Introduction to Chapter 2

This chapter is about graphing data from your experiments with the car and ramp. You will learn that graphs are mathematical models used for making predictions and solving equations.

Investigations for Chapter 2

2.1 Using a Scientific Model to Predict Speed

How can you predict the speed of the car at any point on the ramp?

In this Investigation you will create a graphical model that you can use to predict the speed of the car at any point on the ramp. You will do this by determining the speed of the car at six points along the ramp and then graphing the speed of the car against the distance traveled.

2.2 Position and Time

How do you model the motion of the car?

In this Investigation you will make a distance vs. time graph from the data you collect with the car and ramp. You are going to model the motion of one trip of the car down the ramp. To get enough data to model motion, you will collect data at 10 or more points along the ramp. Your teacher will assign your group’s ramp angle.

2.3 Acceleration

How is the speed of the car changing?

Since acceleration depends on the angle of the hill, a car and ramp make a good tool to discover the behavior of uniform acceleration, or when speed changes at a constant rate. Shallow (nearly level) angles will give very little acceleration, and the increasing speed is easy to observe. Steep (nearly vertical) angles resemble free fall or motion that is entirely under the influence of gravity.
### Learning Goals

In this chapter, you will:

- Construct a speed vs. distance graph.
- Use a graph to make a prediction that can be quantitatively tested.
- Calculate the percent error between a measurement and a prediction.
- Create and analyze a distance vs. time graph.
- Determine the slope of a line.
- Distinguish between linear and nonlinear graphs.
- Distinguish between speed and acceleration.
- Calculate acceleration from a formula.
- Calculate acceleration from the slope of a speed vs. time graph.

### Vocabulary

<table>
<thead>
<tr>
<th>accelerate</th>
<th>deceleration</th>
<th>graphical model</th>
<th>instantaneous speed</th>
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<tbody>
<tr>
<td>acceleration</td>
<td>dependent variable</td>
<td>gravity</td>
<td>physical model</td>
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<tr>
<td>average speed</td>
<td>free fall</td>
<td>independent variable</td>
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<td>conceptual model</td>
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2.1 Using a Scientific Model to Predict Speed

In this section, you will learn how to make a model that will accurately predict the speed of a car. Making models is an important part of science and engineering. For a given situation, models tell us how all the variables, like speed, distance, and time, fit together. If we have a model, we can predict what will happen because we know how changes in one variable affect the others.

Why make models?

Suppose it is your job to design a train to go from New York to Los Angeles in the shortest possible time. Your train would have to go up and down hills and across flat plains carrying 1,000 people.

**Why make models?**

- **How powerful a motor do you need?**
- **How powerful do the brakes need to be?**
- **How much fuel do you need to carry?**

There are many things you have to know. You want the answers to the questions before you build the train. How do you get answers to a complicated problem such as how to design a high-speed train?

The way we answer complicated questions is to break them down into smaller questions. Each smaller question can be answered with simple experiments or research. One question might be how fast a train will roll down a hill of a given angle. You might do an experiment with a miniature train to get some data on downhill speeds that would help you design the brakes for the train. Other questions might be answered with research, in order to learn how other people solved similar problems.

You can often use the results of an experiment to produce a model that tells how each of the variables in the experiment are related. One model you might make is a graph showing how fuel efficiency depends on the size of the engine. If you know the engine size needed to climb the steepest hill, the model tells you how much fuel you have to carry on the train. Once you have models for each part of the train, you can evaluate different choices for your design.
Scientific models

What do experiments tell us?

An experiment tells us about the relationship between variables. If we roll a car downhill to learn about its motion, we will need to measure its speed at several distances from the top. Speed and distance are the variables. We will be looking for a way to connect these variables. We need to know exactly how much speed is gained by the car for every centimeter it rolls down the ramp. We collect experimental data to figure out the relationship between the variables.

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<th>Distance (cm)</th>
<th>Speed (cm/sec)</th>
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<td>40</td>
<td>120</td>
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<td>60</td>
<td>160</td>
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</table>

What is a scientific model?

We then take the results and make a scientific model that shows how each variable relates to another. For example, how does the distance traveled relate to the speed? The data above shows that for every 20 centimeters traveled, the speed increased by 40 cm/sec. If we graph this data, we can use it to make predictions about the speed of the car at other places along the ramp. A similar process could be used in the train design. A graphical model could answer the question “If the hill is longer by a kilometer, how much faster will the train go if the brakes fail?”

Solving the big question

Once we have models for the smaller relationships, we can put them together to solve the bigger question of how to design the train. Experiments and research have given us enough information to create and test models that tell us how each part will work. Once we know how each part of the train will work, we can design a train where all the parts work together.

Accurate measurements

Instruments like an electronic timer allow you to make very accurate measurements of speed. The more accurate your measurements, the better your model will be. By using very accurate data to make the graphical model, you can be sure that your predictions will be accurate also.

Figure 2.2: A model is something we make that identifies the relationships between the variables. The model can answer questions like “If I change the distance down the ramp, how much will the speed change?”

Variables

- force
- angle
- time
- weight
- speed
- distance

Experiment

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<tr>
<th>Distance (cm)</th>
<th>Speed (cm/sec)</th>
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<td>10</td>
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<td>60</td>
<td>290</td>
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<td>70</td>
<td>260</td>
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Model

<table>
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<tr>
<th>Speed vs. distance</th>
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<tr>
<td>Speed (cm/sec)</td>
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<td>Distance (cm)</td>
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</table>

Prediction

At 55 centimeters, the speed of the car will be 231 cm/sec.
Chapter 2

2.1 Using a Scientific Model to Predict Speed

Our models of nature can take many forms. For example, suppose you want to kick a soccer ball into the goal. In your mind, you know how the ball moves on the grass of the field or through the air because of your previous experience. This mental image is a kind of model you use to make adjustments in how you kick the ball toward the goal.

Physical models

Some models are physical. Physical models are models that we can look at, touch, feel, and take measurements from. Engineers often construct scale models of bridges and evaluate them for strength and design. The word scale means that lengths on the model are proportional to lengths on the real object. For example, a scale of 1 inch = 10 feet (120:1) means that every inch on the model represents 10 feet in real life. It is much easier to do experiments on scale models than it is to build full-size bridges! If properly constructed, models tell the engineers about the behavior of the real bridge, and help them avoid dangerous mistakes.

Conceptual models

Much of our scientific understanding of nature is expressed in the form of conceptual models. These types of models are descriptive, that is, we use them to describe how something works. For example, in 1543, Nicholas Copernicus, the great astronomer, described a conceptual model of the heavens in which the Earth revolves in an orbit around the sun. Copernicus’s conceptual model was a major revolution in our understanding of astronomy, since most people of his time believed in Ptolemy’s model in which the sun moved around the Earth. Other astronomers added to Copernicus’ model. Galileo invented the telescope in 1609, and Johannes Kepler used the telescope to work out detailed orbits for other planets. In 1687, Isaac Newton’s law of universal gravitation finally provided a model that explained why planets move in orbits. Our models improve as our understanding grows.

Figure 2.3: Mental models help us imagine how something will happen. Soccer players make accurate models in their minds when they shoot the ball at the goal.

Figure 2.4: Some models are physical, like this model of a bridge. Models can tell engineers and architects a lot about how a project will be built.
Making a graphical model

Graphical models While conceptual models are very useful, often they are only the first step toward making a model that can make predictions. The next step is often a graph. A graph shows how two variables are related with a picture that is easy to understand. A graphical model uses a graph to show a relationship between the variable on the \( x \)-axis and the variable on the \( y \)-axis. Because a graph uses numbers it is also known as a mathematical model.

The dependent variable The graph shows how the speed of a rolling car changes as it rolls downhill. We expect the speed to change. Speed is the dependent variable because we think the speed depends on how far down the ramp the car gets.

The independent variable The distance is the independent variable. We say it is independent because we are free to make the distance anything we want by choosing where on the ramp to measure.

Choosing \( x \) and \( y \) People have decided to always put the independent variable on the horizontal (\( x \)) axis. You should too, since this is how people will read any graph you make. The dependent variable goes on the vertical (\( y \)) axis.

How to Make a Graph

1. Decide what to put on \( x \) and \( y \).

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>Speed (cm/sec)</th>
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<tbody>
<tr>
<td>20</td>
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<td>198</td>
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<td>60</td>
<td>242</td>
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<tr>
<td>80</td>
<td>280</td>
</tr>
<tr>
<td>90</td>
<td>297</td>
</tr>
</tbody>
</table>

   Letting each box = 20 fits the biggest data point (297 cm/sec)

2. Make a scale for each axis by counting boxes to fit your largest value. Count by multiples of 1, 2, 5, or 10 to make it easier to plot points. Make the graph big, try to use as much of the graph paper as you can.

3. Plot your points by finding the \( x \) value, and drawing a line up until you get to the right \( y \) value. Put a dot for each point.

4. Draw a smooth curve that shows the pattern of the points. Don’t simply connect the dots.

5. Make a title for your graph.
Reading a graph

Why are graphs useful?
One purpose of making a graph is to organize your data into a model you can use to make predictions. Pictures are much easier to understand than tables of data (figure 2.5). By making a graph, you are making a picture that shows the exact relationship between your variables.

Making predictions from a graph
Suppose you want to find out what the speed of the car would be 50 centimeters from the start. You did not measure the speed there. Yet the graph can give you an answer.

1 To predict the speed, start by finding 50 centimeters on the x-axis.
2 Draw a line vertically upward from 50 centimeters until it hits the curve you drew from your data.
3 Draw a line horizontally over until it reaches the y-axis.
4 Use the scale on the y-axis to read the predicted speed.
5 For this example, the model graph predicts the speed to be 220 cm/sec.

Checking the accuracy of a model
If the graph is created from accurate data, the prediction will also be accurate. You could check by doing another experiment and measuring the speed of the car at 50 centimeters. You should find it to be very close to the prediction from your graph.

Figure 2.5: Some different shapes for ramps and their corresponding speed vs. distance graphs.
Chapter 2

Cause and effect relationships

In many experiments we are looking for a cause and effect relationship. How does changing one variable effect another? Graphs are a good way to see whether there is a connection between two variables or not. You cannot always tell from looking at tables of data. With a graph, the connection is clear.

Patterns indicate relationships

When there is a relationship between the variables the graph shows a clear pattern. The speed and distance variables show a strong relationship. When there is no relationship the graph looks like a collection of dots. No pattern appears. The number of musical groups a student listed in one minute and the last two digits of his or her phone number are an example of two variables that are not related.

Strong and weak relationships

You can tell how strong the relationship is from the pattern. If the relationship is strong, a small change in one variable makes a big change in another. If the relationship is weak, even a big change in one variable has little effect on the other. In weak relationships, the points may follow a pattern but there is not much change in one variable compared to big changes in the other (figure 2.6).

Inverse relationships

Some relationships are inverse. When one variable increases, the other decreases. If you graph how much money you spend against how much you have left, you see an inverse relationship. The more you spend, the less you have. Graphs of inverse relationships always slope down to the right (figure 2.7).

Figure 2.6: In a strong relationship (top), a big change in distance creates a big change in speed. In a weak relationship (bottom), a big change in mass makes almost no change in speed.

Figure 2.7: A typical graph for an inverse relationship.
2.2 Position and Time

Graphical models like the speed vs. distance graph are good for organizing data so you can make predictions. In this section, you will learn how to model motion with another graph: position vs. time. The position vs. time graph offers a new way to find the speed of a moving object. The position vs. time graph will also be our example as we explore different ways to use and interpret graphs. The techniques you learn in this section will help you understand acceleration, the next important idea in motion.

Position

Position

In physics, the word position means where something is compared with where it started, including direction. As things move their position changes. If you walked in a straight line away from your school, your position would keep getting larger (figure 2.8). If you stopped walking, your position would stop changing.

Distance

Distance is an interval of length without regard to direction. You can walk a distance of 10 miles in a circle and end up exactly where you started. If you walk a curved path, the distance you walk could be much greater than the distance between where you started and where you end up (figure 2.9).

Position and distance

Position and distance are different. If you are 7 kilometers north of school, that is a statement of your position. If you walk back towards your school, your position decreases. If you get back to where you started, your position is zero even though the distance you walked is 14 kilometers (7 km away plus 7 km back)!
The position vs. time graph

What does the graph tell you?
The position vs. time graph shows where things are at different times. If things have moved, it is easy to see from the graph. You might think giving the speed is enough description of how things have moved. But speed does not always give you enough information.

A car trip with a rest
For example, suppose you take a car trip that includes 1.5 hours of driving and a half-hour rest stop, for a total time of 2 hours. You drive a total distance of 90 miles in a straight line. At the end you call your friends to tell them it took you 2 hours and they calculate your speed to be 45 mph (90 miles divided by 2 hours).

Actually, you drove a lot faster than 45 mph to make up for the half-hour rest stop. You really covered the 90 miles in 1.5 hours, at a speed of 60 mph. You stopped (with zero speed) for a half hour.

The graph is a better picture of the trip
The position vs. time graph shows your trip much more accurately than saying you covered 90 miles in 2 hours. For the first hour, your position gradually increases from zero (start) until you are 60 miles away. Your position stays the same between 1 hour and 1.5 hours because you stopped. Then you get going again and cover the last 30 miles in a half hour. The position vs. time graph shows a complete history of your trip including your stop.

Figure 2.10: Examples of graphs showing different speeds. Graph A shows movement away from start. Graph B shows movement back toward start. Graph C shows no motion. The object is stopped with zero speed.
Determining speed from the slope of a graph

Look at the distance vs. time

Let’s take a closer look at the first hour of your driving trip (figure 2.11). You drove at a constant speed of 60 mph. The position vs. time graph shows the position of your car on the highway as it changes with time. The line on the graph represents the motion of the car. If the graph is a complete description of the motion, you should be able to figure out the speed of the car from the graph.

The definition of slope

The definition of slope is the ratio of “rise” (vertical change) to the “run” (horizontal change) of the line. The rise is determined by finding the height of the triangle shown. The run is determined by finding the length along the base of the triangle. For this graph, the $x$-values represent time and the $y$-values represent position.

Speed is the slope of the position vs. time graph

Speed is the distance traveled divided by the time taken. The distance is really the difference in position between where you finished and where you started. This is equal to the rise (vertical distance) on the graph. The run on the graph is the time taken for the trip. The slope is rise over run, which is the distance traveled over the time taken, which is the speed.

Speed is the slope of the position vs. time graph.

The slope of a graph

Speed from the slope of the position vs. time graph

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{10}{5} = 2.0
\]

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ mph}
\]
Instantaneous and average speed

Speed does not usually stay constant

Does your speed stay exactly the same during a real trip? The answer is, of course not. Your speed is almost always changing. You slow down for stop lights, and speed up to pass people. For the next example, consider taking a bicycle trip. You may remain on flat ground for moment, but eventually you come to a hill. As you climb the hill, you slow down. As you go down the hill, you speed up.

Average speed

There are two ways you should think about speed. If it takes you 2 hours to ride 50 kilometers, your average speed is 25 kilometers per hour (25 km/h). To calculate average speed, you simply take the total distance traveled divided by the total time taken.

Instantaneous speed

At some points along the way, you may go slower, or faster than average. The instantaneous speed is the speed you have at a specific point in your journey. You might go uphill at 10 km/h and downhill at 60 km/h, with an average speed of 25 km/h even though your speed was never exactly 25 km/h at any time in the trip!

A bike trip with a hill

The real story is told by the position vs. time graph. The graph captures both the instantaneous speed and the average speed. If the slope of the graph is steep (C), you have lots of position changing in little time (figure 2.12) indicating a high speed. If the slope is shallow (B), relatively little position changes over a long time, giving a slow speed. If the graph is level the slope is zero, so the speed is also zero, indicating you have stopped and are not moving.

Figure 2.12: Calculating the speed of each part of the trip.
2.3 Acceleration

The speed of things is always changing. Your car speeds up and slows down. If you slow down gradually, it feels very different from slamming on the brakes and stopping fast. In this section we will learn how to measure and discuss changes in speed. Specifically, we will investigate objects rolling downhill. You already know that an object rolling downhill speeds up. The rate at which its speed changes is called **acceleration**.

**Acceleration**

**You accelerate coasting downhill**

What happens if you coast your bicycle down a long hill without pedaling? You accelerate, that is your speed increases steadily. If your bike has a speedometer you find that your speed increases by the same amount every second!

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<th>Time</th>
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<td>1 second</td>
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<td>2 seconds</td>
<td>2 mph</td>
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<td>3 seconds</td>
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<tr>
<td>4 seconds</td>
<td>4 mph</td>
</tr>
<tr>
<td>5 seconds</td>
<td>5 mph</td>
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**Steeper hills**

On a steeper hill, your findings are similar. Your speed increases every second, but by a bigger amount. On the first hill your speed increased by 1 mph every second. On the steeper hill you find your speed increases by 2 mph every second.

Acceleration is the amount that your speed increases, compared to how long it takes. Increasing speed by 1 mph every second means you accelerated at 1 mph per second. Every second your speed increased by 1 mile per hour. It is common to describe acceleration in units of speed (changed) per second.

**Figure 2.13**: How much of the acceleration of gravity you experience depends on the angle of the hill.
Chapter 2

Acceleration when speed is in miles per hour

Acceleration

Acceleration is the rate of change in the speed of an object. Rate of change means the ratio of the amount of change divided by how much time it took to change.

The acceleration of the car is 10 mph/sec

An example of acceleration

Suppose you are driving and your speed goes from 20 mph to 60 mph in four seconds. The amount of change is 60 mph minus 20 mph, or 40 miles per hour. The time it takes to change is 4 seconds. The acceleration is 40 mph divided by 4 seconds, or 10 mph/sec. Your car accelerated 10 mph per second. That means your speed increased by 10 miles per hour each second. Table 2.1 shows how your speed changed during the four seconds of acceleration.

Table 2.1: Watching your speed while accelerating

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<thead>
<tr>
<th>Time</th>
<th>Speed</th>
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<tbody>
<tr>
<td>0 (start)</td>
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<td>30 mph</td>
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<tr>
<td>2 seconds</td>
<td>40 mph</td>
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<tr>
<td>3 seconds</td>
<td>50 mph</td>
</tr>
<tr>
<td>4 seconds</td>
<td>60 mph</td>
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</table>
## Acceleration in metric units

### The units of acceleration

The units of acceleration can be confusing. Almost all of the calculations of acceleration you will do will be in metric units. If we measure speed in cm/sec, then the change in speed is expressed in cm/sec as well. For example, 2 cm/sec is the difference between a speed of 3 cm/sec and a speed of 1 cm/sec.

### Calculating acceleration

Acceleration is the change in speed divided by the change in time. The units for acceleration are units of speed over units of time. If speed is in cm/sec and time in seconds, then the units for acceleration are cm/sec/sec, or centimeters per second per second. What this means is that the acceleration is the amount that the speed changes in each second. An acceleration of 50 cm/sec/sec means that the speed increases by 50 cm/sec every second. If the acceleration persists for three seconds then the speed increases by a total of 150 cm/sec (3 seconds × 50 cm/sec/sec).

### What do units of seconds squared mean?

To make matters confusing, an acceleration in cm/sec/sec is written cm/sec² (centimeters per second squared). Likewise, an acceleration of m/sec/sec is written m/sec² (meters per second squared). If you use the rules for simplifying fractions on the units of cm/sec/sec, the denominator ends up having units of seconds times seconds, or sec². Saying seconds squared is just a math-shorthand way of talking. The units of square seconds do not have physical meaning in the same way that square inches mean surface area. It is better to think about acceleration in units of speed change per second (that is, centimeters per second per second).

### How we get units of cm/sec²

\[
\text{Acceleration} = \frac{\text{Change in speed}}{\text{Change in time}}
\]

1. **Plug in values**
   \[
   \frac{50 \text{ cm sec}}{\text{sec}} \times \frac{\text{sec}}{\text{sec}} = \frac{50 \text{ cm sec}}{\text{sec}^2}
   \]
2. **Clear the compound fractions**
   \[
   \frac{50 \text{ cm sec}}{\text{sec}^2} \times \frac{\text{sec}}{\text{sec}} = \frac{50 \text{ cm sec}}{\text{sec}^2}
   \]
3. **Final units**
   \[
   \frac{50 \text{ cm sec}}{\text{sec}^2}
   \]

### Acceleration in m/sec²

Many physics problems will use acceleration in m/sec². If you encounter an acceleration of 10 m/sec², this number means the speed is increasing by 10 m/sec every second.

### Example

A car rolls down a ramp and you measure times and distances as shown. Calculate the acceleration in cm/sec².

- **Change in speed**
  \[
  150 \text{ cm/sec} - 50 \text{ cm/sec} = 100 \text{ cm/sec}
  \]
- **Change in time**
  \[
  0.60 \text{ sec} - 0.10 \text{ sec} = 0.50 \text{ sec}
  \]

\[
\text{Acceleration} = \frac{100 \text{ cm/sec}}{0.50 \text{ sec}} = 200 \text{ cm/sec}^2
\]

**Figure 2.14:** An example of calculating acceleration for a car on a ramp.
Different examples of acceleration

Any change in speed means acceleration

Acceleration means changes in speed or velocity. *Any* change in speed means there is acceleration. If you put on the brakes and slow down, your speed changes. In the example of slowing down, the acceleration is in the negative direction. We also use the term *deceleration* to describe this situation. *Acceleration occurs whenever the speed changes, whether the speed increases or decreases.*

Zero acceleration

An object has zero acceleration if it is traveling at constant speed in one direction. You might think of zero acceleration as “cruise control.” If the speed of your car stays the same at 60 miles per hour, your acceleration is zero.

Acceleration when turning

If you change direction, some acceleration happens. When you turn a sharp corner in a car you feel pulled to one side. The pull you feel comes from the acceleration due to turning. To explain this, you need to remember velocity encompasses speed and direction. Any time you change either speed or direction, you are accelerating.

Steep hills and acceleration

You have probably noticed that the steeper the hill, the faster you accelerate. You may already know this effect has to do with *gravity*. Gravity pulls everything down toward the center of the Earth. The steeper the hill, the greater the amount of gravity pulling you forward, and the greater your acceleration.

Free fall

If you drop something straight down it accelerates in free fall. The speed of a free falling object in a vacuum increases by 9.8 meters per second for every second it falls (figure 2.15). This special acceleration is called the acceleration of gravity because it is the acceleration of objects under the influence of the Earth’s gravity. The acceleration of gravity would be different on the moon or on other planets.
2.3 Acceleration

**Acceleration and the speed vs. time graph**

The speed vs. time graph

Another motion graph we need to understand is the graph of speed vs. time. This is the most important graph for understanding acceleration because it shows how the speed changes with time.

The graph below shows an example from an experiment with a car rolling down a ramp. The time is the time between when the car was first released and when its speed was measured after having moved farther down the ramp. You can see that the speed of the car increases the longer it rolls down.

The graph shows a straight line. This means that the speed of the car increases by the same amount every second. The graph (and data) also shows that the speed of the car increases by 25 cm/sec every one-tenth (0.1) of a second.

Acceleration

You should be thinking of acceleration. This graph shows an acceleration of 250 cm/sec/sec or 250 cm/sec$^2$. This is calculated by dividing the change in speed (25 cm/sec) by the change in time (0.1 seconds).

Seeing acceleration on a graph

If you see a slope on a speed vs. time graph, you are seeing acceleration. Figure 2.16 shows some examples of graphs with and without acceleration. Any time the graph of speed vs. time is not perfectly horizontal, it shows acceleration. If the graph slopes down, it means the speed is decreasing. If the graph slopes up, the speed is increasing.

**Figure 2.16**: Examples of graphs with different amounts of acceleration.
- Graph **A** shows positive acceleration, or speeding up.
- Graph **B** shows negative acceleration, or slowing down.
- Graph **C** shows zero acceleration.
Calculating acceleration from the speed vs. time graph

**Slope**
From the last section, you know that the slope of a graph is equal to the ratio of rise to run. On the speed vs. time graph, the rise and run have special meanings, as they did for the distance vs. time graph. The rise is the amount the speed changes. The run is the amount the time changes.

**Acceleration and slope**
Remember, acceleration is the change in speed over the change in time. This is exactly the same as the rise over run for the speed vs. time graph. The slope of the speed vs. time graph is the acceleration.

**Acceleration is the slope of the speed vs. time graph**

To determine the slope of the speed vs. time graph, take the rise or change in speed and divide by the run or change in time. It is helpful to draw the triangle shown above to help figure out the rise and run. The rise is the height of the triangle. The run is the length of the base of the triangle.

**Complex speed vs. time graphs**
You can use slope to recognize when there is acceleration in complicated speed vs. time graphs (figure 2.17). Level graphs mean the speed does not change, which means the acceleration is zero.
Chapter 2 Review

Vocabulary review

Match the following terms with the correct definition. There is one extra definition in the list that will not match any of the terms.

<table>
<thead>
<tr>
<th>Set One</th>
<th>Set Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. scientific model  a. A way to show how something works that is descriptive in nature</td>
<td>1. position a. Total distance traveled divided by total time elapsed</td>
</tr>
<tr>
<td>2. conceptual model  b. A variable that changes in response to another variable</td>
<td>2. slope b. The amount of time elapsed during an experiment</td>
</tr>
<tr>
<td>3. graphical model  c. A variable that doesn’t change during an experiment</td>
<td>3. average speed c. How speed changes over time</td>
</tr>
<tr>
<td>4. dependent variable  d. A variable that we set in an experiment</td>
<td>4. instantaneous speed d. A measurement of a line on a graph, equal to vertical change divided by horizontal change</td>
</tr>
<tr>
<td>5. independent variable  e. A way to show how variables are connected</td>
<td>5. acceleration e. Where something is compared with where it started</td>
</tr>
<tr>
<td>f. A graph that shows how variables are connected</td>
<td>f. Speed at one moment in time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set Three</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. deceleration  a. A measurement of a line on a graph, equal to horizontal change divided by vertical change</td>
</tr>
<tr>
<td>2. gravity b. A force that tends to pull things toward the center of the earth</td>
</tr>
<tr>
<td>3. free fall c. A decrease in speed over time</td>
</tr>
<tr>
<td>d. An object that is moving freely towards the center of the Earth exhibits this type of motion</td>
</tr>
</tbody>
</table>
Concept review

1. One of the early conceptual models of the solar system showed the other planets and the sun orbiting around the Earth. Copernicus developed a new model of the solar system that shows the Earth and other planets orbiting around the sun. Draw a picture of these two models of the solar system.

2. The following terms and phrases refer to the two axes of a graph. Divide the terms and phrases according to which group they belong in.

3. Which of the following types of scientific models is frequently used to make numerical predictions that you can test with measurements? You may choose more than one.
   a. a graph
   b. an equation
   c. a conceptual model
   d. a physical model

4. You take a walk from your house to your friend’s house around the block. If you graph your position during your walk, the longest distance on the graph is 15 meters. But you actually walked 20 meters. Explain why your position (distance from start) and the actual distance you walked were different.

5. You know the average speed of a trip, and you have a position versus time graph of the trip. Which gives you more information about the trip? Explain your answer.

6. The slope of a position vs. time graph is equal to __________.

7. What is the difference between average speed and instantaneous speed? Use a real-life example to help you explain.

8. Is it possible for an object to simultaneously have a speed of zero but an acceleration that is not zero? Answer with an example.

9. What is the acceleration of a car that is going at a steady speed of 60 mph?

10. Does a car accelerate when it goes around a corner at a steady speed? Explain your answer.

11. Does the speedometer of a car give you the average speed or the instantaneous speed of the car? Explain your answer.

12. The slope of a speed vs. time graph is equal to __________.
Problems

1. Engineers propose to build a bridge that is 30 meters in length. They build a model of the bridge that is 3 meters in length. What is the scale of the model? Express your answer in the form \( 1:x \), where \( x \) is the corresponding number of meters on the bridge, when compared with 1 meter on the model.

2. You do an experiment where you measure the height of plants and calculate their growth rate. The growth rate is the amount each plant gets taller per day. You collect the following data on height and growth rate:

<table>
<thead>
<tr>
<th>Week</th>
<th>Height of plant (cm)</th>
<th>Average daily growth rate (mm/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>start</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.9</td>
<td>8.1</td>
</tr>
<tr>
<td>2</td>
<td>11.8</td>
<td>5.6</td>
</tr>
<tr>
<td>3</td>
<td>15.2</td>
<td>4.9</td>
</tr>
<tr>
<td>4</td>
<td>17.7</td>
<td>3.6</td>
</tr>
<tr>
<td>5</td>
<td>19.9</td>
<td>3.1</td>
</tr>
<tr>
<td>6</td>
<td>21.2</td>
<td>1.9</td>
</tr>
<tr>
<td>7</td>
<td>22.1</td>
<td>1.3</td>
</tr>
<tr>
<td>8</td>
<td>22.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

   a. Graph the above data with height on the \( x \)-axis and growth rate on the \( y \)-axis.

   b. Does the data show (you may choose more than one):
   
   1) a strong relationship between variables
   2) a weak relationship between variables
   3) an inverse relationship between variables
   4) a direct relationship between variables

3. A woman goes to a store three blocks away from her home. She walks in a straight line and at a steady pace. Draw a position vs. time graph of her walk. Regard home as start.

4. A woman leaves a store and goes to her home three blocks away. She walks in a straight line and at a steady pace. Draw a position vs. time graph of her walk.

5. A car rolling down a ramp starts with a speed of 50 cm/sec. The car keeps rolling and 0.5 seconds later the speed is 150 cm/sec. Calculate the acceleration of the car in cm/sec\(^2\).

6. Think about the relationship between the amount of gas you have in your car and how far you can travel. Make a graphical model of this relationship. Which is the dependent variable (the effect)? Which is the independent variable (the cause)?

7. The data table below contains information from an experiment where a car was rolling down a ramp. You suspect some of the numbers are incorrect. Which numbers are suspect? Make a graph that demonstrates how you found the bad data.

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>Speed (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>110</td>
</tr>
<tr>
<td>20</td>
<td>154</td>
</tr>
<tr>
<td>30</td>
<td>205</td>
</tr>
<tr>
<td>40</td>
<td>218</td>
</tr>
<tr>
<td>50</td>
<td>243</td>
</tr>
<tr>
<td>60</td>
<td>264</td>
</tr>
<tr>
<td>80</td>
<td>275</td>
</tr>
<tr>
<td>90</td>
<td>327</td>
</tr>
</tbody>
</table>
8. Use the graph below to predict the speed of the car at the following distances: 20 cm, 35 cm, 60 cm, 80 cm

9. Arrange the four points on the distance vs. time graph in order from slowest to fastest.

10. A bicyclist, traveling at 30 miles per hour, rides a total of 48 miles. How much time did it take?

11. A turtle is moving in a straight line at a steady speed of 15 cm/sec for 3 hours. How far did the turtle travel?

12. Match each of the three distance vs. time graphs with the corresponding speed vs. time graph. All three distance vs. time graphs contain only straight-line segments.

13. Calculate speed from the position vs. time graph on the left. Show all of your work.

Applying your knowledge

1. Research the following: What is the fastest acceleration in a human in a sprint race? What is the fastest acceleration of a race horse? Which animal is capable of the fastest acceleration?

2. How fast do your fingernails grow? Devise an experiment to determine the answer. How would you represent your measurement? What units would you use to represent the speed?