

# Westside

## *Algebra 2 PreAP*

Name \_\_\_\_\_ Period \_\_\_\_\_

### **IMPORTANT INSTRUCTIONS FOR STUDENTS!!!**

**We understand that students come to Algebra II with different strengths and needs. For this reason, students have options for completing the packet and getting assistance!**

- Students should try to answer all the questions; you must show all work.
- Khan Academy video tutorials may be very helpful to you. HISD has aligned Khan Academy with Algebra I, are available by clicking this link:  
[http://www.houstonisd.org/cms/lib2/TX01001591/Centricity/Domain/8050/Khan\\_Acad\\_Video\\_Algmt\\_Al1.pdf](http://www.houstonisd.org/cms/lib2/TX01001591/Centricity/Domain/8050/Khan_Acad_Video_Algmt_Al1.pdf)
- Finally, honor and integrity is at the heart of a Westside Wolf! Smart wolves never cheat. You are only hurting yourself by attempting to copy someone else's work. This packet is to help you be ready for Algebra II, and help your teachers know what you can do.
- Need face-to-face help with packet? Go to tutorials the first week of school!
- There will be an assessment over this material on or before 2 week of school.
- All student that are not newly enrolled must have their summer packets complete by the end of the first week of school.
- Students who enrolled at Westside High School on or after the first day of school, must submit their summer packet within two weeks of their enrollment date at Westside.

**Now! Get Ready, Get Set, and Do Your Best!**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Algebra 2 Pre-AP Summer Packet Due: Friday August 26, 2016****Solving Equations**

|  |  |
|--|--|
| <b>Addition Property of Equality</b>       | For any numbers a, b, and c, if $a = b$ , then $a + c = b + c$ .                               |
| <b>Subtraction Property of Equality</b>    | For any numbers a, b, and c, if $a = b$ , then $a - c = b - c$ .                               |
| <b>Multiplication Property of Equality</b> | For any numbers a, b, and c, if $a = b$ , then $ac = bc$ .                                     |
| <b>Division Property of Equality</b>       | For any numbers a, b, and c, with $c \neq 0$ , if $a = b$ , then $\frac{a}{c} = \frac{b}{c}$ . |

**Example 1**Solve:  $3\frac{1}{2}p = 1\frac{1}{2}$ 

$$\frac{7}{2}p = \frac{3}{2}$$

Original equation

Rewrite each mixed number as an improper fraction

$$2\left(\frac{7}{2}\right) \quad \left(\frac{3}{2}\right)2$$

$$7\left(\frac{p}{2}\right) = \left(\frac{3}{2}\right)2$$

Multiply each side by the reciprocal of  $7/2$ .

$$p = \frac{3}{7}$$

Simplify

$$\text{Check: } 3\frac{1}{2}\left(\frac{3}{7}\right) = 1\frac{1}{2}$$

$$\frac{7}{2}\left(\frac{3}{7}\right) = \frac{3}{2}$$

Substitute solution for variable

Rewrite each mixed number as an improper fraction

$$\frac{3}{2} = \frac{3}{2}$$

Left Hand Side = Right Hand Side

LHS = RHS correct

**Example 2**Solve:  $-5n = 60$ 

$$\frac{-5n}{-5} = \frac{60}{-5}$$

Original equation

Divide both sides by -5 or multiply both sides by  $-1/5$ 

$$n = -12$$

Simplify

$$\text{Check: } -5(-12) = 60$$

$$60 = 60$$

Substitute solution for variable  
Left Hand Side = Right Hand Side

LHS = RHS correct

### Solving Equations with the Variable on Each Side

To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to write an equivalent equation that has the variable on just one side of the equation. Then use the Multiplication or Division Property of Equality to solve the equation.

#### **Example 1**

**Solve:**  $5y - 8 = 3y + 12$

$$5y - 8 - 3y = 3y + 12 - 3y$$

$$2y - 8 = 12$$

$$2y - 8 + 8 = 12 + 8$$

$$2y = 20$$

$$\frac{2y}{2} = \frac{20}{2}$$

$$y = 10$$

**Check:**  $5y - 8 = 3y + 12$

$$5(10) - 8 = 3(10) + 12$$

$$50 - 8 = 30 + 12$$

$$42 = 42$$

$$\text{LHS} = \text{RHS} \quad \text{correct}$$

#### **Example 2**

**Solve:**  $-11 - 3y = 8y + 1$

$$-11 - 3y + 3y = 8y + 1 + 3y$$

$$-11 = 11y + 1$$

$$-11 - 1 = 11y + 1 - 1$$

$$-12 = 11y$$

$$\frac{-12}{11} = \frac{11y}{11}$$

$$\frac{-12}{11} = y$$

$$y = -\frac{12}{11}$$

$$-1 \frac{1}{11} = y$$

**Check:**  $-11 - 3v = 8v + 1$

$$-11 - 3\left(-\frac{12}{11}\right) = 8\left(-\frac{12}{11}\right) + 1$$

$$-11 + \frac{36}{11} = -\frac{96}{11} + 1$$

$$-\frac{121}{11} + \frac{36}{11} = -\frac{96}{11} + \frac{11}{11}$$

$$-\frac{85}{11} = -\frac{85}{11}$$

#### **Example 3**

**Solve:**  $4(2a - 1) = -10(a - 5)$

$$8a - 4 = -10a + 50$$

$$8a - 4 + 10a = -10a + 50 + 10a$$

$$18a - 4 = 50$$

$$18a - 4 + 4 = 50 + 4$$

$$18a = 54$$

$$\frac{18a}{18} = \frac{54}{18}$$

$$a = 3$$

**Check:**  $4(2a - 1) = -10(a - 5)$

$$4(2(3) - 1) = -10(3 - 5)$$

$$4(6 - 1) = -10(-2)$$

$$4(5) = 20$$

$$20 = 20$$

$$\text{LHS} = \text{RHS} \quad \text{correct}$$

**Exercises: Solve each equation. Then check your solution.**

1.  $20 = y - 8$

2.  $w - \frac{1}{2} = \frac{5}{8}$

3.  $-17 = b + 4$

4.  $\frac{h}{3} = -2$

5.  $\frac{1}{8}m = 6$

6.  $\frac{1}{5}p = \frac{3}{5}$

7.  $3h = -42$

8.  $-\frac{1}{2}m = 16$

9.  $-3t = 51$

10.  $6 - b = 5b + 30$

11.  $5y - 2y = 3y + 2$

12.  $5x + 2 = 2x - 10$

13.  $4n - 8 = 3n + 2$

14.  $1.2x + 4.3 = 2.1 - x$

15.  $4.4s + 6.2 = 8.8s - 1.8$

16.  $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$

17.  $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$

18.  $8 - 5p = 4p - 1$

19.  $-3(x + 5) = 3(x - 1)$

20.  $2(7 + 3t) = -t$

21.  $3(a + 1) - 5 = 3a - 2$

## Solving Equations and Formulas

Solve for variables: sometimes you may want to solve an equation such as  $V = lwh$  for one of its variables. For example, if you know the values of  $V$ ,  $w$ , and  $h$ , then the equation  $l = \frac{V}{wh}$  is more useful for finding the value of  $l$ .

**Example 1** Solve  $2x - 4y = 8$  for  $y$ .

$$\begin{aligned} 2x - 4y &= 8 \\ 2x - 4y - 2x &= 8 - 2x \\ -4y &= 8 - 2x \\ \frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\ y &= \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4} \end{aligned}$$

**Example 2** Solve  $3m - n = km - 8$

$$\begin{aligned} 3m - n &= km - 8 \\ 3m - n - km &= km - 8 - km \\ 3m - n - km &= -8 \\ 3m - n - km + n &= -8 + n \\ 3m - km &= -8 + n \\ m(3 - k) &= -8 + n \\ \frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \end{aligned}$$

$$m = \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}$$

Since division by 0 is undefined,  $3 - k \neq 0$ , or  $k \neq 3$ .

**Exercises:** Solve each equation or formula for the variable specified.

22.  $ax - b = c$  for  $x$

23.  $15x + 1 = y$  for  $x$

24.  $(x + f) + 2 = j$  for  $x$

25.  $xy + z = 9$  for  $y$

26.  $x(4 - k) = p$  for  $k$

27.  $7x + 3y = m$  for  $y$

28.  $xy + xz = 6 + a$  for  $x$

## Describe Number Patterns

**Write Equations:** Sometimes a pattern can lead to a general rule that can be written as an equation.

**Example:** Suppose you purchased a number of packages of blank CDs. If each package contains 3 CDs, you could make a chart to show the relationship between the number of packages of compact disks and the number of disks purchased. Use  $x$  for the number of packages and  $y$  for the number of compact disks.

Make a table of ordered pairs for several points of the graph.

|                    |   |   |   |    |    |
|--------------------|---|---|---|----|----|
| Number of packages | 1 | 2 | 3 | 4  | 5  |
| Number of CDs      | 3 | 6 | 9 | 12 | 15 |

The difference in the  $x$  values is 1, and the difference in the  $y$  values is 3. This pattern shows that  $y$  is always three times  $x$ . This suggests the relation  $y = 3x$ . Since the relation is also a function, we can write the equation in functional notation as  $f(x) = 3x$ .

**Exercises:**

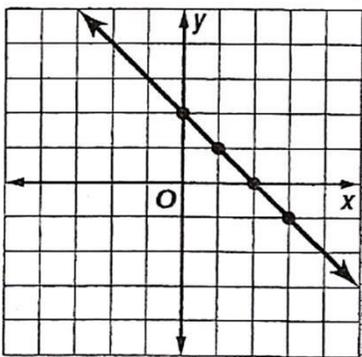
29. Write an equation for the function in functional notation. Then complete the table.

|     |    |   |   |   |   |
|-----|----|---|---|---|---|
| $x$ | -1 | 0 | 1 | 2 | 3 |
| $y$ | -2 | 2 | 6 |   |   |

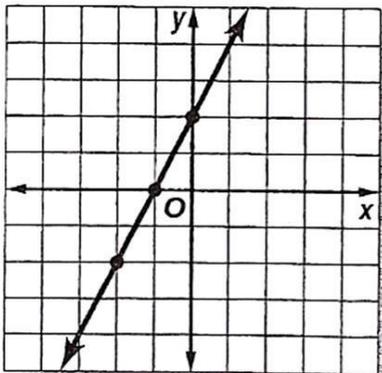
30. Write an equation for the function in functional notation. Then complete the table.

|     |    |    |   |   |   |
|-----|----|----|---|---|---|
| $x$ | -2 | -1 | 0 | 1 | 2 |
| $y$ | 10 | 7  | 4 |   |   |

31. Write an equation in functional notation.



32. Write an equation in functional notation.



## Equations of Linear Function

|                      |   |
|----------------------|---|
| Standard Form        | $Ax + By = C$   |
| Slope-Intercept Form | $y = mx + b$ , where $m$ is the given slope and $b$ is the $y$ -intercept                 |
| Point-Slope Form     | $y - y_1 = m(x - x_1)$ , where $m$ is the given slope and $(x_1, y_1)$ is the given point |

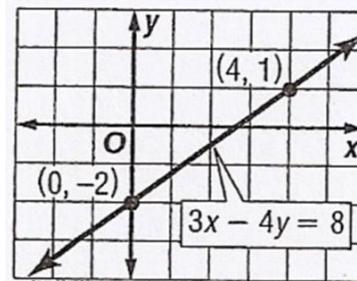
**Example 1:** Write an equation of a line in standard form whose slope is  $-4$  and whose  $y$ -intercept is  $3$ .

$$\begin{aligned}
 y &= mx + b \\
 y &= -4x + 3 \\
 +4x & \quad +4x \\
 4x + y &= 3
 \end{aligned}$$

**Example 2:** Graph  $3x - 4y = 8$

$$\begin{aligned}
 3x - 4y &= 8 \\
 -4y &= -3x + 8 \\
 \frac{-4y}{-4} &= \frac{-3x + 8}{-4} \\
 y &= \frac{3}{4}x - 2
 \end{aligned}$$

Original equation  
 Subtract  $3x$  from each side  
 Divide each side by  $-4$   
 Simplify



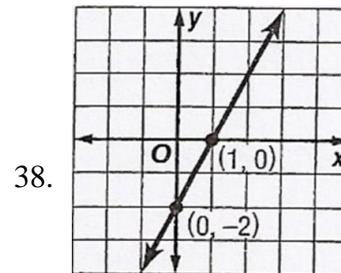
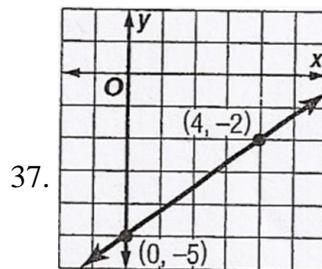
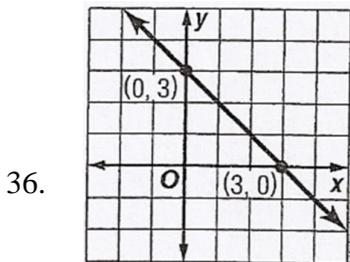
The  $y$ -intercept of  $y = \frac{3}{4}x - 2$  is  $-2$  and the slope is  $\frac{3}{4}$ . So graph the point  $(0, -2)$ . From this point, move up 3 units and right 4 units. Draw a line passing through both points.

**Exercises:**

Write an equation of the line in Standard Form with the given information.

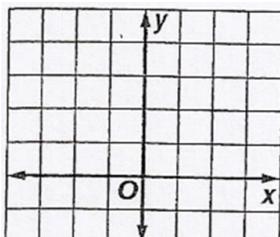
33. Slope:  $8$ ,  $y$ -intercept  $-3$       34. Slope:  $-2$ , point  $(5, 3)$       35. Slope:  $-1$ ,  $y$ -intercept  $-7$

Write an equation of the line in Standard Form in each graph.

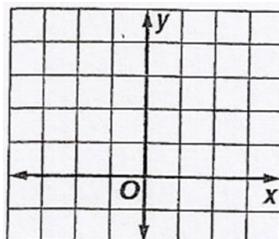


Graph each equation.

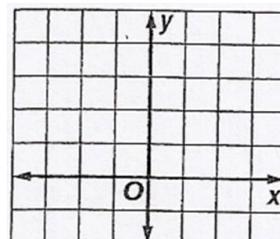
39.  $2x - y = -1$



40.  $3x + y = 2$



41.  $x + y = -1$



## Graphing Systems of Equations

**Solve by Graphing** One method of solving a system of equations is to graph the equations on the same coordinate plane.

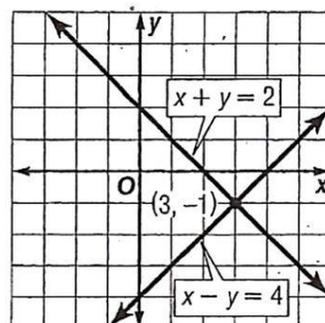
**Example:** Graph each system of equations. Then determine whether the system has *no solution*, *one solution*, or *infinitely many solutions*. If the system has one solution, name it.

a.  $x + y = 2$   
 $x - y = 4$

The graphs intersect. Therefore, there is one solution. The point  $(3, -1)$  seems to lie on both lines. Check this estimate by replacing  $x$  with 3 and  $y$  with -1 in each equation.

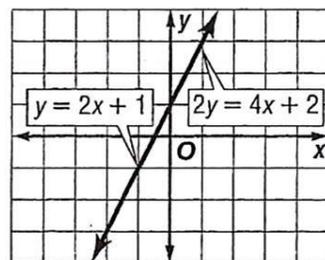
$$\begin{aligned} x + y &= 2 \\ 3 + (-1) &= 2 \quad \checkmark \\ x - y &= 4 \\ 3 - (-1) &= 3 + 1 \text{ or } 4 \quad \checkmark \end{aligned}$$

The solution is  $(3, -1)$ .



b.  $y = 2x + 1$   
 $2y = 4x + 2$

The graphs coincide. Therefore there are infinitely many solutions.



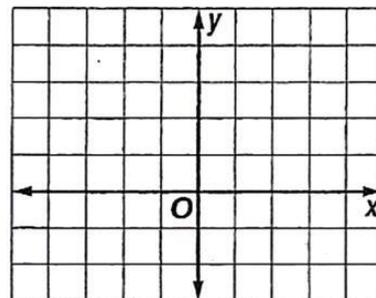
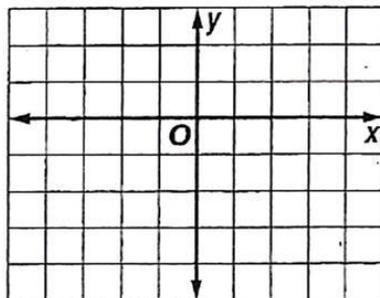
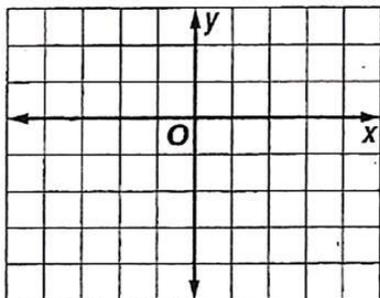
**Exercises:**

**Graph each system of equations. Then determine whether the system has *no solution*, *one solution*, or *infinitely many solutions*. If the system has one solution, name it.**

42.  $y = -2$   
 $3x - y = -1$

43.  $x = 2$   
 $2x + y = 1$

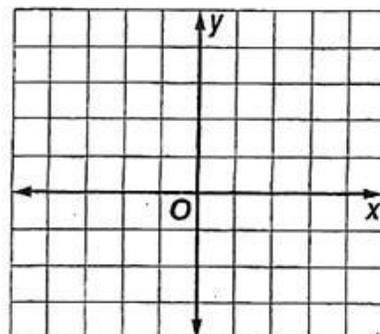
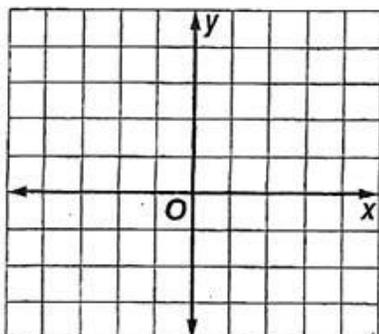
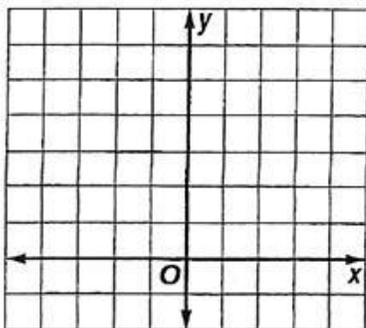
44.  $y = \frac{1}{2}x$   
 $x + y = 3$



45.  $2x + y = 6$   
 $2x - y = -2$

46.  $3x + 2y = 6$   
 $3x + 2y = -4$

47.  $2y = -4x + 4$   
 $y = -2x + 2$



## Solving Systems of Equations by Substitution

**Example 1:** use substitution to solve they system of equations.

$$y = 2x$$

$$4x - y = -4$$

Substitute  $2x$  for  $y$  in the second equation.

$$4x - y = -4 \quad \text{second equation}$$

$$4x - 2x = -4 \quad y = 2x$$

$$2x = -4 \quad \text{combine like terms}$$

$$x = -2 \quad \text{Divide each side by 2 and simplify.}$$

Use  $y = 2x$  to find the value of  $y$ .

$$y = 2x \quad \text{First equation}$$

$$y = 2(-2) \quad x = -2$$

$$y = -4 \quad \text{simplify}$$

The solution is  $(-2, -4)$ .

**Example 2:** Solve for one variable, then substitute.

$$x + 3y = 7$$

$$2x - 4y = -6$$

Solve the first equation for  $x$  since the coefficient of  $x$  is 1.

$$x + 3y = 7 \quad \text{First equation}$$

$$x + 3y - 3y = 7 - 3y \quad \text{Subtract } 3y \text{ from each side}$$

$$x = 7 - 3y \quad \text{Simplify}$$

Find the value of  $y$  by substituting  $7 - 3y$  for  $x$  in the second equation.

$$2x - 4y = -6 \quad \text{Second equation}$$

$$2(7 - 3y) - 4y = -6 \quad x = 7 - 3y$$

$$14 - 6y - 4y = -6 \quad \text{Distributive Property}$$

$$14 - 10y = -6 \quad \text{Combine like terms.}$$

$$14 - 10y - 14 = -6 - 14 \quad \text{Subtract 14 from each side.}$$

$$-10y = -20 \quad \text{Simplify.}$$

$$y = 2 \quad \text{Divide each side by -10 and simplify.}$$

Use  $y = 2$  to find the value of  $x$ .

$$x = 7 - 3y$$

$$x = 7 - 3(2)$$

$$x = 1$$

The solution is  $(1, 2)$ .

**Exercises:** Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has *no solution* or *infinitely many solutions*.

48.  $y = 4x$   
 $3x - y = 1$

49.  $x = 2y$   
 $y = x - 2$

50.  $x = 2y - 3$   
 $x = 2y + 4$

## Elimination Using Addition and Subtraction

**Elimination Using Addition:** In systems of equations in which the coefficients of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called elimination.

**Example 1:** Use addition to solve the system of equations

$$x - 3y = 7$$

$$3x + 3y = 9$$

Write the equations in column form and add to eliminate y.

$$\begin{array}{r} x - 3y = 7 \\ (+) 3x + 3y = 9 \\ \hline 4x = 16 \\ \underline{4x} = \underline{16} \\ 4 \quad 4 \\ x = 4 \end{array}$$

Substitute 4 for x either equation and solve for y.

$$\begin{array}{r} 4x - 3y = 7 \\ 4 - 3y - 4 = 7 - 4 \\ \underline{-3y = 3} \\ -3 \quad -3 \\ y = -1 \end{array}$$

The solution is (4, -1).

**Example 2:** The sum of two numbers is 70 and their difference is 24. Find the numbers.

Let x represent one number and y represent the other number.

$$\begin{array}{r} x + y = 70 \\ (+) x - y = 24 \\ \hline 2x = 94 \\ \underline{2x} = \underline{94} \\ 2 \quad 2 \\ x = 47 \end{array}$$

Substitute 47 for x in either equation.

$$\begin{array}{r} 47 + y = 70 \\ 47 + y - 47 = 70 - 47 \\ y = 23 \end{array}$$

The numbers are 47 and 23.

**Exercises:** Use elimination to solve each system of equations.

51.  $x + y = -4$   
 $x - y = 2$

52.  $2m - 3n = 14$   
 $m + 3n = -11$

53.  $3a - b = -9$   
 $-3a - 2b = 0$

## Multiplying a Polynomial by a Monomial

**Product of Monomial and Polynomial:** The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

**Example 1: Find  $-3x^2(4x^2 + 6x - 8)$ .**

$$\begin{aligned} & -3x^2(4x^2 + 6x - 8) \\ &= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8) \\ &= -12x^4 + (-18x^3) - (-24x^2) \\ &= -12x^4 - 18x^3 + 24x^2 \end{aligned}$$

**Example 2: Simplify  $-2(4x^2 + 5x) - x(x^2 + 6x)$**

$$\begin{aligned} & -2(4x^2 + 5x) - x(x^2 + 6x) \\ &= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x) \\ &= -8x^2 + (-10x) + (-x^3) + (-6x^2) \\ &= (-x^3) + [-8x^2 + (-6x^2)] + (-10x) \\ &= -x^3 - 14x^2 - 10x \end{aligned}$$

**Exercises: Find each product.**

54.  $x(5x + x^2)$

55.  $x(4x^2 + 3x + 2)$

56.  $-2xy(2y + 4x^2)$

57.  $-2g(g^2 - 2g + 2)$

58.  $3x(x^4 + x^3 + x^2)$

59.  $-4x(2x^3 - 2x + 3)$

## Factoring Using the Greatest Common Factor

**Example 1: Use GCF to factor  $12mn + 80m^2$**

Find the GCF of  $12mn$  and  $80m^2$

$$12mn = 2 \cdot 2 \cdot 3 \cdot m \cdot n$$

$$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$$

$$\text{GCF} = 2 \cdot 2 \cdot m \text{ or } 4m$$

Write each term as the product of the GCF and its remaining factors.

$$12mn + 80m^2 = 4m(3 \cdot n) + 4m(2 \cdot 2 \cdot 5 \cdot m)$$

$$= 4m(3n) + 4m(20m)$$

$$= 4m(3n + 20m)$$

$$12mn + 80m^2 = 4m(3n + 20m)$$

**Example 2: Factor  $6ax + 3ay + 2bx + by$  by grouping.**

$$6ax + 3ay + 2bx + by$$

$$= (6ax + 3ay) + (2bx + by)$$

$$= 3a(2x + y) + b(2x + y)$$

$$= (3a + b)(2x + y)$$

Check using the FOIL method.

$$(3a + b)(2x + y)$$

$$= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y)$$

$$= 6ax + 3ay + 2bx + by$$

**Exercises: Factor each polynomial.**

60.  $24x + 48y$

61.  $30mn^2 + m^2n - 6n$

62.  $q^4 - 18q^3 + 22q$

63.  $9x^2 - 3x$

64.  $4m + 6n - 8mn$

65.  $45s^3 - 15s^2$

66.  $14c^3 - 42c^5 - 49c^4$

67.  $55p^2 - 11p^4 + 44p^5$

68.  $14y^3 - 28y^2 + y$

## Multiplying Polynomials

**Multiply Binomials:** To multiply two binomials, you can apply the Distributive Property twice. You can use FOIL (First, Outer, Inner and Last) method.

**Example 1:** Find  $(x + 3)(x - 4)$

$$\begin{aligned}(x + 3)(x - 4) &= x(x - 4) + 3(x - 4) \\ &= (x)(x) + x(-4) + 3(x) + 3(-4) \\ &= x^2 - 4x + 3x - 12 \\ &= x^2 - x - 12\end{aligned}$$

**Example 2:** Find  $(x - 2)(x + 5)$  using FOIL method.

$$\begin{aligned}(x - 2)(x + 5) & \quad \text{First Outer Inner Last} \\ &= (x)(x) + (x)(5) + (-2)(x) + (-2)(5) \\ &= x^2 + 5x + (-2x) - 10 \\ &= x^2 + 3x - 10\end{aligned}$$

**Exercises: Find each product.**

69.  $(x + 2)(x + 3)$

70.  $(x - 4)(x + 1)$

71.  $(x - 6)(x - 2)$

72.  $(p - 4)(p + 2)$

73.  $(y + 5)(y + 2)$

74.  $(2x - 1)(x + 5)$

75.  $(3n - 4)(3n - 4)$

76.  $(8m - 2)(8m + 2)$

77.  $(k + 4)(5k - 1)$

## Factoring Trinomials: $x^2 + bx + c$

**Factor  $x^2 + bx + c$  :** To factor a trinomial of the form  $x^2 + bx + c$ , find two integers  $m$  and  $n$ , whose sum is equal to  $b$  and whose product is equal to  $c$ .

**Example 1: Factor each trinomial.**

a.  $x^2 + 7x + 10$

In this trinomial,  $b = 7$  and  $c = 10$ .

| Factors of 10 | Sum of Factors |
|---------------|----------------|
| 1, 10         | 11             |
| 2, 5          | 7              |

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

b.  $x^2 - 8x + 7$

In this trinomial,  $b = -8$  and  $c = 7$ .

Notice that  $m + n$  is negative and  $mn$  is positive, so  $m$  and  $n$  are both negative.

Since  $-7 + (-1) = -8$  and  $(-7)(-1) = 7$ ,  $m = -7$  and  $n = -1$ .

$$x^2 - 8x + 7 = (x - 7)(x - 1)$$

**Example 2: Factor  $x^2 + 6x - 16$**

In this trinomial,  $b = 6$  and  $c = -16$ . This means  $m + n$  is positive and  $mn$  is negative. Make a list of the factors of  $-16$ , where one factor of each pair is positive.

| Factors of -16 | Sum of Factors |
|----------------|----------------|
| 1, -16         | -15            |
| -1, 16         | 15             |
| 2, -8          | -6             |
| -2, 8          | 6              |

Therefore,  $m = -2$  and  $n = 8$ .

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

**Exercises: Factor each trinomial.**

78.  $x^2 + 4x + 3$

79.  $m^2 + 12m + 32$

80.  $r^2 - 3r + 2$

81.  $x^2 - x - 6$

82.  $x^2 - 4x - 21$

83.  $x^2 - 22x + 121$

84.  $c^2 - 4c - 12$

85.  $p^2 - 16p + 64$

86.  $9 - 10x + x^2$

87.  $x^2 + 6x + 5$

88.  $a^2 + 8a - 9$

89.  $y^2 - 7y - 8$

90.  $x^2 - 2x - 3$

91.  $y^2 + 14y + 13$

92.  $m^2 + 9m + 20$

## Factoring Trinomials: $ax^2 + bx + c$

**Factor  $ax^2 + bx + c$  :** To factor a trinomial of the form  $ax^2 + bx + c$ , find two integers  $m$  and  $n$ , whose sum is equal to  $b$  and whose product is equal to  $ac$ . If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

### **Example 1: Factor $2x^2 + 15x + 18$ .**

In this example,  $a = 2$ ,  $b = 15$ , and  $c = 18$ . You need to find two numbers whose sum is 15 and whose product is  $2 \cdot 18$  or 36. Make a list of the factors of 36 and look for the pair of factors whose sum is 15.

| Factors of 36 | Sum of Factors |
|---------------|----------------|
| 1, 36         | 37             |
| 2, 18         | 20             |
| 3, 12         | 15             |

Use the pattern  $ax^2 + mx + nx + c$  with  $a = 2$ ,  $m = 3$ ,  $n = 12$  and  $c = 18$ .

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

### **Example 2: Factor $3x^2 - 3x - 18$**

Note that the GCF of the terms  $3x^2$ ,  $3x$ , and 18 is 3. First factor out this GCF.

$$3x^2 - 3x - 18 = 3(x^2 - x - 6).$$

Now factor  $x^2 - x - 6$ . Since  $a = 1$ , find the two factors of  $-6$  whose sum is  $-1$ .

| Factors of -6 | Sum of Factors |
|---------------|----------------|
| 1, -6         | -5             |
| -1, 6         | 5              |
| -2, 3         | 1              |
| 2, -3         | -1             |

Now use the pattern  $(x + m)(x + n)$  with  $m = 2$  and  $n = -3$ .

$$x^2 - x - 6 = (x + 2)(x - 3)$$

The complete factorization is

$$3x^2 - 3x - 18 = 3(x + 2)(x - 3).$$

**Exercises: Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.**

93.  $2x^2 - 3x - 2$

94.  $3m^2 - 8m - 3$

95.  $16r^2 - 8r + 1$

96.  $6x^2 + 5x - 6$

97.  $3x^2 + 2x - 8$

98.  $18x^2 - 27x - 5$

99.  $2a^2 + 5a + 3$

100.  $18x^2 + 9x - 5$

101.  $4c^2 + 19c + 21$