Coordinate Geometry and Proof

Objective:
Students will prove theorems using coordinate representations of geometric figures.

1. Given: Quadrilateral $ABCD$, with vertices $A (0, 0), B (2, 4), C (7, 4), D (5, 0)$, prove: $ABCD$ is a parallelogram.

2. The given parallelogram $PQRS$ has vertices $P (0, 0), Q (a, 0), R (?, ?)$ and $S (b, c)$.
   a) Determine the coordinates of $R$.
   b) Find the midpoint of $QS$.
   c) Find the midpoint of $PR$.
   d) What do you notice?
   e) Show that if the diagonals are equal in length then the parallelogram is a rectangle.

3. Prove the diagonals of a rhombus are perpendicular.
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1. Prove the diagonals of a rectangle are equal.

2. Prove the diagonals of an isosceles trapezoid are equal.

3. Show that the triangle with vertices (0, 0), (6, 4), and (4, 7) is a right triangle.

4. Prove that the diagonals of a parallelogram bisect each other.

5. Show that the segments connecting consecutive midpoints of the sides of a quadrilateral form a parallelogram.

6. In the diagram below, suppose that $M$ and $N$ are midpoints of $PQ$ and $RQ$, respectively, and $PN = RM$.

   a) Find the coordinates of $M$ and $N$.

   b) Using the distance formula, write and simplify an equation expressing the fact that $PN = RM$.

   c) Use the equation you found in part (b) to express $b$ in terms of $a$ alone. What does this tell you about $\triangle PQR$?