \sqrt{x^4}$

(A) I only (B) II only (C) II and III only (D) I, II and III

Ans

13. Let $F(x)$ be an antiderivative of $f(x) = \frac{4\ln x}{e^{\sqrt{x}}}$. If $F(1) = 2$, then $F(3)=$

A) 2.837 (B) 3.007 (C) 3.177 (D) 3.347

Ans

14. The following table lists the known values of a function f.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoid Rule is used to approximate $\int_1^5 f(x)\,dx$ the result is

(A) 4.1 (B) 4.3 (C) 4.5 (D) 4.7

Ans

15. If the function f is defined by $f(x) = \sum_{k=0}^{\infty}\left[(1 - \sin(x))^2\right]^k$, then $f\left(\frac{\pi}{6}\right) =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$

Ans

EXAM IV
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let f be the function defined by $f(x) = e^{x/2} - \ln(x^3 + 1)$ for $x > -1$.

   (a) Find the x-coordinate of all relative maximum and minimum points. Justify your answers.

   (b) Find the intervals on which f is increasing.

   (c) Find the intervals on which the graph of f is concave down.

   (d) Find the area of the first quadrant region bounded by the graph of f and the lines $x = 0$ and $x = 1$.

2. Consider the curve C given by the parametric equations

$$x = 2 - 3\cos t \quad \text{and} \quad y = 3 + 2\sin t \ , \quad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) The curve C intersects the y-axis twice. Find the length of the curve between the two y-intercepts.

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

3. Car A has positive velocity $v(t)$ as it travels along a straight road, where v is a differentiable function of t. The velocity of the car is recorded for several selected values of t over the interval $0 \le t \le 60$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60
$v(t)$ (feet per second)	5	14	7	11	12	40	44

(a) Use the data from the table to approximate the acceleration of Car A at $t = 25$ seconds. Show the computation that lead to your answer. Indicate units of measure.

(b) Use the data from the table to approximate the distance traveled by Car A over the time interval $0 \le t \le 60$ seconds by using a midpoint Riemann sum with 3 subdivisions of equal length. Show the work that lead to your answer.

(c) Car B travels along the same road with an acceleration of $a(t) = \frac{1}{\sqrt{t+9}}$ ft / sec^2.

At time $t = 0$ seconds, the velocity of Car B is 3 ft/sec. Which car is traveling faster at $t = 40$ seconds? Show the work that lead to your answer

4. Let f be the function defined by the power series $f(x) = \sum_{n=0}^{\infty} a_n x^{2n}$ where $a_0 = 1$ and $a_n = \frac{a_{n-1}}{n}$ for $n \geq 1$.

(a) Write the first 5 terms of the series and the general term.

(b) Determine the radius of convergence for the series in part (a). Show your reasoning.

(c) Show that $f'(x) = 2x \cdot f(x)$.

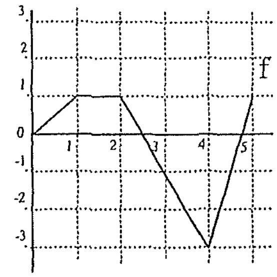
5. When the valve at the bottom of a cylindrical tank is opened, the rate at which the level of liquid in the tank drops is proportional to the square root of the depth of the liquid. Thus, if $y(t)$ is the liquid's depth at time t minutes after the valve is opened, water drains from the tank according to the differential equation $\frac{dy}{dt} = -k\sqrt{y}$ for some positive constant k that depends on the size of the drain.

   (a) Find a general solution for the differential equation.

   (b) Suppose that $y(0) = 9$ feet and $y(20) = 4$. Find an equation for $y(t)$.

   (c) At what time is the water level dropping at a rate of 0.1 feet per minute?

6. Let f be a function defined the closed interval $[0, 5]$ with zeros at $x = 0$, $5/2$ and $19/4$. The graph of f is shown at the right.



Consider the function G defined by $G(x) = \int_0^x f(t)\,dt$.

(a) Find $G(3)$.

(b) On what intervals is the graph of G increasing and concave down? Show your reasoning.

(c) Show that G has exactly one zero between $x = 3$ and $x = 4$.

(d) Find an equation of the tangent line to the graph G at the point where $x = 3$.

(e) Sketch a graph of the function G over its domain.

EXAM IV

Part A				Part B	
1.	D	18.	B	1.	D
2.	B	19.	D	2.	D
3.	C	20.	D	3.	C
4.	B	21.	D	4.	C
5.	D	22.	A	5.	C
6.	C	23.	A	6.	A
7.	C	24.	B	7.	C
8.	D	25.	B	8.	D
9.	D	26.	D	9.	B
10.	C	27.	D	10.	C
11.	C	28.	B	11.	D
12.	D	29.	C	12.	D
13.	B	30.	D	13.	C
14.	C			14.	C
15.	A			15.	B
16.	B				
17.	C				

EXAM IV Section II Part A

1. (a) rel min at $x = -0.364$ and $x = 1.977$
   rel max at $x = 0.487$
   (b) $-0.364 < x < 0.487$ and $x > 1.977$
   (c) $0.043 < x < 1.092$ (d) 1.097 units^2

2. (a) $\frac{2}{3}\cot t$ (b) $2x - 3y + 5 + 6\sqrt{2} = 0$
   (c) 3.757

EXAM IV Section II Part B

3. (a) 0.4 ft / sec^2
   (b) 1300 feet
   (c) Car A is traveling faster

4. (a) $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{n!}$
   (b) converges for all x (radius is infinite)

5. (a) $y = \left(C - \frac{kt}{2}\right)^2$ (b) $y(t) = \left(3 - \frac{t}{20}\right)^2$
   (c) 40 min

6. (a) 1.5 (b) $2 < x < 2.5$
   (c) $G(3) > 0$ and $G(4) < 0$ so by the Intermediate Value Theorem, G has a zero in $[3,4]$. $G'(x) < 0$ on $[3,4]$, so G is monotone, decreasing and has only one zero in $[3,4]$.
   (d) $y - 1.5 = -1(x - 3)$