

Writing Equations Using Sequences

Sequences can be described using a formula. If the position of the term in the sequence is represented by n , if $n = 1, 2, 3, 4, \dots$ the n th term in the sequence 2, 4, 6, 8, ... may be described as $2n$. Other sequences are more difficult to recognize. If $n = 1, 2, 3, 4, \dots$ the n th term in the sequence 3, 5, 7, 9, ... may be described as $2n + 1$. We needed to add one to the first formula because each term in this 2nd sequence is one more than the corresponding term in the 1st sequence.

Given any sequence, how can an equation be written to describe the n th term?

In the examples above, find the difference between each term. Notice that the common difference of the terms is 2. It is not a coincidence that the coefficient of n is 2 (the common difference). Find the common difference in the sequences below.

3, 6, 9, 12, 15

5, 9, 13, 17, 21

9, 7, 5, 3, 1, -1

-3, 3, 9, 15, 21

The common difference is the multiplier of n .

To find out what should be added or subtracted, you can find the zero term.

We are now going to develop a formula. First, let us define our variables:

T = the n th term of the sequence d = the common difference
 n = position in the sequence z = the zero term

The equation will be $T = dn + z$. How does this formula compare to $y = mx + b$?

Sequence	Difference of terms	Difference multiplied by position n	What is the zero term?*	Equation
3, 6, 9, 12, 15	3	$3n$	$3 - 3 = 0$	$T = 3n + 0$
5, 9, 13, 17, 21	4	$4n$	$5 - 4 = 1$	$T = 4n + 1$
9, 7, 5, 3, 1, -1	-2	$-2n$	$9 - -2 = 11$	$T = -2n + 11$
-3, 3, 9, 15, 21	6	$6n$	$-3 - 6 = -9$	$T = 6n - 9$

* subtract the difference from the first term

1. Write an equation to describe the sequences below.
 - a) 9, 8, 7, 6, 5, ...
 - b) 13, 17, 21, 25, 29, ...
 - c) 6, 11, 16, 21, 26, ...
 - d) 112, 100, 88, 76 ...

2. Describe two different methods to find the 100th term of the sequences in problem 1 b). Which will be easier. Calculate this 100th term.

3. Given the sequence 12, ____, 20, ____ 28
 - a) Find the missing terms of the sequence.
 - b) Write an equation for the n th term T .
 - c) What is the 20th term?

4. Given the sequence, 21, 20, 22, 19, 23, 18, 24, 17, 25, 16, 26, ...
 - a) What comes next in the sequence?
 - b) How can you write an equation for this sequence?
 - c) What is the 75th term?
 - d) What is the 50th term?

5. Fill in the table below:

n	$n^2 + 1$	1st difference of terms	2 nd difference of terms
1	2	-	-
2	5	3	-
3	10	5	2
4			
5			
6			
7			

6. Make a table like the one above for $3n^2 - 2$.

n	$3n^2 - 2$	1 st difference	2 nd difference

7. Make a table like the one above for x^3 . (Use the table feature on your calculator.) Describe the pattern that you observe.

n	x^3	1 st difference	2 nd difference

8. The sequence $-1, 2, 5, 8, 11 \dots$ can be described by the equation $T = 3n - 4$.

- Plot the points $(1, -1)$ $(2, 2)$ $(3, 5)$ $(4, 8)$ $(5, 11)$.
- Plot the line on the same graph.
- How do these points relate to the sequence and the line?

