Mathcounts
Things to Know

1. **Primes less than 200**

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

2. **Perfect squares less than 1,000**

$0^2 = 0$	$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$	$25^2 = 625$	$30^2 = 900$
$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$	$21^2 = 441$	$26^2 = 676$	$31^2 = 961$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$	$22^2 = 484$	$27^2 = 729$	
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$	$23^2 = 529$	$28^2 = 784$	
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$	$24^2 = 576$	$29^2 = 841$	

3. **Powers of natural numbers**

$2^0 = 1$	$3^0 = 1$	$4^0 = 1$	$5^0 = 1$	$6^0 = 1$	$7^0 = 1$	$8^0 = 1$	$9^0 = 1$
$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$	$6^1 = 6$	$7^1 = 7$	$8^1 = 8$	$9^1 = 9$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$	$6^4 = 1296$	$7^4 = 2401$		
$2^5 = 32$	$3^5 = 243$	$4^5 = 1024$					
$2^6 = 64$	$3^6 = 729$						
$2^7 = 128$							
$2^8 = 256$							
$2^9 = 512$							
$2^{10} = 1024$							

4. **Fractions and Decimal Equivalents**

   $\frac{1}{2} = .5 \qquad \frac{1}{6} = .1\overline{6} \qquad \frac{1}{11} = .\overline{09}$

   $\frac{1}{3} = .\overline{3} \qquad \frac{5}{6} = .8\overline{3} \qquad \frac{2}{11} = .\overline{18}$

   $\frac{2}{3} = .\overline{6} \qquad \frac{1}{7} = .\overline{142857}$

   $\frac{1}{4} = .25 \qquad \frac{1}{8} = .125$

   $\frac{3}{4} = .75 \qquad \frac{3}{8} = .375$

   $\frac{1}{5} = .2 \qquad \frac{5}{8} = .625$

   $\frac{2}{5} = .4 \qquad \frac{7}{8} = .875$

   $\frac{3}{5} = .6 \qquad \frac{1}{9} = .\overline{1}$

   $\frac{4}{5} = .8 \qquad \frac{2}{9} = .\overline{2}$

5. **Factorials**

   0!=1
   1!=1
   2!=2
   3!=6
   4!=24
   5!=120
   6!=720
   7!=5040
   8!=40,320
   9!=362,880
   10!=3,628,800

6. **Combinatorics Formulas**
$_nC_r = \frac{n!}{r!(n-r)!}$ (when order does not matter)
$_nP_r = \frac{n!}{(n-r)!}$ (when order does matter)

7. **Divisibility rules**
$2 \rightarrow$ iff units digit is even
$3 \rightarrow$ iff sum of digits is divisible by 3
$4 \rightarrow$ iff last two digits form a number divisible by 4
$5 \rightarrow$ iff units digit is 0 or 5
$6 \rightarrow$ iff number is divisible by both 2 and 3
$7 \rightarrow$ iff result of subtracting twice the last digit from the number remaining when the last digit is removed is divisible by 7
$8 \rightarrow$ iff last three digits of number form a number divisible by 8
$9 \rightarrow$ iff sum of digits is divisible by 9
$10 \rightarrow$ iff units digit is 0
$11 \rightarrow$ iff result of alternately adding and subtracting the digits is divisible by 11.
$12 \rightarrow$ iff number is divisible by both 4 and 3.

8. **Special Factorizations**
$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$
$a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$
$a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$
$a^2 - b^2 = (a+b)(a-b)$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

9. **Quadratic Formula**
If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

10. **Units Digits Patterns** $(a^1, a^2, a^3..)$
$1 \rightarrow 1$ $6 \rightarrow 6$
$2 \rightarrow 2, 4, 8, 6$ $7 \rightarrow 7, 9, 3, 1$
$3 \rightarrow 3, 9, 7, 1$ $8 \rightarrow 8, 4, 2, 6$
$4 \rightarrow 4, 6$ $9 \rightarrow 9, 1$
$5 \rightarrow 5$

11. **Geometry**

General
Things that may be assumed from a geometric figure:

1. Lines that appear to be straight are straight.
2. Points of intersection are depicted accurately.
3. Points shown on a line are collinear.
4. All the points in the figure are coplanar unless specified otherwise.
5. Relative positions of points are accurate.

Some that may <u>not</u> be assumed from a given geometric figure:

1. Exact measurement or relative size of figures.
2. Congruence or similarity.
3. Parallel or perpendicular lines.

Miscellaneous

- Number of diagonals in an n-gon $\rightarrow \frac{n(n-3)}{2}$

- Sum of measures of internal angles in an n-gon $\rightarrow 180°(n-2)$

- Sum of measures of external angles in an n-gon $\rightarrow 360°$
- If the ratio of the same linear dimension of two similar two-dimensional figures is k, then the ratio of their areas is k^2
- If the ratio of the same linear dimension of two similar three-dimensional figures is k, then the ratio of their volumes is k^3

Circle

- Area=πr^2, Circumference=$2\pi r$, where r is the radius

Triangle

- Area=$\frac{1}{2}bh$, where b is a side (the base), and h is the length of the altitude drawn to that side (the height)
- Area= $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the side lengths and s is the semiperimeter (Heron's Formula)
- Area= $\frac{1}{2}ab\sin C$, where a, b are two sides and C is the angle formed by those sides
- Area= rs, where r is the inradius of the triangle and s is the semiperimeter
- Area= $\frac{s^2\sqrt{3}}{4}$, where s is the length of a side of an equilateral triangle

Quadrilaterals

- Parallelogram
    - Area= bh, where b is the base and h is the height of the parallelogram
- Rectangle
    - Area=lw, Perimeter=$2(l+w)$, where l and w are the length and width of the rectangle
- Square
    - Area=s^2, where s is the side length
    - Area= $d^2/2$, where d is the diagonal
- Rhombus
    - Area=$\frac{1}{2}d_1 d_2$, where d_1 and d_2 are the two diagonals of the rhombus

Prism

- Volume=Bh, where B is the area of the base and h is the height
- Surface area= $2B + Ph$, where B is the area of the base, P is the perimeter of the base, and h is the height of the prism

Pyramid

- Volume=$\frac{1}{3}Bh$, where B is the area of the base and h is the height

Rectangular Prism

- Volume=lwh
- Surface area= $2(lw + wh + lh)$
- Length of space diagonal= $\sqrt{l^2 + w^2 + h^2}$, where l, w, and h are the lengths of the edges of the prism

Cube

- Volume = e^3
- Surface Area = $6e^2$
- Length of Space Diagonal: $e\sqrt{3}$, where e is the edge length

Cylinder

- Volume= $\pi r^2 h$
- Surface Area= $2\pi rh + 2\pi r^2$, where r is the radius and h is the height

Cone

- Volume = $\frac{1}{3}\pi r^2 h$
- Surface Area = $\pi r^2 + \pi r\sqrt{r^2 + h^2}$

Sphere

- Volume = $\frac{4}{3}\pi r^3$
- Surface Area= $4\pi r^2$, where r is the radius

12. **Triangles**

Triangle Inequality
$a + b < c$, where a and b are the shorter sides and c is the largest side

Pythagorean Theorem
$a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse of a right triangle

Law of Cosines
$c^2 = a^2 + b^2 - 2ab\cos C$, where a, b, and c are the sides of a triangle, and C is the measure of the angle formed by sides a and b

Congruency and Similarity Theorems

- Congruency
  For any triangles: SSS, SAS, ASA, AAS
  For right triangles: HL, LL, SA
  Similarity
  For any triangles: AA, SAS, SSS

Pythagorean Triples

- 3, 4, 5
- 5, 12, 13
- 8, 15, 17
- 7, 24, 25
- 12, 35, 37
- 9, 40, 41
- ka, kb, kc, where (a, b, c) is a Pythagorean triple and k is any positive real number

13. Trigonometry

Basic trig function mnemonic device: "SOHCAHTOA"

Definitions of trigonometric functions

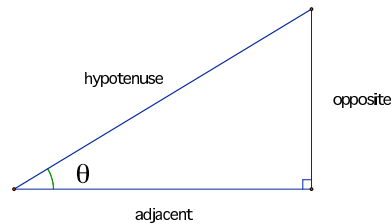$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan\theta}$$

Values of trigonometric functions for common angle measures

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

14. Some Approximations

- $\pi \approx 3.14159$
- $\sqrt{2} \approx 1.414$
- $\sqrt{3} \approx 1.732$
- $\sqrt{5} \approx 2.236$

15. Arithmetic Sequences

- $a_1 + a_2 + \cdots + a_n = \frac{n(a_1 + a_n)}{2}$
- $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
- $1 + 3 + 5 + \cdots + (2n + 1) = n^2$
- $2 + 4 + 6 + 8 + \cdots + (2n) = n(n + 1)$

16. Conversion Factors

Length and Area
12 in. = 1 ft.
3 ft. = 1 yd.
5280 ft. = 1 mi.
1760 yd. = 1 mi.
2.54 cm = 1 in.
640 acres = 1 mi.^{2}
Weight
16 oz. = 1 lb.

Volume
3 ts. = 1 tbsp.
2 tbsp. = 1 fl. oz.
8 fl. oz = 1 cup
2 cups = 1 pt.
2 pt. = 1 qt.
4 qt. = 1 gal.

1 mL = 1 cm^3

Metric Prefixes
kilo-	10^3
hecto-	10^2
deca-	10^1
— -	10^0
deci-	10^{-1}
centi-	10^{-2}
milli-	10^{-3}