Linear Patterns

Unit Overview

In this unit you will explore rate of change situations and learn to represent such situations with tables, graphs, and equations. You will use and graph equations, solve one-step and two-step equations, and solve real-world problems by using models to write equations.

Academic Vocabulary

As you work through this unit, add these terms to your vocabulary notebook.

- inverse operations
- linear
- ordered pair
- rate of change
- variable

Essential Questions

? Why are tables, graphs, and equations useful for representing relationships?

? How can you use equations to solve real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 3.2 and 3.4, will give you an opportunity to demonstrate how you can write, graph, and solve linear equations and use formulas to solve problems.

Embedded Assessment 1

Embedded Assessment 2

Write your answers on notebook paper. Show your work.

1. Copy and complete the table.

Input	3	7	11	14	17	21
Output	0		8			

   Write the rule you used.

2. Copy and complete the table.

Input	Output
3	18
7	
9	
11	
20	120
35	

   Write the rule you used.

3. Make a grid like the one below. Then plot each point on it.
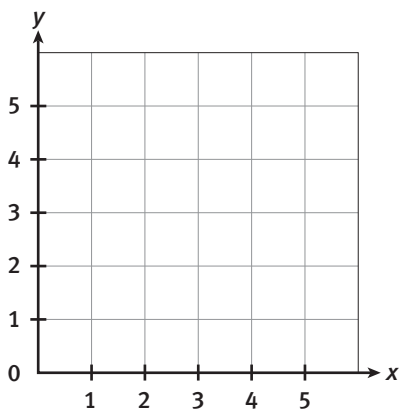
   a. $(0, 2)$ **b.** $(2, 1)$
   c. $(1, 3)$ **d.** $(4, 4)$
   e. $(5, 0)$

Evaluate each expression for $n = 7$.

4. $n + 9$

5. $(n - 3) \div 4$

6. $\frac{35}{n}$

7. Tell how to undo each operation and explain why it works.
   a. adding 28
   b. dividing by 17

8. Write the reciprocal of each number.
   a. 7
   b. $\frac{1}{2}$
   c. $\frac{2}{5}$
   d. $\frac{4}{3}$

   Explain how a number and its reciprocal are related.

Linear Patterns $y = ax$
Centipede and Bug

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/ Retell, Create Representations, Look for a Pattern, Identify a Subtask, Quickwrite, Think/Pair/Share

One sunny day, Sen T. Pede and his friend Lady Bug start out from the elm tree and move toward a rose bush that is 45 feet away. Sen crawls at 5 feet per minute and Lady crawls at 3 feet per minute.

1. Use the diagram below to show where each critter is exactly three minutes after they start their journey. Place the letter *S* at Sen's location and the letter *L* at Lady's location.
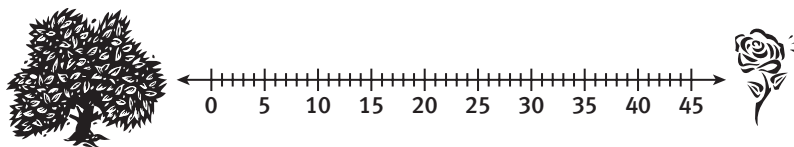
2. Complete this table to show how far Sen and Lady are from the elm tree for each time value.

Time (min) Since Leaving Elm Tree	0	1	2	3	4	5
Sen's Distance (ft) from Elm Tree						
Lady's Distance (ft) from Elm Tree						

3. Describe a pattern you see in Sen's row of the table. You may use words and/or mathematical symbols.

4. How far will Sen be from the elm tree after seven minutes?

5. How long will it take Sen to crawl 40 feet?

6. Describe a pattern you see in Lady's row of the table. You may use words and/or mathematical symbols.

7. How far will Lady be from the rose bush after eight minutes?

**8.** How long will it take Lady to crawl 36 feet?

My Notes

MATH TERMS

The vertical axis and horizontal axis are number lines that intersect at the point (0, 0). The plural of axis is **axes**.
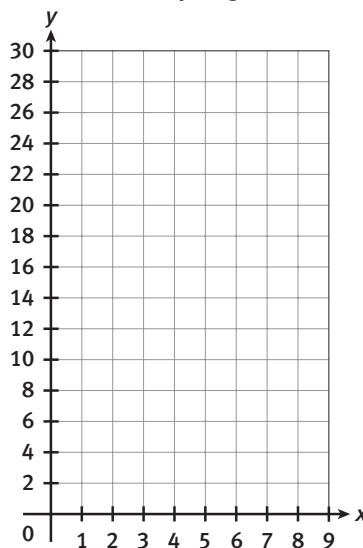
9. After Sen reaches the rose bush, how long will he wait for Lady to arrive? Explain how you determined your answer.

10. Plot the data from Question 2 on the grids below and label the **axes**. Let the vertical axis show distance and the horizontal axis show time.

Sen T. Pede
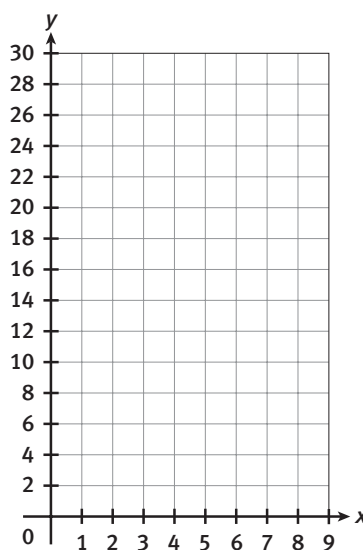
Lady Bug

SUGGESTED LEARNING STRATEGIES: Debriefing, Think/Pair/Share, Look for a Pattern, Guess and Check

11. The data points appear to be **linear**. What do you think this means?

12. Look back at Sen's graph in Question 10. If you connect all of the points on Sen's graph with a line, the line contains the **ordered pair** (2.5, 12.5).

   a. What does this ordered pair mean in this problem situation?

   b. Does the meaning of this ordered pair make sense? Why or why not?

   c. Should the data points on the graphs in question 10 be connected with a line? Explain your reasoning.

13. Use your graph to determine how far Sen would travel in 8 minutes.

14. Use Lady's graph to determine how long it would take Lady to travel 30 ft. Explain how you did this.

15. If you extended Sen's graph, it would contain the point (12, 60). Explain what the coordinates in this ordered pair tell you about Sen.

ACADEMIC VOCABULARY

An **ordered pair** is a pair of numbers that locate a point in the coordinate plane. The first number describes the horizontal position, and the second number describes the vertical position. The numbers are the *coordinates* of the point.

WRITING MATH

Ordered pairs are written as (x, y); for example, (5, 7) is an ordered pair.

My Notes

ACADEMIC VOCABULARY

Variables, or letters, are used in place of unknown numbers in expressions and equations.

ACADEMIC VOCABULARY

Rate of change is a relationship that compares the change in one variable to the corresponding change in a related variable. For example, two variables in text messaging are the number of messages sent and the cost per message. If it costs 15 cents to send each message, then the rate of change is 15 cents per 1 message.

MATH TERMS

If the rate of change remains the same in a problem situation, it is called a **constant rate of change**.

CONNECT TO AP

The notion of rate of change is a fundamental principle in advanced mathematics courses.

On another day, Sen and Lady want to head to the lake for a picnic. However, the lake is much farther away than the rose bush. They need to find a better way than making a table or using a graph to determine how long it will take them to reach the lake. They decide to use **variables** to stand for the distance crawled and the amount of time it takes.

16. Sen crawls d feet in m minutes. Use variables to write an equation that relates d and m for Sen's pace.

17. Use your equation from Question 16 to determine how far Sen can crawl in one hour.

18. Think about the distances Sen crawls.

   a. How far does he crawl between time $m = 20$ minutes and $m = 21$ minutes?
   b. How far does he crawl between time $m = 25$ minutes and $m = 26$ minutes?
   c. Describe the distance Sen crawls as a **rate of change**.

19. Lady crawls d feet in m minutes. Write an equation that shows the relationship between d and m for Lady's pace.

20. Use your equation from Question 19 to determine how far Lady can crawl in one hour.

21. Describe how far Lady crawls as a rate of change between time:

   a. $m = 32$ minutes and $m = 33$ minutes
   b. $m = 32$ minutes and $m = 34$ minutes
   c. Is the rate of change in Part a the same as the rate of change in Part b? Explain why or why not.

22. If the lake is 540 feet from the elm tree, how long will it take each critter to reach the lake? Show your work or explain how you determined your answer.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Group
Presentation, Create Representations

Sen and Lady have a turtle friend, Archimedes, who sometimes
goes along on their adventures.

23. Archimedes crawls *d* feet in *m* minutes. The equation that
shows the relationship between *d* and *m* for his pace is $d = 7.5m$.
Use appropriate units to describe what the **coefficient** 7.5 tells
you about how Archimedes moves.

24. What question is answered by the solution to the equation
$540 = 7.5\ m$?

25. What question is answered by the solution to the equation
$d = 7.5 \times 540$?

Archimedes takes a box of crackers to share at the picnic. He thinks
that each critter will eat 0.5 oz of crackers.

26. Complete this table.

Number of Critters	0	1	2	3	4	5
Crackers Needed (oz)	0	0.5	1			

27. Use the grid in the My Notes spaces to make a graph on
which you plot the data from the table in Question 26. Label
both axes and title your graph.

28. Add the ordered pair (3.5, 1.75) to Archimedes' graph, and
connect all of the data points with a line.

   a. What does this ordered pair mean in this problem situation?

   b. Does the meaning of this ordered pair make sense? Explain
   why or why not.

My Notes

> **MATH TERMS**
> A **coefficient** is a number multi-
> plied with a variable in an alge-
> braic expression or equation.

> **WRITING MATH**
> It is common in mathematics to
> drop the multiplication symbol
> when writing equations with
> variables. The equation
> $d = 7.5 \times m$ is usually written as
> $d = 7.5m$.

SUGGESTED LEARNING STRATEGIES: Self Revision/ Peer Revision

My Notes

29. If Archimedes extended the data in his table or graph, it would contain the point (20, 10).

a. Explain what the coordinates in this ordered pair mean in this problem situation.

b. Does the data point make sense in this problem situation? Explain why or why not.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Itsy Spyder walks every morning. He travels 4 feet per minute. Make a table and graph that show how far Itsy goes during the first 5 minutes of his walk.

2. Should you connect the data points in the graph for Question 1 with a line? Explain.

3. How long will it take Itsy to travel 32 feet? Explain how you got your answer.

4. Itsy Spyder travels d feet in m minutes. Write an equation for this relationship and use it to show how far Itsy travels in 15 minutes.

5. How far does Itsy travel between $m = 18$ min and $m = 21$ min?

6. If Julie rides her bike 1001 feet in 7 minutes, how far does she travel in 21 minutes?

7. The equation $b = 3m$ represents the number of books, b, that Sam can read each month, m.

a. Tell what the coefficient 3 represents.

b. What question is answered by the solution to the equation $36 = 3m$?

c. Make a table and graph to show how many books Sam can read in a 3-month period. Use the vertical axis for books and the horizontal axis for months.

d. Should you connect the data points with a line? How do you know?

8. **MATHEMATICAL REFLECTION** How can you use patterns to write equations? How can you use patterns and equations to make predictions?

Graphing Linear Equations
What's My Line?

SUGGESTED LEARNING STRATEGIES: Marking the Text, Guess and Check

You have made graphs to represent several situations. In this activity, you will graph linear equations.

EXAMPLE 1

Graph the equation $y = x + 2$.
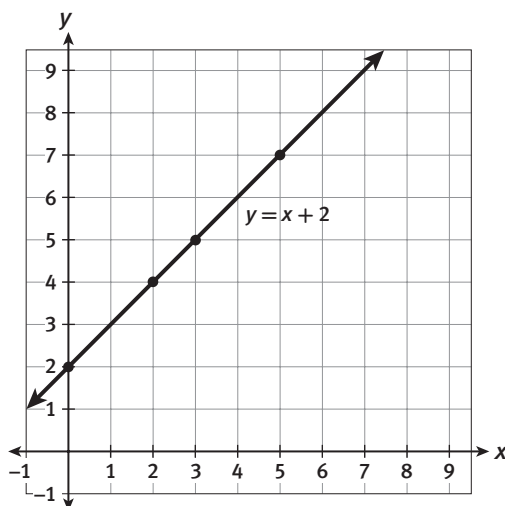
Step 1: *Make an input/output table for the equation.*

Input, x	$x + 2$	Output, y
0	$0 + 2$	2
2	$2 + 2$	4
3	$3 + 2$	5
5	$5 + 2$	7

Step 2: *Write the input and output values as ordered pairs.*

$(0, 2), (2, 4), (3, 5) (5, 7)$

Step 3: *Graph the ordered pairs and then use a straightedge to connect the data points. Label the line with its equation.*

Solution:

1. Could you have used $3\frac{1}{2}$ as an input value? Explain.

2. What other numbers could you use as input values and still be able to graph on this grid? Explain your choices.

My Notes

ACADEMIC VOCABULARY

A **linear equation** is an equation whose solutions lie on a straight line.

MATH TIP

When graphing a linear equation that is not representing a real-world situation, you may connect the data points with a straight line. The line usually has an arrow at each end to show that it continues in both directions.

My Notes

3. How many data points do you think are needed to plot the graph of a linear equation? Explain your reasoning.

4. How is an ordered pair on the graph of a line related to the linear equation the line represents?

TRY THESE **A**

For each equation, complete the table and make a list of ordered pairs from the table. In the My Notes space, draw a coordinate plane and graph the linear equation.

a. $y = x - 5$

Input, x	Output, y
7	
6	
5	
9	

Ordered pairs:

b. $y = 0.5x + 3$

Input, x	Output, y
0	
2	
4	
6	

Ordered pairs:

c. $y = 3x - 2$

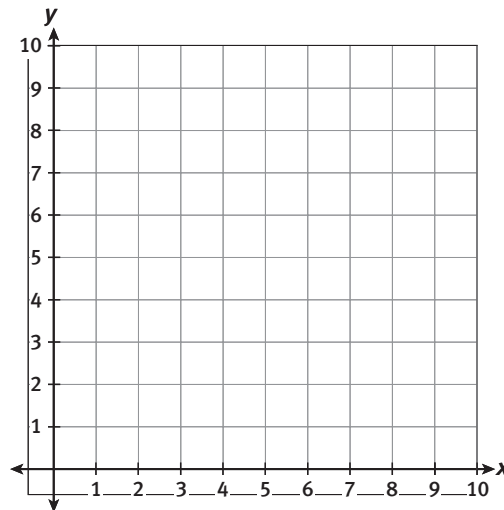| Input, x	Output, y
3	
2.5	
2	
1.5	

Ordered pairs:

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation

My Notes

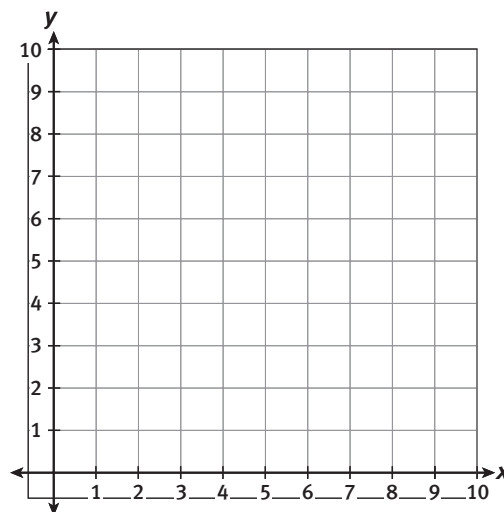Graph each linear equation by choosing input values and then finding the output values.

5. $y = 2x - 1$

Input, x	Output, y

6. $y = 2x + 2$

Input, x	Output, y

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Copy and complete the table for $y = x - 2$, list the ordered pairs, and graph the linear equation.

Input, x	Output, y
5	
2.5	
3	
11	

2. What 3 input values for x would you choose to make an input/output table for $y = \frac{1}{5}x$ Explain your choices.

   Graph each equation. Make an input/output table. Then list the ordered pairs and graph them.

3. $y = \frac{1}{3}x + 1$

4. $y = 2x + 3$

5. **MATHEMATICAL REFLECTION** How many ordered pairs does any linear graph contain? Explain.

Linear Graphs

LIBRARY LINES

Write your answers on notebook paper. Show your work.

Mark and Suzanne were putting away the encyclopedias on a shelf in their school library. They place the books next to each other in a row. All of the books in the set are exactly the same thickness. The table shows the total length of a row of the books for different numbers of books in the row.

1. Copy and complete the table.

Books in Row	0	2	4	6	8	10
Length of Row (in.)	0	6				

2. What is the thickness, in inches, of a single book? Explain how you determined your answer.

3. Plot the data from the table on graph paper. Be sure to label each axis with a title and an appropriate scale.

4. Use the graph to determine the thickness of 12 books from the set. Explain how you determined your answer.

5. Choose a variable for the number of books placed in a row on the shelf and another variable for the total length of these books in a row. Then use your variables to write a rule that gives the total length of these books in terms of the number of books on the shelf placed in a row. Be certain to indicate what your variables mean.

6. Use your equation from Question 5 to find the length of 82 books from the set.

7. Graph the linear equation $y = 3x - 2$. Make a table of ordered pairs first.

Linear Graphs

LIBRARY LINES

	Exemplary	Proficient	Emerging
Math Knowledge #1, #5, #7	The student: • Correctly completes the table (1). • Chooses and defines appropriate variables for the problem situation (5). • Correctly determines ordered pairs given an equation (7).	The student determines answers for questions 1, 5, and 7, but only two of the answers are complete and accurate.	The student determines answers for at least two questions (1, 5, and 7), but the answers may not be complete and accurate.
Problem Solving #2, #4, #6	The student: • Correctly determines the thickness in inches of a single book using a table (2). • Correctly determines the thickness of 12 books from a graph (4). • Correctly determines the length of 82 books using an equation (6).	The student determines answers for questions 2, 4, and 6, but only two of the answers are complete and accurate.	The student determines answers for at least two of the questions (2, 4, and 6), but the answers may not be complete and accurate.
Representation #3, #5, #7	The student: • Correctly plots data on a coordinate grid, including labels and appropriate scales (3). • Writes an equation for the problem situation using appropriate variables and operations (5). • Correctly graphs an equation from the set of ordered pairs generated using the equation (7).	The student represents the information from all three questions (3, 5, and 7), but only two of the representations are complete and accurate.	The student represents the information from at least two of the questions, but only one response is complete and accurate.
Communication #2, #4	The student: • Clearly and completely explains how to determine the thickness of a single book using the table (2). • Clearly and completely explains how to use the graph to determine the thickness of 12 books (4).	The student gives explanations for Questions 2 and 4, but only one of the explanations is clear and complete. The other explanation contains errors or is incomplete.	The student gives an explanation for only one of the questions and the explanation may not be complete and accurate.

Solving One-Step Equations
Becoming Undone

My Notes

Algebraic expressions and equations can be used to represent real-world situations. Solving the resulting equations helps you determine answers to real-life problems.

EXAMPLE 1

Write an equation that represents this question: What number do you add to 15 to get 23?

Step 1: *Write an equation from the information in the question.*

$$15 + number = 23$$

Step 2: *Write the equation using a variable for the unknown number.*

$$15 + x = 23$$

EXAMPLE 2

Rick and Jan went out to lunch. The bill for their lunches came to $18.94. Rick knows that his lunch cost $11.95. How much does Jan's lunch cost?

Step 1: *To write an equation to represent this situation, start with a* ***verbal model***.

Rick wrote this model:

$$\boxed{\text{Cost of Rick's Meal}} + \boxed{\text{Cost of Jan's Meal}} = \boxed{\text{Total bill}}$$

Jan wrote this model:

$$\boxed{\text{Total bill}} - \boxed{\text{Cost of Rick's Meal}} = \boxed{\text{Cost of Jan's Meal}}$$

Step 2: *Write the variable or number that represents each part of the verbal model.*

$$\boxed{\text{Cost of Rick's Meal}} = 11.95$$
$$\boxed{\text{Cost of Jan's Meal}} = p$$
$$\boxed{\text{Total bill}} = 18.94$$

Step 3: *Replace each part of the verbal model with a number or variable to create an equation.*

Rick's equation: $11.95 + p = 18.94$

Jan's equation: $18.94 - 11.95 = p$

MATH TERMS

In algebra a **verbal model** is the first step in creating an equation from the information in a real-life problem.

CONNECT TO AP

Throughout your study of mathematics, you will be required to model written descriptions of physical situations with algebraic representations.

Solving One-Step Equations
Becoming Undone

My Notes

TRY THESE A

Write equations for each of the following.

a. What number do you subtract from 39 to get 31?

b. What number do you multiply by 4 to get 36?

c. What number do you divide by 5 to get 8?

The value of the variable that makes an equation a true statement is called the **solution** to the equation. You can find the solution for some equations using mental math.

MATH TiP

Fact families are very useful when using mental math to solve simple equations.

Solve: $p + 3 = 9$

Fact family:

$p + 3 = 9 \qquad 3 + p = 9$
$9 - 3 = p \qquad 9 - p = 3$

You would use $9 - 3 = p$ to solve for the variable p.

EXAMPLE 3

Solve $p - 5 = 32$ using mental math.

Step 1:   Think of a question to help you solve the equation.

From what number can you subtract 5 and get 32?

Step 2:   Find the value that makes the equation true.

Solution: $p = 37$

Whenever you solve an equation, always check your solution. To check a solution, substitute it for the variable in the original equation. If you get a true statement, the solution is correct.

Check: $p - 5 = 32$

$(37) - 5 \overset{?}{=} 32$

$32 = 32$ ✓

TRY THESE B

Write your answers in the My Notes space. Write a verbal question for each equation and solve using mental math. Check your solutions.

a. $x + 14 = 27$ **b.** $3p = 27$ **c.** $\dfrac{42}{y} = 7$

d. $31 - b = 28$ **e.** $z - 7 = 28$

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward, Close Reading, Marking the Text

When solving equations, it helps to have a systematic way to solve them. What do you notice about both sides of a scale when the scale is balanced?

Suppose you want to solve the equation $x + 2 = 5$. Since you want to find the value of x, you need to get the x to one side of the scale by itself. This is called "isolating the variable."

$x + 2$ ——— 5
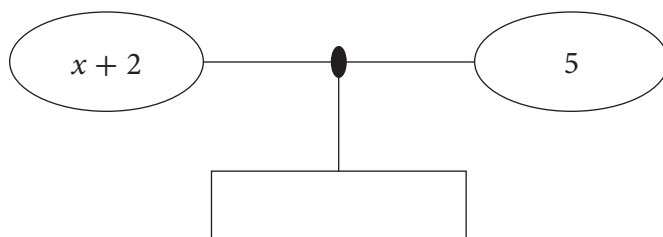
A scale is balanced when the two sides are equal. How can you keep the scale balanced if you add something to one side? If you subtract something from one side? If you add or subtract something from one side, you must also add or subtract the same thing from the other side.

$x + 2 - 2$ ——— $5 - 2$
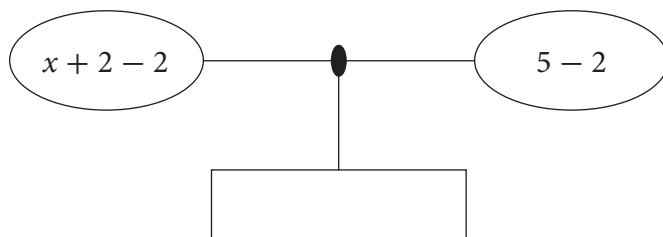
You can subtract 2 from the left side to isolate the x, because $+2$ and -2 are a *zero pair*. To keep the scale balanced, you must also subtract 2 from the right side.

Use the additive identity to simplify $x + 0$.

$x + 0$ ——— 3
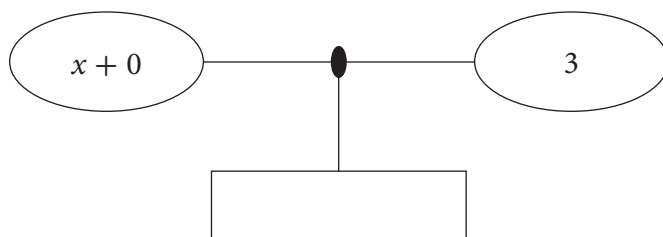
My Notes

> **MATH TiP**
>
> When you "isolate the variable," you put it by itself on one side of the equation.

> **MATH TiP**
>
> A *zero pair* is a pair of numbers that add up to zero.
>
> $+3$ and -3 are a zero pair.
>
> $+12$ and -12 are a zero pair.

Solving One-Step Equations
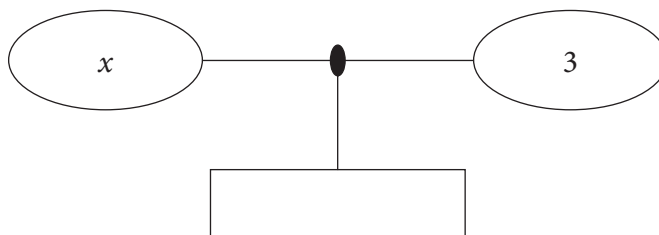## Becoming Undone

My Notes

> **MATH TIP**
>
> Check: $x + 2 = 5$
>
> $(3) + 2 \stackrel{?}{=} 5$
>
> $5 = 5$ ✓

The solution is $x = 3$. Check it as shown in the Math Tip.

You do not have to draw a scale to solve every equation. Just imagine that the equal sign in the equation is the center of a balance. Work through Question 1 to solve $w + 4 = 11$.

1. Fill in each circle with an operation and each box with a number so that you keep the scale balanced and isolate the w. Be sure to check your answer.

    $w + 4 = 11$

    $w + 4 \bigcirc \square = 11 \bigcirc \square$

    $w + \square = \square$

    $w = \square$

2. Solve the equation $y - 8 = 5$ and check your solution.

    $y - 8 = 5$

    $y - 8 \bigcirc \square = 5 \bigcirc \square$

    $y - \square = \square$

    $y = \square$

3. Compare the operations you used in Questions 1 and 2. Explain why you did or did not use the same operations.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Graphic Organizer, Create Representations, Work Backward
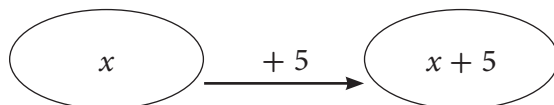
You can also use flowcharts to solve equations by putting each expression in the equation into a flowchart and then working backward to find the value of each variable.

> **EXAMPLE 4**
>
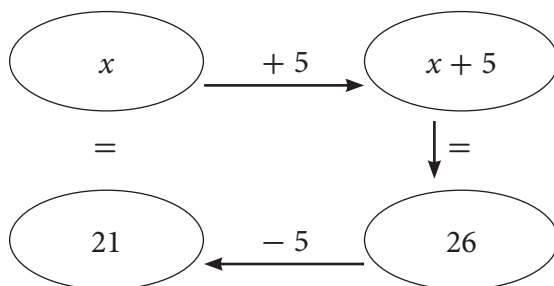> Solve the equation $x + 5 = 26$.
>
> If you know that the expression $x + 5$ equals 26, you can work backward to find the value of x that makes this true.
>
> *Step 1:* Start with the flowchart that you would use to evaluate the expression.
>
> $$\boxed{x} \xrightarrow{+5} \boxed{x+5}$$
>
> You already know that $x + 5 = 26$ so you can work backward to find x. To "undo" the "plus 5" you subtract 5.
>
> *Step 2:* Complete the flow chart to find the value of x.
>
> $$\begin{array}{ccc} x & \xrightarrow{+5} & x+5 \\ \Big\| = & & \Big\downarrow = \\ 21 & \xleftarrow{-5} & 26 \end{array}$$
>
> Solution: $x = \underline{\quad 21 \quad}$.
>
> To check, substitute this value back into the equation and see whether the equation is true.
>
> $$x + 5 \stackrel{?}{=} 26$$
> $$\underline{21} + 5 \stackrel{?}{=} 26$$
> $$\underline{21} = 26$$

TRY THESE C

Write your answers in the My Notes space. Show your work. Draw and use a flowchart to solve each equation. Check your solutions.

a. $x + 119 = 321$ **b.** $7.2x = 21.6$ **c.** $\frac{1}{3}b = 15$

My Notes

ACADEMIC VOCABULARY

Inverse operations are operations that "undo" each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

CONNECT TO AP

In AP mathematics, the notion of inverse operations is essential to understanding the relationship between the processes called differentiation and integration.

My Notes

Instead of using flowcharts, you can also solve equations algebraically.

EXAMPLE 5

Find the value of x if $x - 15$ equals 25.

Step 1:	*Write the problem.*	$x - 15 = 25$
Step 2:	*Add 15 to both sides to isolate x.*	$x - 15 + 15 = 25 + 15$
Solution:		$x = 40$

Solve and check $23n = 115$.

Step 1:	*Write the problem.*	$23n = 115$
Step 2:	*Divide by 23 to undo multiplying by 23.*	$\dfrac{23n}{23} = \dfrac{115}{23}$
Step 3:	*Simplify each side of the equation.*	$1(n) = 5$
Solution:	Use the multiplicative identity to isolate n.	$n = 5$

Check: $23n = 115$

$23 \times 5 \stackrel{?}{=} 115$

$115 = 115$

TRY THESE D

Solve each equation. Then check your solution.

a. $27x = 1377$ **b.** $3.72 = \dfrac{y}{1.5}$

c. $2.48 = 3.1z$ **d.** $\dfrac{3}{4}x = 6$

4. Your school's cheerleaders are raising money to buy new uniforms. Their goal is to earn $1257.50. So far, they have $625.76. Write and solve an equation to determine how much more the cheerleaders must earn to reach their goal.

5. Two thirds of the Patel family came to the family picnic. If 34 family members came to the picnic, how many people are in the Patel family?

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward

6. More people came to the Jackson family picnic than to the Patel family picnic. How many people could have come to the Jackson picnic?

An **inequality** is a mathematical statement that compares two quantities using $<$, $>$, $\geq$, or $\leq$.

7. Write an inequality that compares the number of people who came to the Jackson family picnic to the number of people who came to the Patel family picnic. Let the variable x represent the number of people who came to the Jackson family picnic.

You can use inverse operations to solve inequalities.

My Notes

EXAMPLE 6

Find the value of n if $n + 16$ is less than 57.

Step 1: Write the problem. $n + 16 < 57$
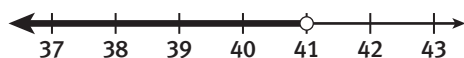
Step 2: Subtract 16 from both sides to isolate n. $n + 16 - 16 < 57 - 16$

Solution: $n < 41$

You can represent the solution $n < 41$ on a number line.

There is an open circle on 41 because it is not part of the solution. The ray to the left of 41 means all numbers less than 41. If the solution had been $n \leq 41$ the circle would be filled in.

You can check your solution by choosing any number less than 41 and substituting it in the original inequality. If the resulting statement is true, the solution checks. For example, choose 39 and substitute it for n.

$$n + 16 < 57$$
$$39 + 16 \overset{?}{<} 57$$
$$55 < 57$$

TRY THESE E

Solve and graph each inequality. Check your work.

a. $y - 28 > 42$ **b.** $x + 13 \leq 36$

MATH TIP

When an inequality has $>$ or $<$, use an open circle on the graph.

When an inequality has $\geq$ or $\leq$, use a closed circle on the graph.

EXAMPLE 7

Find the value of x if $\frac{x}{6}$ is greater than or equal to 8. Graph the solution.

Step 1: Write the problem. $\qquad\qquad\qquad\qquad\qquad$ $\frac{x}{6} \geq 8$

Step 2: Multiply both sides by 6 to undo dividing by 6. $\quad \frac{x}{6} \times 6 \geq 8 \times 6$

Solution: $\qquad\qquad\qquad\qquad\qquad$ $x \geq 48$

Check: Choose any number greater than or equal to 48, such as 54, and substitute it for x in the original inequality.

$$\frac{x}{6} \geq 8$$
$$\frac{54}{6} \overset{?}{\geq} 8$$
$$9 \geq 8$$

TRY THESE F

Solve and graph each inequality. Check your work.

a. $7x < 9.1$ $\qquad\qquad$ **b.** $\frac{3}{4}x > 27$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write a question that will help solve each equation. Then use mental math to solve.

   a. $43 - x = 35$

   b. $w + 9 = 17$

   c. $9p = 36$

2. Write an equation for each question. Solve the equation and check your solution.

   a. What number can you divide by 4 to get 6?

   b. What number is added to 13 to get 63?

3. Solve and check each equation.

   a. $w - 1.23 = 4.72$ $\qquad$ **b.** $\frac{7}{6} = y + \frac{2}{3}$

   c. $2.7x = 8.64$ $\qquad\qquad$ **d.** $29 = \frac{p}{14}$

   e. $\frac{2}{3}x = 18$

4. The Math Club wants to buy math history books for the library. The total cost is $105. So far they have raised $72. How much more do they need to buy the books?

   a. Write a verbal model.

   b. Use numbers and variables to write expressions.

   c. Write an equation.

   d. How much more money does the Math Club need to earn?

5. Solve and graph each inequality.

   a. $1.2x \leq 9.6$ $\qquad\qquad$ **b.** $y - 11 > 11$

   c. $w + 1.5 < 5$ $\qquad\qquad$ **d.** $\frac{n}{4} \geq 8$

6. **MATHEMATICAL REFLECTION** Why is it important to have a systematic way to solve equations rather than just relying on "mental math"?

Solving Two-Step Equations
Class Trip

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Work Backward, Think/Pair/Share

Jamie wants to go on a class trip during spring vacation. His teacher and some classmates will be biking and hiking through the state park. Jamie needs $498 to go on the trip. He received $168 for his birthday. If he uses his birthday money and saves $15 each week, when will Jamie have enough money for the trip?

**1.** Jamie decided to write an equation to model the situation.

  a. Write a verbal model.

  b. Use numbers and variables to write expressions.

  c. Write an equation.

In the previous activity, you solved equations by using a single inverse operation. Two-step equations can also be solved using a flowchart and working backwards.

When solving an equation such as $4x + 7 = 23$, think of x as the input and $4x + 7$ as the output. Start by making a flowchart to show the operations needed to go from the input to the output.

$$x \xrightarrow{\times 4} 4x \xrightarrow{+7} 4x + 7$$
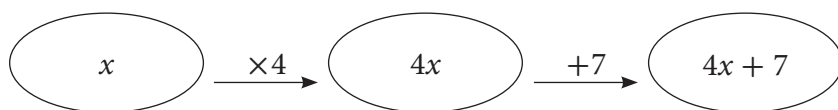
My Notes

> **READING MATH**
>
> $4x$ means "4 times x". In algebra, a multiplication sign is not used between a number and a variable.
>
> $3n$ is the same as "$3 \times n$" or $3(n)$. Not using the multiplication symbol helps avoid confusion between x and $\times$.

SUGGESTED LEARNING STRATEGIES: Work Backward, Create Representations

My Notes

To work backward, start from the output value and undo each operation using its inverse operation until you find the input. Since the original equation says that $4x + 7 = 23$, $4x + 7$ in the output oval can be replaced with 23.
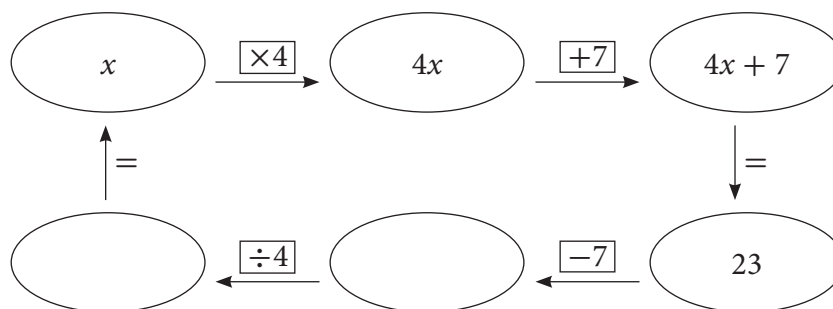
The last step to reach the output was adding 7. Subtract 7 to undo that step. Notice where -7 is placed in the flow chart. As you continue backward, the next step is multiplying by 4. Divide by 4 to undo that step. Notice where $\div 4$ is placed in the flow chart.

2. Complete the flow chart above. Start with 23 and use the inverse operations in the flow chart to fill in the last two ovals. What is the solution for x? Explain.

3. Use this flow chart with the backward method to solve the problem $\frac{x}{3} - 2 = 10$.
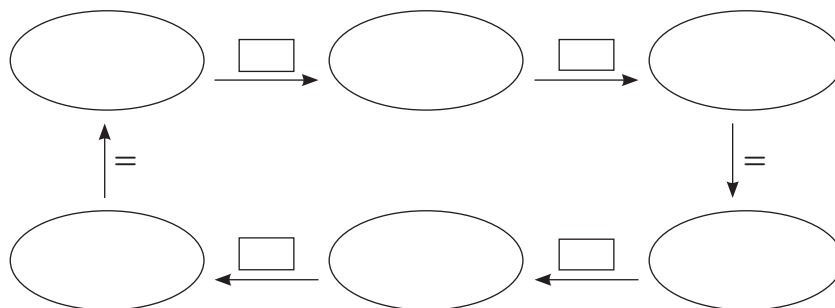
4. Now use the flowchart method to find a solution to the equation you wrote in Question1c. How long will it take Jamie to save up for the trip?

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward

5. Draw a flowchart and work backward to solve each of the following equations. Check your solutions.

a. $3x + 8 = 17$ **b.** $\dfrac{b}{5} - 2 = 13$

In addition to using flowcharts, you can also solve equations algebraically. An equation that requires two arithmetic operations to solve is called a *two-step equation*. Arithmetic operations are addition, subtraction, multiplication, and division.

EXAMPLE 1

Solve $4x + 7 = 23$.

Step 1: Subtract 7 from both sides and simplify.

$4x + 7 - 7 = 23 - 7$

$4x = 16$

Step 2: Divide each side by 4 and simplify.

$\dfrac{4x}{4} = \dfrac{16}{4}$

$1x = 4$

Solution:

$x = 4$

TRY THESE A

Solve these equations algebraically.

a. $3x + 5 = 35$ **b.** $20 = 3x + 8$

c. $\dfrac{x}{4} - 2 = 11$ **d.** $12 = \dfrac{x}{2} + 3$

Sometimes you may need to apply the distributive property to solve an equation.

$3(x + 2) = 12$

$3x + 6 = 12$

$3x + 6 - 6 = 12 - 6$

$\dfrac{3x}{3} = \dfrac{6}{3}$

$x = 2$

My Notes

6. Now that you have examined two methods for solving equations, compare and contrast them.

   a. Describe the ways in which solving equations algebraically is the same as working backward.

   b. Describe the ways in which solving algebraically is different from working backward.

Jamie wanted to choose clothes to take on the class trip. He looked up the average temperature for the state park during the time of year they were going on the trip. He found that the mean temperature at the park was 30° Celsius.

Jamie only knew temperature in degrees Fahrenheit, so he did not know whether this was hot or cold. He looked for a **formula** to convert Celsius to Fahrenheit temperatures and found one based on cricket chirps. The number of chirps that a cricket will make in a minute is dependent on the temperature in degrees Fahrenheit.

To use a formula, replace all variables that have given numbers and then solve the resulting equation for the remaining variable.

7. The formula using cricket chirps is $n = 4F - 128$, where n is the number of chirps per minute and F is the temperature in degrees Fahrenheit. If a cricket chirps 184 times in a minute, what is the temperature? Show your work.

MATH TERMS

A **formula** is an equation that shows a relationship between two or more quantities.

Some formulas are based on scientific standards, such as those used to convert between units of measure. An example is the formula for comparing Celsius and Fahrenheit temperatures.

Other formulas are based on observed measurements, such as counting the seconds between seeing lightning and hearing thunder to estimate how many thousand feet away the lightning struck.

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward, Quickwrite, Think/Pair/Share, Group Presentation, Close Reading, Marking the Text

Jamie also found two versions of another formula for converting temperature:

$$C = \frac{5}{9}F - \frac{160}{9} \qquad\qquad C = \frac{5}{9}(F - 32)$$

- The variable C represents temperature in degrees Celsius.

- The variable F represents temperature in degrees Fahrenheit.

Jamie decides to use the first version of the equation to convert the Celsius temperature to Fahrenheit. Both versions result in the same answer.

8. When Jamie replaced the C in the formula with the number 30, he obtained the following equation:

$$30 = \frac{5}{9}F - \frac{160}{9}$$

a. Solve the equation for F algebraically.

b. What kind of clothing should Jamie take? Explain.

Jamie's teacher told the students to leave their cell phones at home because there is no reception in the park. They can use a pay phone at the ranger's station, so Jamie will take a phone card with him.

9. Jamie's phone card is worth $2.80. To call home from the park there is $0.40 connection fee and its costs $0.12 each minute you talk.

a. Write an equation to determine how many minutes Jamie can talk.

b. Solve the equation and check your answer.

My Notes

CONNECT TO SOCIAL STUDIES

In many cultures the House Cricket is considered to be good luck. People believe that keeping one or more of these little insects in the house is sure to bring good fortune.

MATH TIP

- Write a verbal model for the problem.

- Use numbers and variables to write expressions.

- Write an equation.

- Find the answer.

My Notes

You can also use inverses to solve two-step inequalities.

EXAMPLE 2

Solve $\frac{x}{3} - 4 > 8$ and graph the solution.

Step 1: Add 4 to both sides and simplify. $\frac{x}{3} - 4 + 4 > 8 + 4$
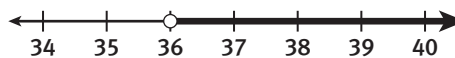
$\frac{x}{3} > 12$

Step 2: Multiply each side by 3 and simplify. $\frac{x}{3} \times 3 > 12 \times 3$

Solution: ◄——┼——┼——○——┼——┼——┼——► $x > 36$
                34   35   36   37   38   39   40

TRY THESE B

a. $2x + 6 > 20$ **b.** $5x - 6 \geq 14$

c. $\frac{x}{7} + 3 < 12$ **d.** $\frac{x}{3} - 5 \leq 10$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Work backward to solve the equation $8x - 9 = 15$ using a flow chart.

2. Work backward to solve the equation $\frac{n}{4} + 3 = 14$ using a flow chart.

3. Solve each equation or inequality. Check your solution.

   a. $12 = 5w - 3$ **b.** $7x + 5 = 19$

   c. $\frac{2}{3}y + 7 = 17$ **d.** $5.12 = \frac{n}{6.3} - 11.1$

   e. $8x - 7 < 25$ **f.** $\frac{4}{5}n + 3.5 \geq 19.5$

   g. $\frac{y}{3} + 5 \geq 24$ **h.** $1.2x - 6 > 18$

   i. $2(x + 3) = 16$ **j.** $28 = 4(x - 3)$

4. Graph the solutions of the inequalities in 3e and 3f.

5. A taxi ride costs $2.85 plus $0.35 for each quarter-mile. How many quarter miles can you go for $8.10?

6. The formula for finding the perimeter of a rectangle is $P = 2l + 2w$, where P is perimeter, l is length, and w is width. Find the length of a rectangle if its perimeter is 18 cm and its width is 6 cm.

7. **MATHEMATICAL REFLECTION** Describe how first learning the flowchart method of solving equations helped you understand the algebraic method.

Solving Linear Equations

FUNCTIONAL FORMULAS

Write your answers on notebook paper. Show your work.

1. Zena paid $5.18 at Quick Delivery to send a package to a friend. She paid a flat fee of $1.10 plus $0.17 for each ounce the package weighed.

   a. Write a verbal model.

   b. Write an equation.

   c. Solve the equation to find the weight of the package.

2. Scientists measure temperature in degrees Kelvin. The formula below can be used to convert between degrees Fahrenheit and degrees Kelvin.

$$F = \frac{9}{5}K - 459.67$$

   If the temperature of a gas is 71.33°F, what is the temperature of the gas in degrees Kelvin?

3. Kayla noticed that her pet cricket was chirping very fast. She knew from science class that she could figure out how fast he was chirping using the formula $\frac{n}{4} + 32 = F$, where n is the number of chirps per minute and F is the temperature in degrees Fahrenheit. If the temperature in the house is 72°F, how fast is Kayla's pet cricket chirping?

Solving Linear Equations

FUNCTIONAL FORMULAS

	Exemplary	Proficient	Emerging
Math Knowledge # 2, 3	The student uses the formula and substitutes the problem information for the correct variable to determine the solution (2 and 3).	The student attempts both problems but only one solution is complete and correct.	The student attempts at least one of the problems, but the solution may be incomplete or incorrect.
Problem Solving #1c	The student correctly solves the equation created in part 1b (1c).	The student correctly solves the equation created in page 1b (1c).	The student attempts to find a solution to the equation, but it is incomplete or incorrect.
Representation #1a, #1b	The student: • Writes an accurate verbal model that includes the problem information and appropriate operations (1a). • Writes a correct equation that includes appropriate variables, numbers and operations (1b).	The student writes both a verbal model and equation for the problem situation, but only one of the models is complete and correct.	The student writes either a verbal model or an equation, but it may be incomplete or incorrect.

ACTIVITY 3.1

1. Allister can mow two lawns (l) each day (d). Copy and complete the table.

Days	1	2	3	4
Lawns	2			

2. Use grid paper to graph the data. Place the number of lawns mowed on the vertical axis and the number of days on the horizontal axis. Remember to label the axes.

3. Should you connect the data points with a line? Explain.

4. Use your table and graph from Questions 1 and 2.

   a. Write an equation to represent Allister's data.

   b. How many days will it take Allister to mow 14 lawns? Show your calculations.

5. A car can travel 60 miles every hour.

   a. Write an equation to determine the distance d that a car can travel in t hours.

   b. How long will it take the car to travel 540 miles? Explain how you arrived at your answer.

   c. How far will the car travel in 12 hours? Show your work.

6. Joey can ride his skate board 50 feet every minute.

   a. Write an equation that determines his distance d in m minutes.

   b. Use your equation to tell how far Joey can ride his skate board in an hour.

7. Shelby walks at a constant rate. The equation that represents this relationship is $d = 4t$, where d is distance in miles and t is the time in hours.

   a. What does the coefficient 4 tell you about Shelby?

   b. What question is answered by the equation $d = 4 \times 32$?

8. Samari's dad owns a toy store and Samari helps him replenish the inventory on Saturdays. On a recent Saturday, Samari was asked to stack jigsaw puzzles on the shelf. One puzzle box has a height of 3 inches.

   a. Complete the table below.

Number of Puzzle Boxes	0	1	2	3	4	5
Height of Stack (in.)						

   b. Plot the data points from the table on the grid. Be sure to label and use an appropriate scale for each axis. Use the vertical axis to represent the height of the stack and the horizontal axis to represent the number of puzzle boxes.

   c. Write a rule for the height H of a stack of puzzle boxes that has b boxes.

   d. Use your rule in part (c) to determine the height of 32 puzzle boxes.

   e. If Samari's graph were large enough, it would contain the point (40, 120). Explain what the coordinates in this ordered pair mean.

   f. Should you connect the data points with a line? Explain why or why not.

ACTIVITY 3.2

Graph each equation. Make an input-output table for each one.

9. $y = \frac{1}{2}x - 4$

Input, x	Output, y
8	
10	
11	
15	

10. $y = 3x + 4$

11. $y = x - 3$

12. $y = 2.1x + 1$

ACTIVITY 3.3

13. Write a verbal question for each equation and then solve using mental math

   a. $\frac{63}{x} = 21$

   b. $29 - z = 21$

14. Write an equation for each question. Solve the equation to get the answer. Check your solution.

   a. What number must be multiplied to 7 to get 35?

   b. What number is subtracted from 12 to get 9?

15. Solve and check each equation or inequality.

   a. $9.36 = z + 3.78$

   b. $w - \frac{3}{4} = \frac{7}{12}$

   c. $24 = \frac{2}{5}x$

   d. $\frac{x}{2.8} = 25$

   e. $x - 3.5 < 9.3$

   f. $x + \frac{2}{7} > \frac{5}{7}$

   g. $\frac{x}{9} + 2 \leq 6$

16. The Recycling Club has a goal to recycle 2000 pounds of newspaper this year. They have already recycled 1585 pounds. How many more pounds do they need to recycle to meet their goal?

   a. Write a verbal model.

   b. Assign numbers and variables.

   c. Write an equation.

   d. How many more pounds do they need to recycle?

17. Mrs. Smith is having a graduation party for all of the eighth grade students in her school. She is making 120 cupcakes to serve at the party. She needs to buy trays to hold the cupcakes. Each tray will hold 24 cupcakes. How many trays will she have to buy?

   a. Write a verbal model.

   b. Assign numbers and variables.

   c. Write an equation.

   d. How many trays will she have to buy?

ACTIVITY 3.4

18. Solve the equation $3x + 2 = 65$. Work backward and use a flow chart.

19. Solve and check each equation or inequality. Graph the solutions of the inequalities.

   a. $106 = 6x - 8$

   b. $\frac{n}{5} + 10 = 21$

   c. $0.13w + 5 = 6.17$

   d. $3x + 7 > 13$

   e. $\frac{5}{8}x < 10$

   f. $2x - 3.7 \geq 9.5$

20. The body surface area of an infant weighing between 3 and 30 kg is dependent on its weight. The formula is $B = 1321 + 0.34W$, where B is the infant's body surface area in square centimeters and W is the infant's weight in kilograms. If an infant's body surface area is 1326.1 square centimeters, how much does the infant weigh in kilograms?

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

   - Why are tables, graphs, and equations useful for representing relationships?

   - How can you use equations to solve real-world problems?

Academic Vocabulary

2. Look at the following academic vocabulary words:

   - inverse operations
   - rate of change
   - linear
   - variable
   - ordered pair

   Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

   a. What will you do to address each weakness?

   b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

1. Which equation describes the relationship shown in this table?

x	y
−1	3
0	1
1	−1
2	−3

A. $y = -x + 2$ **C.** $y = -2x + 1$
B. $y = -x - 2$ **D.** $y = -2x - 1$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. What is the value of x in the equation $7\frac{1}{2} = x + 2\frac{1}{2}$?

2.

**Read
Solve
Explain**

3. Carlisle has a 1200-word book report due in five days. So far, she has written 500 words of the report. She wants to develop a writing plan to know how many words she needs to write each day to complete her assignment on time.

Part A: Write an equation to determine how many words she needs to write each day to finish the book report in 5 days. Explain why the equation represents the situation.
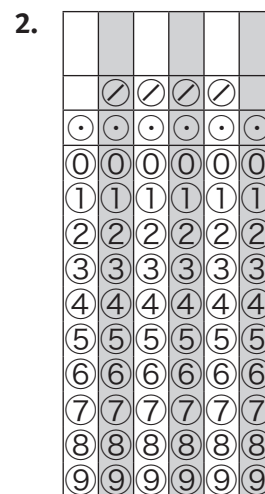
Solve and Explain

_____

_____

_____

_____

_____

Part B: How many words does she need to write each day? Show how you found your answer.

Solve and Explain

_____

_____

Read

Solve

Explain
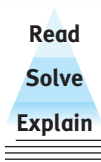
4. The function table shows a relationship between x and y.

x	y
0	0
1	3
2	6
3	

Part A: Complete the function table.

Part B: Graph this function.
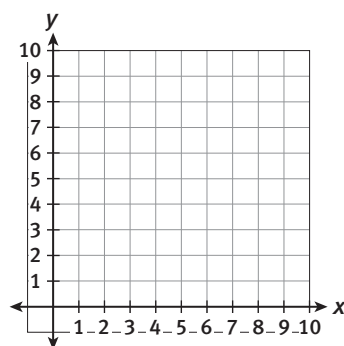
Part C: The function you just graphed is $y = 3x$. On the same grid, graph the function, $y = \frac{x}{3}$. Then describe how the two graphs are similar and different.

Solve and Explain

_____

_____

_____

_____

_____

_____

_____

_____