Unit Overview
In this unit you will study angle relationships in parallel lines and polygons. You will also study similar figures, proportions, and unit conversions.

Academic Vocabulary
Add these words to your vocabulary notebook.
- angle
- similar figures
- transformation (geometric)

Essential Questions
- How is proportional reasoning used to solve real-world problems?
- What are transformations and how are they useful in solving real-world problems?

Embedded Assessments
This unit has 2 Embedded Assessments. The first embedded assessment allows you to demonstrate your understanding of angle relationships in parallel lines and polygons and similar figures. In the second, you will demonstrate understanding of proportions and unit conversions.

Embedded Assessment 1
Lines, Angles, Transformations
p. 211

Embedded Assessment 2
Proportions, Similarity, and Conversions
p. 241
Write your answers on notebook paper. Show your work.

1. Use a compass to measure the following angles.

2. Give the coordinates of points A, B, C, and D on the graph below.

3. Solve the following equations.
   a. \( 4x + 7 = 10 \)
   b. \( 2x + 4.1 = 5.7 \)
   c. \( 3x + \frac{1}{2} = 2x + \frac{5}{8} \)

4. Write and solve an equation that represents the statement “four more than three times a number is twenty-two.”

5. Name the equivalent ratios in each set of numbers. Justify your answer.
   a. \( \frac{2}{5}, \frac{3}{6}, \frac{4}{9}, \frac{6}{15} \)
   b. \( \frac{1.2}{7}, \frac{2.4}{8.4}, \frac{3.6}{21} \)
   c. 2:3, \( \frac{4}{6}, \) 6 to 9

6. Give the characteristics of each of the following triangles and angles.
   a. scalene triangle
   b. isosceles triangle
   c. equilateral triangle
   d. acute angle
   e. obtuse angle
   f. complementary angles
   g. supplementary angles

7. Tell which of the following graphs are about 33% shaded, which are about 50% shaded, and which are about 75% shaded. Explain how you made your decisions.
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

8. Explain how to convert \( 5\frac{1}{2} \) feet to inches.
Bob Toose, football coach and geometry teacher at Johnny Unitas High School, names his football plays after different geometric terms. He knows that the players from other schools will not know what is coming at them with these names.

He gives playbook quizzes to make sure his players know their plays. A portion of one of his quizzes follows. For this quiz, the scrimmage line is shown at the 20 yard line.

1. Match the play with the mathematical term that best describes it.

Angle

Perpendicular Lines

Parallel Lines

Right Angle
Coach Toose is very particular about the routes that his players run. He told his receiver that this “corner” route needed to be run at a 50° angle to the sideline of the end zone.

2. What is the measure of angle \( b \) in the diagram above?

3. What is the relationship between these two angles called? Explain your choice.

4. Coach Toose wanted his players to run other corner routes as well. Identify the angle complementary to the one listed. Then draw and label each route.
   \[ \text{a. } 20° \quad \text{b. } 73° \]

Another route that Coach Toose has his players run is a “post” route. The route can be used to show supplementary angles.

5. Tell what it means for angles to be supplementary and sketch an example below.
ACTIVITY 4.1 continued

Angle Pair Relationships

The Winning Angle

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Visualization, Create Representations, Think/Pair/Share

6. This “post” route is seen below as it passes over the goal line. Give the measure of angle $d$.

![Diagram of a football field with angle $d$ marked]

7. Coach Toose’s team runs a variety of “post” routes. Draw and label the routes below and identify the angle supplementary to the one listed.

| a. 20° | b. 153° |

TRY THESE A

Give the measure of the complementary and supplementary angles of each:

| a. 57° | b. 93° |

Are these angles complementary? Explain why or why not.

| c. 47° and 53° | d. 12° and 78° |

Are these angles supplementary? Explain why or why not.

| e. 37° and 143° | f. 118° and 52° |
The coach uses a diagram like the one below to show plays to his team. Your teacher will give you tape to recreate these same play lines on your desk or on a piece of paper.

Now using the tape, add a “slant” route to your diagram and label the angles as seen below.

8. What mathematical term do you think Coach Toose uses for this play?

9. Measure angle \( j \) on your diagram.

10. Without measuring, predict which other angles have the same measure as angle \( j \) and list them below.

11. Now measure these angles. Were your predictions correct?

12. What term is used to identify angles that have the same measure?
13. What is true about the measures of the remaining angles in your diagram?

14. Using the diagram that you made on your desk and your observations in the previous questions, what can you say about the angles formed by two parallel lines cut by a transversal?

### TRY THESE B

**a.** Find the missing angle measures on the diagram below.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠RHC</td>
<td>125°</td>
</tr>
<tr>
<td>∠SKU</td>
<td></td>
</tr>
<tr>
<td>∠HKO</td>
<td></td>
</tr>
<tr>
<td>∠JHK</td>
<td></td>
</tr>
<tr>
<td>∠RHJ</td>
<td></td>
</tr>
<tr>
<td>∠CHK</td>
<td></td>
</tr>
<tr>
<td>∠OKU</td>
<td></td>
</tr>
<tr>
<td>∠SKH</td>
<td></td>
</tr>
</tbody>
</table>

15. What does the term exterior mean in everyday language? Give at least two examples.

16. Which angles in the figure above do you think are exterior angles? Explain.
17. What does the term interior mean in everyday language? Give at least two examples.

18. Which angles in the figure on the left do you think are interior angles? Explain.

19. There are two pairs of angles in the diagram on the left that are referred to as alternate exterior angles and two pairs of angles that are referred to as alternate interior angles.

   a. Explain what it means for angles to be alternate exterior angles.

   b. Name the two pairs of alternate exterior angles and tell what you notice about the measure of these angles.

   c. Explain what it means for angles to be alternate interior angles.

   d. Name the two pairs of alternate interior angles and tell what you notice about the measures of these angles.
20. Another type of angle found in parallel lines cut by a transversal are **corresponding angles**. What do you think is meant by the term corresponding?

21. Name the pairs of corresponding angles in the diagram above and tell what you notice about the measures of these angles.

22. Two pairs of **vertical angles** are formed when two lines intersect. List the pairs of vertical angles in the diagram above and tell what you notice about the measures of these angles.

23. Using what you now know about alternate exterior, alternate interior, corresponding and vertical angles, find the missing measures in the diagram below without using a protractor.
24. Choose an activity you are interested in and find at least three ways in which the relationship between angles and lines are related to this activity.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Give a real-life example of parallel lines.
2. Give a real-life example of perpendicular lines.
3. What must be true for an angle to be a right angle?
4. Are angles with measures of 11° and 89° complementary? Why or why not?
5. Can two obtuse angles be supplementary? Explain why or why not.
6. What is the measure of an angle supplementary to an angle that measures 101°?
7. What is the measure of an angle complementary to an angle that measures 105°?
8. Give the missing angle measures in the diagram below.

9. **MATHEMATICAL REFLECTION** Give at least five examples of how math and your daily routine are related. Explain the math concept that is related to each activity and what the relationship is.
Chip designs games for his computer. One of his current projects is called *Down the Chute*. In the game, triangles appear at the top of a long U-shaped chute with parallel sides and slowly descend to rest at the bottom. The object of the game is to completely fill the region between the parallel sides of the chute.

Before each triangle appears, the player must provide the measure (in degrees) of one angle in the triangle. As the triangle descends down the chute, the player is allowed to transform the triangle with the following commands:

- reflect the triangle vertically
- reflect the triangle horizontally
- rotate the triangle $x$ degrees clockwise

**Once the triangle comes to rest**

1. Use the triangular pieces given to you by your teacher to fill in the rectangular chute you have. Is there more than one way to fill the chute? Explain.
2. If a triangle is not changed as it slides down the chute, this move (or transformation) is called a translation. A translation changes only a figure’s position. Translate the triangle below two units down and five units to the right.

3. To perform a reflection, each point of a pre-image is copied on the opposite side of a given line and remains equidistant from the line. Sketch the reflection, called the image, of the triangle over the line $y = 4$.

4. List the coordinates of the vertices of the image in Item 3.
5. If a pre-image is reflected over a horizontal line, the transformation is a **vertical reflection**. If a pre-image is reflected over a vertical line, the transformation is a **horizontal reflection**. Label each transformation below as a vertical or horizontal reflection.

6. To perform a **rotation**, each point of the pre-image travels along a circle the same number of degrees. The direction of this path can be either clockwise or counter clockwise. Label each of the following rotations as 90° clockwise, 90° counterclockwise, 180°, or not a rotation.

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**CONNECT TO AP**

Translations and reflections of figures in the coordinate plane are preparing you to successfully translate and reflect functions. This is a helpful tool for visualizing and setting up the graphs for many problems you will solve in calculus.
7. Draw a sketch of the image of the triangle to the right when it is:

a. rotated 180° clockwise around point A

b. rotated 90° counterclockwise around point A

c. rotated 90° clockwise around point A

d. rotated 45° counterclockwise around point A

e. rotated 45° clockwise around point A

8. How many and what types of reflections would have to be performed on a pre-image to get the same image as a 180° rotation?

9. The grid at the left shows triangle $ABC$ and a 90° clockwise rotation of $ABC$ about the origin. Sketch the 180° rotation of $ABC$ and the 90° counterclockwise rotation of $ABC$. How do the images compare with triangle $ABC$?
10. What transformation(s) would you need to perform on each of the following right triangles so that you would end up with a triangle that satisfies both of the following?
   - Both points A and B are at the bottom of the figure.
   - Point A is horizontally to the right of point B.

   a. 
   b. 
   c. 
   d. 

11. How do the sides of the image of a polygon after a translation, reflection, or rotation compare with the corresponding sides of the original figure? How do you know?

12. In one game, Chip’s first triangle with a 90° angle came to rest and displayed the measure of \( \angle CAB \) to be 32°.
   a. Determine the measure of \( \angle CAD \).
   b. Explain why the measure of \( \angle CAD \) is equal to the measure of \( \angle ACB \).
13. When \( \triangle ACD \) came down the chute, Chip selected the 58° and the computer selected the 43° angle. Find the measure of each of the following angles.

a. \( \angle ECD \)

b. \( \angle CDA \)

c. \( \angle FDC \)

14. List the measures of the three angles in \( \triangle ACD \) and list the measures of the three non-overlapping angles whose vertex is at \( C \). How do the two lists compare?

15. Find the measure of each of the following angles.

a. \( \angle FCE \)

b. \( \angle CFD \)

c. \( \angle CEF \)

d. \( \angle EFG \)

e. \( \angle FGE \)
16. Every triangle has three sides and three angles. Use your responses to Items 12, 13 & 15 to complete the following table. For each triangle, list the angle measures and find the sum of the measures of the three angles.

<table>
<thead>
<tr>
<th>Triangle Name</th>
<th>Angle Measures</th>
<th>Sum of the Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ABC</td>
<td>90°, 32°, 58°</td>
<td></td>
</tr>
<tr>
<td>△ACD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>△DCF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>△ACF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>△CEF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>△GEF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

17. Write a statement that appears to be true about the sum of the measures of the angles of a triangle.

18. Determine the measure of the unknown angle in the triangle.
19. The measures of three angles in a triangle are 44°, 3x°, and 5x°. Write an equation and solve for x. Determine the missing angles.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Let \( \triangle MOV \) have vertices: \( M(-2,-2) \), \( O(4,-2) \), and \( V(4,3) \). Determine the coordinates of the vertices for each image of \( \triangle MOV \) after each of the following transformations are performed.
   a. \( \triangle MOV \) is translated 3 units up and 2 units to the right.
   b. \( \triangle MOV \) is reflected over the line \( y = 3 \).
   c. \( \triangle MOV \) is reflected over the line \( x = 4 \).

2. Which transformation(s) have been performed on the pre-image to obtain the image?

   \[ \begin{array}{c}
   \text{Pre-image} \\
   \begin{array}{c}
   \text{Image} \\
   \end{array}
   \end{array} \]

   a. Rotate 180°.
   b. Shift down two units and reflect over the line \( y = 2 \).
   c. Reflect over \( x \)-axis and shift up 4 units.
   d. Reflect over \( y \)-axis and shift up 4 units.

3. If one of the acute angles of a right triangle has a measure of 22°, calculate the measure of the other acute angle.

4. Suppose the measures of the angles in a triangle are \((2x + 20)°, 3x°, \) and \(5x°\). Write an equation, solve for \( x \), and determine the measure of each angle.

5. In \( \triangle FAR \angle F \cong \angle A \) and \( m \angle R = 68° \). Calculate \( m \angle F \). Show your work.

6. **Mathematical Reflection** Does the order in which multiple transformations are performed on a pre-image have an effect on the image?
Skye Gaiser loves stars, and she decorates everything with them. She has become interested in the star patterns used in quilting and knows they are very precisely created and also that the angle measure is very important.

Skye is particularly interested in stars that are inscribed inside regular polygons. A regular polygon is a polygon with all sides congruent and all angles congruent.

1. Write the name of each of the following polygons and tell which appear to be regular polygons.
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

2. An equilateral triangle is an example of a regular polygon.
   a. Use the definition of a regular polygon to show how to find the measure of each angle of an equilateral triangle.
   b. If a triangle is equilateral it is also equiangular. Explain what equiangular means.

3. A square is also a regular polygon.
   a. Write a statement about the four sides and four angles of a square.
   b. Tell the measure of each angle in a square.
4. To draw a four-pointed star, Skye begins by drawing a square and then sketches a second, smaller square that has been rotated $45^\circ$ inside the first square and has the same center as the larger square.

a. Complete the 4-pointed star by drawing the segment that joins each vertex of the large square to the nearest two vertices of the smaller square. The first two of these segments have already been drawn. All eight segments will be congruent and they will form two sets of congruent triangles.

b. Which triangles formed by the four-pointed star are congruent?

c. What is the best name for the triangles?

5. If the measure of $\angle QPW$ is $27.5^\circ$ and quadrilaterals $PQRS$ and $WXYZ$ are squares, then find the measure of each of the following angles.

a. $\angle PWQ$

b. $\angle SPZ$

c. $\angle WPZ$

d. $\angle PWZ$
6. Pentagon $ABCDE$ is a regular polygon with diagonal $AC$.

   a. Explain how you know from the definition of regular polygon that $\triangle ABC$ is isosceles.

   b. Name the congruent angles in $\triangle ABC$.

   c. If the measure of $\angle B$ is $108^\circ$, determine the measures of the other two angles in $\triangle ABC$.

7. To complete the 5-pointed star, draw the remaining diagonals in the pentagon.

8. Use the regular pentagon and completed star above to determine the measure for each of the following angles.

   a. $\angle BAE$
   b. $\angle DAE$
   c. $\angle CAE$
   d. $\angle CAD$

9. What kind of triangle is $\triangle BAE$? Explain how you made your decision.
10. Hexagon \( PQRSTU \) is a regular polygon.

a. List the diagonals that can be drawn containing vertex \( P \).

b. Are any of these diagonals congruent? Explain how you made your decision.

11. When a regular hexagon is inscribed in a circle, the circle and the hexagon have the same center. The longest diagonal drawn from a vertex passes through the center of the hexagon.

a. Draw the three longest diagonals in the hexagon below.

b. How can you tell \( PO, QO, RO, SO, TO, \) and \( UO \) are congruent?

c. When the diagonals \( PS, QT, \) and \( RU \) are drawn, 6 congruent triangles are formed. Are these triangles isosceles, equilateral, or scalene? Explain your reasoning.
12. Tell the measure of the following angles and explain how you determined each measure.

   a. \( \angle POU \)

   b. \( \angle OQP \) and \( \angle OQR \)

   c. \( \angle PQR \)

13. In a regular hexagon the 6 diagonals that do not pass through the center of the hexagon are congruent.

   a. Complete the 6-pointed star by drawing all diagonals that are congruent to \( QS \).

   b. What type of triangles are \( \triangle SUQ \) and \( \triangle TPR \)? Explain your reasoning.

   c. Tell the measure of each angle of \( \triangle SUQ \) and \( \triangle TPR \).

   d. What kind of triangles are \( \triangle TUP \) and \( \triangle PQR \)? Explain your reasoning.
14. In hexagon $PQRSTU$, the diagonals intersect at 6 points:
   - $A$ is the point where $\overline{TP}$ and $\overline{US}$ intersect;
   - $B$ is the point where $\overline{TP}$ and $\overline{UQ}$ intersect;
   - $C$ is the point where $\overline{PR}$ and $\overline{UQ}$ intersect;
   - $D$ is the point where $\overline{PR}$ and $\overline{QS}$ intersect;
   - $E$ is the point where $\overline{RT}$ and $\overline{QS}$ intersect;
   - $F$ is the point where $\overline{RT}$ and $\overline{US}$ intersect.

   a. Label the points of intersection.

   b. What type of polygon is $ABCDEF$?

15. Review your work in this activity and complete the table below as you review.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Sides</th>
<th>Measure of Each Interior Angle</th>
<th>Sum of the Measures of the Interior Angles</th>
<th>Sum as a Factor Times $180^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td></td>
<td></td>
<td></td>
<td>$1 \cdot 180^\circ$</td>
</tr>
<tr>
<td>Regular pentagon</td>
<td>4</td>
<td>$360^\circ$</td>
<td></td>
<td>$2 \cdot 180^\circ$</td>
</tr>
<tr>
<td>Regular hexagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. What patterns can you find in the above table?
Angles in Polygons
A Star Is Drawn

SUGGESTED LEARNING STRATEGIES:

17. Use your patterns to complete the next two rows of the table.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Sides</th>
<th>Measure of Each Interior Angle</th>
<th>Sum of the Measures of the Interior Angles</th>
<th>Sum as a Factor Times 180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular Heptagon</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular Octagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

18. Use a protractor and straight edge to draw a regular octagon in the space below. Decide which diagonals should be used to draw an 8-pointed star and draw the star.

19. Make a list of the steps that would be needed to calculate the measure of one interior angle of a regular nine-sided polygon.

When you extend the side of a polygon, an exterior angle of the polygon is formed. In the figure to the right, triangle $PQR$ has exterior angle $PQS$.

20. What is the measure of angle $PQS$? How do you know?

21. How is the measure of angle $PQS$ related to the measures of angles $P$ and $R$?
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper.
Show your work.

1. True or False? Explain your choice.
   a. As the number of sides of a regular polygon increases, the measure of the interior angles decreases.
   b. If a regular polygon has more than four sides, at least one of the diagonals is parallel to one of its sides.
   c. The sum of the measures of the interior angles of a polygon is always a multiple of 180°.
   d. As the perimeter of a regular hexagon increases, the measure of each interior angle increases.
   e. If a regular polygon has more than three sides, it has at least one pair of congruent diagonals.

2. Given the regular pentagon PENTA, determine the measure of each of the following angles.
   a. \( \angle PAT \)
   b. \( \angle PAG \)
   c. \( \angle APG \)
   d. \( \angle EPN \)
   e. \( \angle EPA \)
   f. \( \angle EGO \)
   g. \( \angle EGP \)
   h. \( \angle PEG \)

3. Given Quad ABCD and Quad WXYZ (not shown) are squares with isosceles triangles AZD, ABW, BCX, and CYD. Determine the measure of each of the following angles.
   a. \( \angle ABC \)
   b. \( \angle ABW \)
   c. \( \angle AWB \)
   d. \( \angle ZWX \)
   e. \( \angle AWZ \)
   f. \( \angle DYC \)

4. Design a 3-pointed star. Write the steps needed to reproduce your design.

5. Given regular hexagon ACFDBE. Draw \( \overline{AB}, \overline{AD} \) and \( \overline{BC} \). Label the point of intersection of \( \overline{AD} \) and \( \overline{BC} \) as \( X \). Determine the measure of each of the following angles.
   a. \( \angle GCA \)
   b. \( \angle CGB \)
   c. \( \angle ACB \)
   d. \( \angle BDA \)
   e. \( \angle EAB \)
   f. \( \angle BDF \)

6. Use the figure in Item 5 to determine the best name for each of the following quadrilaterals (trapezoid, parallelogram, rhombus, rectangle, or square).
   a. Quadrilateral \( ABDC \)
   b. Quadrilateral \( AEBD \)
   c. Quadrilateral \( AEBX \)
   d. Quadrilateral \( ACGD \)
   e. Quadrilateral \( BDFC \)

7. Calculate the measure of an interior angle in a regular decagon (10-sided polygon).

8. MATHMATICAL REFLECTION What is the largest possible value for the measure of an interior angle in a regular polygon?
Honeybees build combs in which they store honey. If you look at a slice of honeycomb, you will see hexagonal sections. Suppose all hexagons in the figure to the right are regular hexagons.

1. Determine the measure of \( \angle FAB \). Show your work.

2. Draw \( \overline{AD} \). Determine the measure of \( \angle FAD \) and \( \angle ADC \).

3. Is \( \overline{AF} \parallel \overline{CD} \)? Explain how you know.

4. If \( \overrightarrow{ED} \) is extended to contain point \( G \), name an angle that is corresponding to \( \angle ADG \).

5. Given \( \triangle WXY \) in the figure.

   \[ \begin{array}{c}
   W \quad 30^\circ \\
   X \\
   Y \quad 45^\circ
   \end{array} \]

   a. Determine the measure of \( \angle X \). Show your work.
   b. Sketch the image of \( \triangle WXY \) if it is reflected over the dotted line.

6. Given isosceles \( \triangle PQR \) with \( \overline{PR} \cong \overline{QR} \) and \( m \angle R = 30^\circ \).

   \[ \begin{array}{c}
   P \\
   Q \\
   R \quad 30^\circ
   \end{array} \]

   a. Determine the measure of \( \angle P \). Show your work.
   b. If side \( PQ \) were extended to the left through point \( T \), what would be the measure of angle \( RPT \)?
   c. Sketch the image of \( \triangle PQR \) if it is rotated 90° in a clockwise direction.
7. In the figure to the left, $m$ is parallel to $n$.

a. If $m\angle 1 = 32.5^\circ$, then $m\angle 2 = \underline{\hspace{2cm}}$.

b. If $m\angle 1 = 32.5^\circ$, then $m\angle 3 = \underline{\hspace{2cm}}$.

c. If $m\angle 1 = 73^\circ$, then $m\angle 6 = \underline{\hspace{2cm}}$.

d. List all angles in the figure congruent to $\angle 4$. 

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<table>
<thead>
<tr>
<th>Math Knowledge #1, 2, 5a, 6a, 7a–c, d</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Determines the correct measure of angle $FAB$ (1)</td>
<td>• Determines only one of the correct measures of the two angles</td>
<td>• Gives an incorrect measure of angle $FAB$</td>
<td></td>
</tr>
<tr>
<td>• Determines the correct measures of angles $FAD$ and $ADC$ (2)</td>
<td>• Determines the correct measure of only one of the angles</td>
<td>• Determines the correct measures of neither of the angles</td>
<td></td>
</tr>
<tr>
<td>• Determines the correct measures of angles $X$ and $P$ (5a, 6a)</td>
<td>• Determines the correct measures of only two of the angles</td>
<td>• Determines the correct measures of neither of the angles</td>
<td></td>
</tr>
<tr>
<td>• Determines the correct measures of angles 2, 3, and 6 (7a–c)</td>
<td>• Lists only two of the angles congruent to angle 4</td>
<td>• Determines the correct measures of fewer than two of the angles</td>
<td></td>
</tr>
<tr>
<td>• Lists all angles congruent to angle 4 (7d)</td>
<td></td>
<td>• Lists fewer than two angles congruent to angle 4</td>
<td></td>
</tr>
</tbody>
</table>

| Problem Solving #3, 4 | | |
|----------------------| | |
| • Determines correctly whether or not segment $AF$ is parallel to segment $CD$. (3) | • Determines an incorrect relationship between segments $AF$ and $CD$ | • Determines an incorrect angle |
| • Determines the correct angle that is corresponding to angle $ADG$. (4) | | |

| Representations #2, 5b, 6b | | |
|---------------------------| | |
| • Draws segment $AD$ correctly. (2) | • Sketches a partially correct reflected image. | • Draws $AD$ incorrectly |
| • Sketches a correct reflected image of triangle $WXY$. (5b) | • Sketches a partially correct rotated image. | • Sketches an incorrect reflected image |
| • Sketches a correct image of a rotated triangle $PQR$. (6b) | | • Sketches an incorrect rotated image |

| Communication #1, 3, 5a, 6a | | |
|-----------------------------| | |
| • Shows correct work for the measure of angles $FAB$, $X$, and $P$. (1, 5a, 6a) | • Shows correct work for the measure of two of the angles. | • Shows correct work for fewer than two of the angles |
| • Gives a complete explanation for his/her conclusion about whether segment $AF$ is parallel to segment $CD$. (3) | • Gives an incomplete explanation for a correct conclusion. | • Gives an explanation for an incorrect conclusion |
Martha Rose Kennedy was watching an old black and white movie about World War II, and during the parade scene she noticed that the flags seemed to have a “funny shape.” Martha did a little research and according to the *US Code, Title 4, Chapter 1*, the ratio of the hoist (height) to the fly (width) of the flag should be 1:1.9. However, in the 1950’s, president Dwight D. Eisenhower eased the restrictions on the dimensions of the US Flag to accommodate current “standard” sizes such as $3 \times 5$, $4 \times 6$, and $5 \times 8$.

1. Without using a ruler, predict which of the following rectangles will have a ratio: \( \frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9} \). Explain how you made your decision.

![Image of flags](image)

2. Explain what it means for “two sides of a rectangle to be in the ratio \( \frac{1}{1.9} \)”

3. a. If \( \frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9} \) and the hoist is 1 ft, calculate the fly.

   b. If the ratio remains constant and the hoist of the flag is 2 ft, calculate the fly.
4. Complete the following table which displays the correct hoist and fly of the US Flag according to the US Flag Code. Assume that the units of measure are the same for the hoist and fly and that \( \frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9} \).

<table>
<thead>
<tr>
<th>Hoist</th>
<th>Fly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>38</td>
</tr>
</tbody>
</table>

A useful way to calculate values that are proportional to a given ratio is to set up and solve a proportion. For example, in Items 3 and 4 the proportion \( \frac{1}{1.9} = \frac{2}{x} \) could have been used to find the fly of a flag whose hoist is 2 units.

**EXAMPLE 1**

The lengths of two sides of a rectangle are in the ratio 4:9. If the shorter side has a length of 18 cm, calculate the length of the longer side.

**Step 1:** Write an equation with 2 equivalent ratios.

\[
\frac{4}{9} = \frac{18 \text{ cm}}{x}
\]

**Step 2:** Calculate the cross products.

\[
\frac{4}{9} \cdot 18 \text{ cm} = \frac{4x}{x}
\]

\[4x = 162 \text{ cm}\]

**Step 3:** Solve for the unknown.

\[
\frac{4x}{4} = \frac{162}{4}
\]

Solution: \( x = 40.5 \text{ cm} \)
TRY THESE A

a. Solve for $a$: $\frac{a}{35} = \frac{1}{1.75}$

b. Solve for $n$: $\frac{7}{8} = \frac{n}{100}$

c. The lengths of two sides of a rectangle are in the ratio 2:3. If the shorter side has a length of 24 in., calculate the length of the longer side.

d. The lengths of two sides of a rectangle are in the ratio 1:2.5. If the longer side has a length of 40 mm, calculate the length of the shorter side.

5. Suppose the hoist of a flag is 10 inches and $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$. Write and solve the proportion needed to calculate the fly.

6. Write and solve the proportion needed to calculate the hoist of a flag whose fly is 17 inches if $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$.

7. One of the ratios that President Eisenhower approved for the hoist and fly of the US Flag was 3:5.
   
   a. Write this ratio as a fraction.

   b. Using this ratio, determine the fly of a flag whose hoist is 3 ft.
8. Find the difference in the lengths of the fly for a 3:5 flag and the flag in the table in Item 4 whose hoist is 3 ft. Which flag has the longer fly?

**EXAMPLE 2**

Harry copied a drawing. The original width was 5 in., and the width of Harry’s drawing was 7 in. Calculate the percent increase or decrease of the change in the width of Harry’s drawing.

**Step 1:** Calculate the difference between the quantities.

\[ 7 - 5 = 2 \]

**Step 2:** Write the difference to original as a ratio and set up a proportion to solve for the percent.

\[ \frac{2 \text{ in.}}{5 \text{ in.}} = \frac{p}{100} \]

**Step 3:** Solve for \( p \).

\[ 5p = 200 \]

\[ p = 40 \]

**Solution:** Since Harry’s copy is larger than the original, the 2 inch difference is a 40% increase.

**TRY THESE B**

Solve the following problems.

**a.** Hillary used to deposit $40 in her savings account each month. Now she deposits $30 each month. Calculate the change in Hillary’s deposits as a percent increase or decrease.

**b.** Bill’s dog, Buddy, gained 5 lbs. If Buddy’s original weight was 60 lbs, express Buddy’s weight gain as a percent increase or decrease.

9. Express the difference that you calculated in Item 8 as a percent increase or decrease of the fly length (for a 3 ft hoist) displayed in the table in Item 4.
10. Another dimension approved by President Eisenhower was 5 by 8. Calculate the difference in the fly for a flag that is 5 ft by 8 ft and the fly of the flag in the table in Item 4 whose hoist is 5 feet. Express this difference as a percent increase or decrease of the value from the table. Show your work.

11. Suppose a flag manufacturer chooses to make flags the ratio of whose hoist to fly is 3:5.

   a. Express this ratio as a fraction.

   b. Find the fly of one of these flags if the hoist is 12 feet. Show your work.

   c. Find the fly of one of these flags if the hoist is 1 foot. Show your work.

   d. Write a fraction that is equal to the fraction in part a, but that has a 1 in the numerator.

TRY THESE C

   a. Write a fraction equal to \(\frac{4}{9}\) whose numerator is 1.

   b. Measure the dimensions of the US flag in your classroom, and write them as a ratio of the hoist to the fly with a numerator of 1.

   c. If the ratio of the length of a rectangle to the width is 6:5, find the width of the rectangle when the length is 15 cm.
Creating a Scale Drawing
Martha Rose decides to investigate further by creating a scale drawing of the US Flag including the thirteen stripes and the blue field for the stars. She chooses the following characteristics for her flag.

- \( \frac{\text{height}}{\text{width}} = \frac{3}{5} \).
- There are 7 red and 6 white stripes, all of which have the same height (hoist).
- The height of the blue field equals the height of seven stripes.
- \( \frac{\text{height of blue field}}{\text{width of blue field}} = \frac{2}{3} \).

12. The height of one stripe is what fraction of the height of the entire flag?

13. The height of the blue field is what fraction of the height of the flag?

14. Since all of the information concerning the dimensions of the flag and its parts are given in terms of the height, Martha decides to begin her scale drawing by choosing 13 cm for the height. Explain why 13 cm is a good choice for the height.

15. Determine the width of Martha’s flag if \( \frac{\text{height}}{\text{width}} = \frac{3}{5} \). On a plain piece of paper, draw the rectangle with this width and a height of 13 cm.

16. Use the information given above to calculate the dimensions of the blue field. Show your work. Add the rectangle that represents the blue field to your scale drawing of the flag.
17. The stripes have the same height but two different widths.

   a. Find the dimensions for each of the six longer stripes and add them to your scale drawing.

   b. Find the dimensions for each of the seven shorter stripes and add them to your scale drawing.

18. Create a scale drawing for a US flag with a height of 13 cm that has the following specifications:
   \[
   \frac{\text{height}}{\text{width}} = \frac{1}{1.9}
   \]
   • There are 7 red and 6 white stripes, all of which have the same height.
   • The height of the blue field is the height of 7 stripes.
   \[
   \frac{\text{height of blue field}}{\text{width of blue field}} = \frac{2}{3}
   \]

19. At the beginning of this activity, you were asked to choose the rectangle whose dimensions were in the ratio 1:1.9. Compare the scale drawings of the two flags that you have drawn to the rectangles presented in Item 1.

   a. Which of the rectangles in Item 1 looks most like the flag you drew in Items 12–17?

   b. Which of the rectangles in Item 1 looks most like the flag you drew in Item 18?

   c. Which of the rectangles in Item 1 is most similar to the flag in your classroom?

20. If you owned a company that manufactures US flags, which would you prefer: flags with the ratio \(\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.67}\) or flags with the ratio \(\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}\)? Explain your answer.
### CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Solve for \( x \).
   - a. \( \frac{x}{20} = \frac{52.5}{75} \)
   - b. \( \frac{13}{16} = \frac{x}{100} \)
   - c. \( \frac{18}{25} = \frac{1}{x} \)
   - d. \( \frac{25}{18} = \frac{1}{x} \)

2. The ratio of Thomas' age to his sister’s age is 3:2. Find Thomas' age if his sister is 10 years old.

3. The ratio of the altitude of a triangle to the length of its base is 5:12. Calculate the altitude if the length of the base is 42 mm.

4. Sarah’s hourly wage went from \$4.00 per hour to \$4.50 per hour. Calculate the difference in Sarah’s wages as a percent increase or decrease.

5. Todd’s hourly wage went from \$4.50 per hour to \$4.00 per hour. Calculate the difference in Todd’s wages as a percent increase or decrease.

6. Write a fraction that is equal to each of the given fractions and has a numerator of 1.
   - a. \( \frac{4}{5} \)
   - b. \( \frac{5}{7} \)
   - c. \( \frac{3.15}{6.3} \)

7. Kenyatta is creating a scale drawing of a rectangle whose height is 3 cm and width is 10 cm. Find the width of the rectangle in his drawing if the height is 4.5 inches.

8. Mariela is creating a scale drawing of a triangle such that \( \frac{\text{altitude}}{\text{length of the base}} = \frac{1}{1.75} \). Find the altitude of Mamie’s triangle if the length of the base is 77 mm.

9. Create a scale drawing of the rectangle below. The height of the scale drawing should be 2.5 inches.

![Rectangle Scale Drawing]

10. Find the width of the rectangle that is a scale drawing of the rectangle above if the height is 13 cm.

11. **MATHEMATICAL REFLECTION** When solving for a percent increase or decrease, you set up a proportion. Explain why the ratio \( \frac{p}{100} \) is one of the ratios used in that proportion.
John Abraham Roosevelt is running for student government. He plans to create posters with a patriotic theme: red, white, and blue. John also wants his colors to be proportional to those of the US Flag.

Before John can buy paint or markers for the posters, he needs to know the ratio of the number of red markers to the number of blue markers. John has a flag hanging in his bedroom with the dimensions pictured below.

1. Find the total area of the US Flag. Show your work.

2. Draw in the thirteen stripes: 7 “short” and 6 “long” and label each stripe as “red” or “white.”
3. There are four “short” red stripes and three “long” red stripes.
   
   a. List the dimensions of each “short” stripe and find the area.
   
   b. List the dimensions of each “long” stripe and find the area.
   
   c. Find the total area of the 7 red stripes. Show your work.

4. Write the ratio of the sum of the areas of the red stripes to the total area of the flag. Express your answer as a fraction reduced to lowest terms.

5. Express your answer from Item 4 as a decimal and as a percent.

6. The total area for white on the US Flag can be found by adding the areas of the three “short” stripes, three “long” stripes and fifty stars.
   
   a. Calculate the sum of the areas of the 6 white stripes. Show your work.
   
   b. John researched flag dimensions and estimated that there was as much red as there is white on the US Flag. Calculate the total area of the fifty stars. Be sure to include units and show your work.
7. What percent of this flag is white? Round your answer to the nearest whole percent.

8. Calculate the area of the US Flag that is blue. Include units and show your work.

9. What percent of this flag is blue?

10. Recall that John Abraham Roosevelt wanted to calculate the number of red or blue markers needed for his campaign posters, and his color scheme would be proportional to those in the US Flag.

   a. How does the area of the blue region on the US Flag in this activity compare to the sum of the areas of the red stripes?

   b. If John has enough money to buy 24 red and blue markers, how many of each should he buy? Explain your answer.
TRY THESE A
a. Express $\frac{3}{8}$ as a decimal and a percent.

John has a lot of socks. 6 pairs are white, 6 pairs are brown, 4 pairs are green, and 8 pairs are black.

b. What percent of John’s socks are brown?

c. What percent of John’s socks are black?

d. What percent of John’s socks are black and green?

e. The area of the triangular region is what percent of the rectangle’s area?

![Diagram of a rectangle and a triangle]

**Math Terms**

A **sector** of a circle is formed by two radii and an arc of the circle.

A **central angle** is an angle whose vertex is the center of the circle.

**Circle Graphs**

Circle graphs are used to display data by representing each portion of the whole as a sector of a circle. The size of the sector is determined by the measure of the central angle.
11. Use a protractor to measure the central angle in each of the sectors.

![Circle Graph](image)

12. Find the sum of the measures of the three central angles above.

John has to deliver a campaign speech. He has decided to spend his allotted time as follows.

- 40% talking about dances and pep rallies
- 25% talking about the food in the cafeteria
- 20% talking about the canned food drive for the community food bank
- 15% talking about recycling on campus

13. John is a “visual” person, so he creates a circle graph that represents the four parts of his speech. Calculate the measure of each central angle for each sector in his circle graph by finding a percent of $360^\circ$. Show your work.

**Math Tip**

To find a percent of a number, convert the percent to a decimal (divide by 100) and multiply.

*Example:*

$35\%$ of $150 = 0.35(150) = 52.5$
14. Use a protractor and the circle below to complete a circle graph that represents the four parts of John’s speech.

- Refer to your responses to Item 13 and draw an angle with the first degree measure. The vertex of the angle should be the center of the circle. One side of this angle has already been drawn. Draw the other side of this sector and label this sector.
- Draw the second central angle such that it has a side in common with the first angle. Label this sector.
- Draw the third central angle such that it has a side in common with the second angle. Label this sector.
- Label the remaining sector. Use a protractor to make sure it has the correct measure.
- Create a title for your circle graph.
TRY THESE B

a. Match each central angle measure of a circle graph to its corresponding percent.

<table>
<thead>
<tr>
<th>Central Angle Measure</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>270°</td>
<td>10%</td>
</tr>
<tr>
<td>180°</td>
<td>41.7%</td>
</tr>
<tr>
<td>30°</td>
<td>50%</td>
</tr>
<tr>
<td>216°</td>
<td>75%</td>
</tr>
<tr>
<td>108°</td>
<td>8.3%</td>
</tr>
<tr>
<td>60°</td>
<td>16.7%</td>
</tr>
<tr>
<td>36°</td>
<td>60%</td>
</tr>
<tr>
<td>150°</td>
<td>30%</td>
</tr>
</tbody>
</table>

b. Create a circle graph that represents the area of red, white and blue on the US Flag in this activity. Use your responses to Items 5, 7, and 9.

c. For his science class, John counted the trees in the woods behind his house:
   - 120 maple trees
   - 84 oak trees
   - 24 pine trees
   - 12 other trees

Create a circle graph that represents the number of each kind of tree behind John’s house.

d. Use the My Notes space to make a list of the steps needed to create the circle graph above.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Express each ratio as a percent.
   a. $\frac{1}{2}$
   b. $\frac{5}{8}$
   c. $\frac{9}{10}$
   d. $\frac{9}{20}$

2. At DDE Middle School, there are 280 6th graders, 368 7th graders, and 152 8th graders.
   a. What percent of the students at DDE Middle School are 6th graders?
   b. What percent of the students at DDE Middle School are 7th graders?
   c. What percent of the students at DDE Middle School are 8th graders?

3. Create a circle graph to represent the number of 6th, 7th, and 8th graders at DDE Middle School from #2 above.

4. The area of rectangle $ABCD$ is what percent of the area of rectangle $BCEF$?

5. The radius of the circle is 5 mm and the length of one side of the inscribed square is 7.1 mm. What percent of the area of the circle is occupied by the square? The area of a circle is $A = \pi r^2$. Use 3.14 for $\pi$.

6. Find each of the following percents of $80$.
   a. 30%
   b. 65%
   c. 37.5%
   d. 120%

7. Find each of the following percents of $360^\circ$.
   a. 12%
   b. 25%
   c. 8.5%
   d. 60%

8. In a recent football game, the offense was on the field for 26.4 minutes out of a total of 48 minutes. For what percent of the game was the offense on the field?

9. In a recent survey, Shayla asked her classmates to name their favorite president. The results are displayed below. Create a circle graph to represent the results from Shayla’s survey.

<table>
<thead>
<tr>
<th>Favorite US President</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>George Washington</td>
<td>16</td>
</tr>
<tr>
<td>Abraham Lincoln</td>
<td>21</td>
</tr>
<tr>
<td>Theodore Roosevelt</td>
<td>3</td>
</tr>
<tr>
<td>John F. Kennedy</td>
<td>6</td>
</tr>
<tr>
<td>Other</td>
<td>4</td>
</tr>
</tbody>
</table>

10. **Mathematical Reflection** What is true about the measure of the central angle of any sector that represents more than 50% of the circle?
Carl found an old journal in which the science fiction author documented his adventures to parallel universes through different tunnels.

The triangles below represent two of the triangles described in the journal.

1. Use the lengths of the three pairs of corresponding sides to create three ratios in the form \( \frac{\text{side length in small triangle}}{\text{corresponding length in large triangle}} \).
2. Compare the ratios that you created in Item 1.

3. **Similar figures** are figures in which the lengths of the corresponding sides are in proportion and the corresponding angles are congruent.

4. If \( \triangle ABC \sim \triangle DEF \), label the missing vertices in the triangles below.

5. The triangles below are similar.

   a. What transformation would help you identify the corresponding parts of the triangles?

   b. Use similarity notation to identify all the similar parts of the triangles.
6. Consider the three triangles below.

a. Compare ratios to identify the similar triangles.

b. Identify the similar triangles using similarity notation.

7. The ratio of two corresponding sides of similar triangles is called the scale factor. Find the scale factors for the similar triangles in Items 1, 5, and 6.

**EXAMPLE 1**

Solve for $x$ if $\triangle AIM \sim \triangle LOW$.

**Step 1:** Find the scale factor using known corresponding lengths.

The scale factor is $\frac{20}{16} = \frac{5}{4}$

**Step 2:** Write a proportion using the scale factor.

$$\frac{5}{4} = \frac{15}{x}$$

**Step 3:** Solve the proportion.

$5x = 60$

$x = 12$

Solution: $12$ cm
**TRY THESE A**

Given \( \triangle TIN \sim \triangle CAN \)

**a.** Find the scale factor.

**b.** Solve for \( x \) and \( y \).

---

**8.** If two triangles are similar, then corresponding angles are congruent. If \( \triangle BIG \sim \triangle SKY \), then list three pairs of congruent angles.

**9.** Suppose \( \triangle DOG \sim \triangle CAT \)

Find the measure of each of the following angles.

**a.** \( \angle C \)

**b.** \( \angle D \)

**c.** \( \angle A \)

**d.** \( \angle G \)

---

**10.** If the three angles of one triangle are congruent to the three angles of another triangle, the triangles are similar. Use similarity notation to state which triangles are similar.
**11.** A **dilation** is a transformation where the image is similar (but not congruent) to the pre-image. Given the pre-image of \( \triangle PQR \) below, sketch the image of \( \triangle PQR \) if it is dilated:

- **a.** by a factor of 2.
- **b.** by a factor of \( \frac{1}{2} \).

**12.** Given the similar triangles below.

- **a.** The larger triangle is a dilation of the smaller triangle. By what factor is the smaller triangle enlarged?
- **b.** By what factor is the larger triangle reduced?
- **c.** What is the relationship between the two scale factors?

You can sketch the dilation of a polygon on the coordinate plane by multiplying the coordinates of each vertex by the dilation factor and connecting the new vertices.

**13.** Sketch the image of \( \triangle RST \) to the right if it is dilated by a factor of 2.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Given the triangles to the right,
   a. use similarity notation to identify the two triangles as similar.
   b. find the scale factor.

2. Given the triangles below,
   a. use similarity notation to identify the two similar triangles.
   b. find the scale factor.
   c. If the height of \(\triangle AGE\) is 7.2" find the height of the smaller triangle.
   d. How does the ratio of the perimeters of the triangles compare to the scale factor?

3. If \(\triangle SIX \sim \triangle TEN\) then solve for \(a\) and \(b\).

4. If \(\triangle CAN \sim \triangle CYR\) then solve for \(p\) and \(q\).

5. \(\triangle MON \sim \triangle WED\), \(m\angle M = 37^\circ\) and \(m\angle E = 82^\circ\). Find the measure of each of the following angles.
   a. \(\angle O\)  b. \(\angle W\)  c. \(\angle N\)  d. \(\angle D\)

6. If \(\triangle JOE \sim \triangle AMY\) find the measure of each of the following angles.
   a. \(\angle J\)
   b. \(\angle O\)
   c. \(\angle Y\)
   d. \(\angle M\)

7. Compare ratios and use similarity notation to state which triangles are similar.

8. Sketch the image of the figure below once it is dilated by a factor of \(\frac{2}{3}\).

9. Does the size of a pre-image increase or decrease when
   a. dilated by a factor greater than 1?
   b. dilated by a factor between 0 and 1?

10. MATHEMATICAL REFLECTION
    If two triangles are similar, how does the ratio of their perimeters compare to the scale factor. Justify your answer.
Ramon was excited when he won tickets to see the Great Howie Dudat, an illusionist. One of the Great Howie Dudat’s best tricks is to raise a 7.5 ton truck into the air 15 meters.

1. One ton is equal to 2000 pounds. Calculate the weight of the truck in pounds. Show your work.

2. The average student at Ramon’s school weighs 125 pounds. Find the number of students whose combined weights equal the weight of the truck. Show your work.

3. One meter is approximately 39.4 inches. Calculate the height that the Great Howie levitates the truck in inches.

4. Convert your response from Item 3, which is given in inches, into feet.

5. a. Which of the Items 1–4 required multiplication?

   b. Which of the Items 1–4 required division?

   c. When converting units, explain how you know which operation to use: multiplication or division.
TRY THESE A

a. Convert 80 inches to feet.

b. There are 8 ounces in one cup. How many ounces are in 3.5 cups?

c. Find the latest foreign currency conversions and convert $50 (US) to yen (Japanese currency), pesos (Mexican currency) and Canadian dollars.

d. Ray needs 25.5 feet of wire to complete a project. Wire is sold by the yard, so he ordered 78.5 yards. Did Ray calculate his order correctly? Explain your answer.

On the way to the performance, Ramon passes by a dam, and on the side of the road there is a sign that posts the rate at which water passes over the spillway.

**Water Volume:**

107 cubic feet
(Per Minute)

6. One cubic foot is equal to 1,728 cubic inches and one gallon is equal to 231 cubic inches. How many gallons are in 107 cubic feet? Round to the nearest hundredth. Show your work.
7. Calculate the number of gallons that pass over the spillway in a day. Explain how you got your answer.

8. The speed of the river that flows from the base of the dam is approximately 705 feet per minute. Convert the speed of the river to miles per hour. Explain the steps you used to get your answer.

**TRY THESE**

a. Convert 1.5 cubic feet to gallons.

b. Ramona is thinking about getting a 50-gallon fish tank. How many cubic feet of space will this tank take up?

c. An ant can cover 22 feet in a minute. Convert this speed to miles per hour.

d. Jack rabbits have been clocked at speeds as high as 36 mph. At this speed, how far can a jack rabbit travel in one second?

e. One square yard of carpeting costs $4.50 and 1 square yard = 9 square feet. Calculate the cost of carpeting an area of 945 square feet.

9. John determined that his house has $1.3 \times 10^8$ square inches of living space.

a. Choose a more appropriate unit of area for John to use.

b. Find the approximate area of living space in John’s house using the unit you chose for part a.
During the show, the Great Howie Dudat performs a trick with four raw eggs. A tray is set on four glasses and each egg is placed on a cylinder and on the tray above each glass. As Howie pulls the tray, the eggs drop into the glasses. Ramon is sure he can learn to do this trick, too. Of course, he will practice with golf balls, and before he begins, he calculates the speed at which he needs to pull the tray to be 74 inches per second.

10. Does Ramon have to be “lightning fast” or “slow and steady” as he pulls the tray? Explain your answer.

Since “miles per hour” are more common as a unit for speed than “inches per second”, it may be necessary to convert units before responding to Item 9. Another way to think about the problem of changing a rate expressed as \( \frac{\text{inches}}{\text{second}} \) to a rate expressed as \( \frac{\text{miles}}{\text{hour}} \) is to multiply the first rate by ratios that are equal to 1.

11. Explain why \( \frac{60 \text{ seconds}}{1 \text{ minute}} \) is equal to 1.
12. Multiply \( \frac{74 \text{ inches}}{\text{second}} \times \frac{60 \text{ seconds}}{\text{minute}} \) and simplify the answer, including units.

13. Multiply your response in Item 11 by the appropriate ratio that is equal to 1 so the units of the product are \( \frac{\text{inches}}{\text{hour}} \) and simplify your answer.

14. Multiply your response in Item 13 by two ratios, each of which is equal to 1, so the units of the product are \( \frac{\text{miles}}{\text{hour}} \) and simplify your answer.

15. Revisit your response to Item 10. Does Ramon have to be “lightning fast” or “slow and steady” as he pulls the tray? Explain your answer.
TRY THESE C

a. Convert \( \frac{60 \text{ miles}}{\text{hour}} \) to \( \frac{\text{inches}}{\text{second}} \).

b. Ramona’s coffee maker drips coffee at a rate of 4 ounces per minute. Given 1 cup = 8 ounces, 1 quart = 4 cups, and 1 gallon = 4 quarts, convert \( \frac{4 \text{ ounces}}{\text{minute}} \) to \( \frac{\text{gallons}}{\text{hour}} \).

c. Ralene’s car averages 25 mpg (miles per gallon). Given 1 km = 0.6 mi, 1 gallon = 3.8 liters and 1 km = 1000 meters, convert \( \frac{25 \text{ miles}}{\text{gallon}} \) to \( \frac{\text{meters}}{\text{liter}} \).

d. The Raymonds estimate that they create 200 ounces of trash each day. At this rate, how many tons of trash will they create in a year? 1 ton = 2000 lbs and 1 pound = 16 ounces.

CHECK YOUR UNDERSTANDING

1. Given 1 mile is about 1.6 kilometers, change 65 miles to kilometers.
2. Given 1 mile = 1,760 yards, convert 6,688 yards to miles. Now convert it to feet.
3. How many hours are in a week?
4. Given 1 gallon = 3.8 liters and 1 gallon = 4 quarts:
   a. Convert 2 liters to quarts.
   b. Which is a better buy: $0.89 for 1 liter or $0.89 for 1 quart?
5. Convert \( \frac{500 \text{ feet}}{\text{minute}} \) to \( \frac{\text{miles}}{\text{hour}} \).
6. Given 1 meter = 39.4 inches, 1 kilometer = 1000 meters and 1 mile = 5,280 ft, convert 5 kilometers to miles.
7. If a special hen can lay two eggs each day, and each dozen is sold for $1.50, how much money can be earned by selling all of the hen’s eggs for a year?
8. If a pipe is leaking at a rate of 5 cups per day, how many gallons will leak in a month? 1 quart = 4 cups, 1 gallon = 4 quarts, 1 month = 30 days
9. Over the past three years, Gerry has grown at the rate of \( 6 \times 10^{-1} \) meters per year. Estimate Gerry’s growth rate using a more appropriate unit.
10. Annise wanted to convert 60 miles per hour into miles per minute. She multiplied: 60 miles \( \div \frac{3 \text{ hours}}{\text{hour}} \). Will she end up with the right answer? Explain.
Proportions, Similarity, and Conversions

GOLDEN RECTANGLES

Early mathematicians marveled at rectangles whose sides were in the ratio 1:1.618. They went so far as to call them Golden Rectangles.

1. Verify that the lengths of the sides of the rectangle in Phase I are in the ratio 1:1.618.

2. In Phase II, Quadrilateral $ABFE$ is a square.
   a. Calculate the length of $FC$ and $ED$. Show your work.
   b. The length of $FC$ is what percent of the length of $BC$? Show your work and round to the nearest tenth of a percent.
   c. Is Quadrilateral $CDEF$ a golden rectangle? Show the work that supports your answer and round to three decimal places.

3. If Quadrilateral $DEGH$ is a square, identify the smallest golden rectangle in Phase III. Show your work and round to three decimal places.

4. Draw the figure that would appear in Phase IV of this pattern.

5. Draw $AC$ in Phase I and $DF$ in Phase II.
   a. Use a triangle from Phase II that completes the similarity statement $\triangle ACD \sim \underline{\text{____________}}$.
   b. Find the scale factor.
   c. Calculate $DF$ if $AC = 5.706$ cm. Round to three decimal places.

6. A rod was a common unit for measuring distances until the mid 1800’s. Today, a rod is used to approximate the length of a 2-person canoe. Let the length of one side of a rectangle be 4 rods. Given 1 rod $= 16.5$ feet, calculate the length of the known side of the rectangle in yards.

7. 1 furlong $= 40$ rods and 1 mile $= 5280$ feet. If a race horse can run 4 furlongs in a minute, calculate the speed of this horse in miles per hour (mph).
Shaleetra met with the guidance counselor to plan her high school curriculum and learned that to graduate she would need to take 4 English, 4 math, 4 science, 3 foreign language, 2 Fine Arts classes, 1 Physical Education, and 6 elective classes.

a. What percent of her classes will be Math?

b. What percent of her classes will be Foreign Language?

c. What percent of her classes will be Electives?

d. Make a circle graph that represents Shaleetra’s high school curriculum.

<table>
<thead>
<tr>
<th>Math Knowledge #5c</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correctly calculates the measure of segment DF and rounds correctly. (5c)</td>
<td>Correctly calculates the measure of DF, but does not round or rounds incorrectly</td>
<td>Does not calculate the correct measure of DF</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #2a–c, 3, 5a, b; 6, 7, 8a–c</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculates the correct length of segments FC and ED (2a)</td>
<td>Calculates the correct length of either FC or ED</td>
<td>Calculates the correct length of neither FC nor ED</td>
<td></td>
</tr>
<tr>
<td>Finds the correct percent and rounds correctly (2b)</td>
<td>Finds the correct percent, but rounds incorrectly</td>
<td>Does not find the correct percent</td>
<td></td>
</tr>
<tr>
<td>Answers correctly whether or not CDEF is a golden rectangle (2c)</td>
<td>Uses a correct method to identify the smallest rectangle, but makes a computational error</td>
<td>Answers incorrectly whether or not quadrilateral CDEF is a golden rectangle</td>
<td></td>
</tr>
<tr>
<td>Identifies the correct smallest golden rectangle in Phase III (3)</td>
<td>Uses a correct method to find the scale factor, but makes a computational error</td>
<td>Does not identify the correct rectangle</td>
<td></td>
</tr>
<tr>
<td>Completes the similarity statement correctly (5a)</td>
<td>Finds the correct length of the side of the rectangle in feet</td>
<td>Completes the statement incorrectly</td>
<td></td>
</tr>
<tr>
<td>Finds the correct scale factor (5b)</td>
<td>Finds the correct length of the side of the rectangle in feet</td>
<td>Finds an incorrect scale factor</td>
<td></td>
</tr>
<tr>
<td>Finds the correct length of the side of the rectangle in yards (6)</td>
<td>Correctly calculates the speed of the horse in feet per minute</td>
<td>Finds an incorrect side length</td>
<td></td>
</tr>
<tr>
<td>Correctly calculates the speed of the horse in miles per hour (7)</td>
<td>Correctly calculates only two percents</td>
<td>Finds an incorrect speed</td>
<td></td>
</tr>
<tr>
<td>Correctly calculates the three percents (8a, b, c)</td>
<td></td>
<td>Correctly calculates only one of the percents</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representations #4, 5, 8d</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Draws the correct figure that would appear in Phase IV of the pattern. (4)</td>
<td>Draws a partially correct figure</td>
<td>Draws an incorrect figure.</td>
<td></td>
</tr>
<tr>
<td>Draws segments AC and DF correctly. (5)</td>
<td>Draws segment AC or DF correctly</td>
<td>Draws neither segment correctly</td>
<td></td>
</tr>
<tr>
<td>Makes a circle graph that correctly represents the curriculum. (8d)</td>
<td>Makes a circle graph that has some, but not all correct sectors</td>
<td>Makes a circle graph at least two correct sectors</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communication #1, 2a–c, 3</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Correctly verifies that the lengths are in ratio 1:1.618. (1)</td>
<td>Uses a correct method of verification, but makes a computational error</td>
<td>Does not verify the ratio</td>
<td></td>
</tr>
<tr>
<td>Shows the correct work in finding the lengths of segments FC and ED. (2a)</td>
<td>Shows a correct method to find the lengths, but makes a computational error</td>
<td>Shows work that contains conceptual errors</td>
<td></td>
</tr>
<tr>
<td>Shows the correct work in finding the percent. (2b)</td>
<td>Shows a correct method to find the percent, but makes a computational error</td>
<td>Shows work that contains conceptual errors</td>
<td></td>
</tr>
<tr>
<td>Shows work that adequately supports the answer. (2c)</td>
<td>Shows a correct method to find the percent, but makes a computational error</td>
<td>Shows work that includes a conceptual error</td>
<td></td>
</tr>
<tr>
<td>Shows correct work that supports the identification. (3)</td>
<td>Shows incorrect work that contains no errors</td>
<td>Shows work that contains conceptual errors</td>
<td></td>
</tr>
</tbody>
</table>
**ACTIVITY 4.1**

1. What must be true for lines to be parallel?
2. What must be true for lines to be perpendicular?
3. Give a real life example of a right angle.
4. What has to be true for two angles to be complementary?
5. Are angles with measures of 40° and 50° complementary?
6. What is the measurement of the angle that is supplementary to a 58° angle?
7. Give the missing measurements in the diagram below.

**ACTIVITY 4.2**

8. Complete the following table.

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>Image</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>△</td>
<td></td>
</tr>
<tr>
<td>△</td>
<td>△</td>
<td>Vertical reflection</td>
</tr>
<tr>
<td>△</td>
<td>△</td>
<td></td>
</tr>
<tr>
<td>△</td>
<td>△</td>
<td></td>
</tr>
</tbody>
</table>

9. Determine the coordinates of the vertices for each image of \( \triangle GEO \) after each of the following transformations are performed. For example, if \( \triangle GEO \) is translated up 1 unit the coordinates of the vertices would become \((0, 5), (3, 1), \) and \((0, 1)\).

   a. Rotate \( \triangle GEO \) 180° around the origin.
   b. Rotate \( \triangle GEO \) 90° counterclockwise around the origin.
   c. Rotate \( \triangle GEO \) 90° counterclockwise around point \( E \).
   d. Translate \( \triangle GEO \) 3 units to the left and reflect over the \( x \)-axis.
   e. Reflect \( \triangle GEO \) over the \( x \)-axis and translate 3 units to the left.
   f. Translate \( \triangle GEO \) 2 units to the right and reflect over the \( y \)-axis.
   g. Reflect \( \triangle GEO \) over the \( y \)-axis and translate 2 units to the right.
10. Each triangle in i–iv is an image of the triangle above. Make a list of the transformations that were performed.

i. 

ii. 

iii. 

iv. 

11. Find the measure of the unknown angle.

\[ \angle 114^\circ \]

\[ 18^\circ \]

\[ 114^\circ \]

\[ \text{a. } 42^\circ \quad \text{b. } 48^\circ \]

\[ \text{c. } 98^\circ \quad \text{d. } 132^\circ \]

12. In each of the following, the measures of two angles in a triangle are given. Find the measure of the third angle in each triangle.

\[ \text{a. } 98^\circ \text{ and } 32^\circ \]

\[ \text{b. } 83.5^\circ \text{ and } 24.5^\circ \]

\[ \text{c. } 23^\circ \text{ and } 40.5^\circ \]

\[ \text{d. } 12.25^\circ \text{ and } 57.75^\circ \]

13. In each of the following, the measures of the three angles in a triangle are given. Write an equation, and solve for \( x \). Give the measure of the third angle.

\[ \text{a. } 55^\circ, 107^\circ, \text{ and } 3x \]

\[ \text{b. } 57^\circ, 87^\circ, \text{ and } 8x \]

\[ \text{c. } 36.5^\circ, 83.5^\circ, \text{ and } 5x \]

**ACTIVITY 4.3**

14. Given Quadrilateral \( \text{FOUR} \) is a square and \( \text{PENTA} \) is a regular pentagon. Determine the measure of each of the following angles.

\[ \text{a. } \angle R \]

\[ \text{b. } \angle ATN \]

\[ \text{c. } \angle ATR \]

\[ \text{d. } \angle RAT \]

\[ \text{e. } \angle FAP \]

\[ \text{f. } \angle FPA \]

\[ \text{g. } \angle ETN \]

\[ \text{h. } \angle ETA \]
15. Given regular hexagon \(HEXAGN\) and right triangle \(ARX\). Determine the measure of each of the following angles.

\[
\begin{align*}
&\text{a. } \angle GAX \\
&\text{b. } \angle XAR \\
&\text{c. } \angle AXR \\
&\text{d. } \angle NXR \\
&\text{e. } \angle AXG \\
&\text{f. } \angle XGN \\
\end{align*}
\]

16. Given regular octagon \(DEFGHJKL\). Determine the measure of each of the following angles.

\[
\begin{align*}
&\text{a. } \angle EFG \\
&\text{b. } \angle FEG \\
&\text{c. } \angle DEG \\
&\text{d. } \angle DHL \\
&\text{e. } \angle JHL \\
&\text{f. } \angle GHD \\
&\text{g. } \angle HDL \\
&\text{h. } \angle JHD \\
\end{align*}
\]

17. Use the regular octagon in Question 16 to answer each of the following.

\[
\begin{align*}
&\text{a. } \text{Is } \overline{KJ} \parallel \overline{LH}? \text{ Explain how you know.} \\
&\text{b. } \text{Is } \overline{EG} \parallel \overline{LH}? \text{ Explain how you know.} \\
&\text{c. } \text{Is } \overline{EG} \parallel \overline{DH}? \text{ Explain how you know.} \\
\end{align*}
\]

18. Calculate the measure of an interior angle in a regular dodecagon (12-sided polygon).

19. Which of the following could be the measure of an interior angle in a regular polygon?

\[30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ, 100^\circ, 120^\circ, 125^\circ, 135^\circ\]

ACTIVITY 4.4

20. The ratio of students to teachers at a middle school is 24:1. Calculate the number of teachers if there are 1080 students.

21. Suppose \(\text{radius of circle A} = \frac{4}{3}\). Determine the radius of circle A if the radius of circle B is 36 cm.

22. Jimmy scored a 75 on his first math test and an 80 on the second test. Calculate the difference in Jimmy’s test scores as a percent increase or decrease.

23. Rewrite \(\frac{25}{67}\) as a fraction with a numerator of 1.

24. Rosalind is creating a scale drawing of the triangle below. Calculate the height of Rosalind’s triangle if the length of the base is 180 cm.
25. Create a scale drawing of the figure below. The width of your drawing should be 8 inches.

26. If the height, h, in the figure above is 1.4 meters, calculate the height of the figure in the scale drawing that you created.

27. Express \( \frac{9}{16} \) as a percent.

28. James needs to save $160 to purchase a new amplifier. For the amounts shown below, calculate what percent each is of his total cost.
   a. $40  
   b. $32  
   c. $50  
   d. $80  
   e. $150  
   f. $110

29. The area of the rectangle is what percent of the area of the triangle in the figure below?

30. Zack is recording songs on a disk with space for 110 minutes of music. His computer says he has 40% of the space remaining on a disk. How much time does Zack still have available on the disk?
   a. 40 min  
   b. 44 min  
   c. 66 min  
   d. 70 min

31. Calculate 46.5% of 360°.

32. A chart showing the amount of time Betsy spends on her activities is given below.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>practice the piano</td>
<td>10.5 hrs</td>
</tr>
<tr>
<td>walk the dog</td>
<td>7 hrs</td>
</tr>
<tr>
<td>homework</td>
<td>21 hrs</td>
</tr>
<tr>
<td>chores</td>
<td>3.5 hrs</td>
</tr>
</tbody>
</table>

Create a circle graph to represent the time Betsy spends on each activity.

33. Given the triangles below.
   a. Use similarity notation to identify the two similar triangles.
   b. Find the scale factor.
   c. If the height of \( \triangle RTO \) is 4.5", determine the height of the smaller triangle.

34. If \( \triangle MAH \sim \triangle ATH \), then solve for \( x \) and \( y \).

35. Use the figure above to complete the following similarity statement:
   \( \triangle TAM \sim \triangle \) \underline{_______} \sim \( \triangle \) \underline{_______}
36. If $\triangle TOM \sim \triangle VAL$, determine the measure of each of the following angles.

\[
\begin{array}{c}
\text{a. } \angle T \\
\text{b. } \angle L \\
\text{c. } \angle V \\
\text{d. } \angle A
\end{array}
\]

37. Tell the measure of each angle of $\triangle ABC$ and $\triangle PQR$ if $\triangle ABC \sim \triangle PQR$, $m\angle A = 90^\circ$, and $m\angle B = 56^\circ$.

38. Sketch the image of the figure below once it is dilated by a factor of 1.5.

**ACTIVITY 4.7**

39. Given 1 pound = 16 ounces, convert 88 ounces to pounds.

40. If $1 \text{ US} = 0.92 \text{ Canadian}$, convert $25 \text{ US}$ to Canadian currency and $25 \text{ Canadian}$ to US currency.

41. Given 1 inch = 2.54 centimeters, convert 1 yard to centimeters.

42. Given 1 mile = 5280 feet and 1 rod = 16.5 feet, how many miles are equal to 800 rods?

43. Convert $\frac{432 \text{ miles}}{\text{hour}}$ to $\frac{\text{feet}}{\text{second}}$.

44. Water is flowing over a dam at a rate of $\frac{462 \text{ cubic feet}}{\text{minute}}$. Convert this rate to $\frac{\text{gallons}}{\text{hour}}$. 1 gallon = 231 cubic inches and 1 cubic foot = 1728 cubic inches.
An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How is proportional reasoning used to solve real-world problems?
   - What are transformations and how are they useful in solving real-world problems?

Academic Vocabulary

2. Look at the following academic vocabulary words:
   - angle
   - similar figures
   - transformations (geometric)
   Explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. What is the measure of ∠DEF in the right triangle DEF?
   F. 22°  
   G. 32°  
   H. 68°  
   I. 158°

2. Lines ℓ and m are parallel. What is the value of x?

3. Isaiah’s little brother likes to copy everything Isaiah does. They decide they will fly kites together. Isaiah uses 14 feet of string to fly his kite 8 feet horizontally from where he is standing. If his brother imitates him, as shown below, how much kite string will he need to get his kite 6 feet away horizontally? Explain your reasoning.
Driver Krista Osborne won the 2008 Ultimate Speed Challenge for soap box derby. She drove down the 989-foot hill in 27 seconds.

**Part A:** Find Krista’s speed in feet per minute.

**Answer and Explain**

**Part B:** Convert her speed to meters per minute.

**Answer and Explain**

**Part C:** Scott Mackley won the 2008 Gasparilla 5K by completing the course in 15 minutes. A 5K race is 5 kilometers. Find Scott’s speed in meters per minute.

**Answer and Explain**

**Part D:** Compare Krista’s speed with Scott’s speed.

**Solve and Explain**