Logical Reasoning ("if-then" Statements)

I. Determine if the following conditional is true or false, provide a counterexample if the statement is false. All points are in the Euclidean Plane.

1. If $AM = MB$, then $M$ is the midpoint of $AB$.

2. If $AB = 2PB$, then $P$ is the midpoint of $AB$.

3. If two angles are supplementary then they are equal.

4. If a figure is a square, then it is a quadrilateral.

5. If a quadrilateral has four right angles then it is a square.

6. If a quadrilateral is a parallelogram then it is a rectangle.

7. In a plane, if two lines are perpendicular to the same line, then they are parallel.

8. If a quadrilateral is a square, then it is a rectangle.

II. Write the converse, inverse and contra-positive conditionals for the true problems above. State whether each conditional is true or false and provide a counterexample when the statement is false.

III. Notice in the conditionals in part 2 the contrapositive is true. In general a contrapositive is true if the original if-then statement is true and is false if the original if-then statement is false. Choose one of the false if-then statements above and write the contrapositive. Give a counterexample to confirm the contrapositive is false.