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Calculus Exam – 2014

This exam consists of 27 questions. Write your answers for each question on the answer sheet. Fractions and radicals should be given in lowest terms.

The work shown on your exam *may* be used to help settle tie breakers.

DO NOT detach the answer sheet from your exam.

Calculators are not permitted on this exam.

Good Luck!

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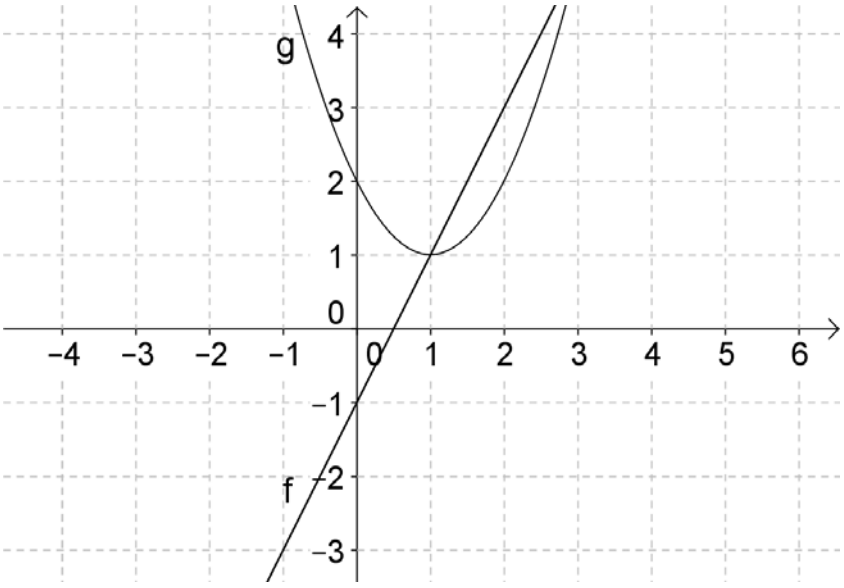
1. The functions f and g are graphed below. Evaluate: $\lim_{x\to 1}\dfrac{f(x)-g(x)}{\sin(2x-2)}=$



2. $f(x)=\dfrac{3x^4-2x+1}{\ln(x)+2}$. Evaluate: $\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1}=$

3. The functions f and g are differentiable. The normal line to the graph of f at $x=1$ is given by $3x+2y=1$, and the tangent line to the graph of g at $x=1$ is given by $2x+7y=3$. Define $H(x)=\dfrac{f(x)}{g(x)}$. Evaluate: $H'(1)=$

4. Use linear approximation to approximate the value of $\sqrt{123}$ from a guess of 11.

5. The average value of the function $f(x)=x^2+x$ on the interval $[0,a]$ is 1. Give the value of a.

6. Give the absolute maximum value of the function $f(x)=\dfrac{3}{2+x+|x|}+\dfrac{3}{2+|x-6|}$.

7. Evaluate: $\int_0^{\pi}\dfrac{(x-2)\sin(x)}{1+\cos^2(x)}dx=$

8. Give the area of the region bounded between the graphs of $x=y^2$, $y=x-2$ and $y=-2$.

9. Give the volume generated when the region bounded between the graphs of $f(x)=x^2$ and $g(x)=x+2$ is rotated around the line $x=-2$.

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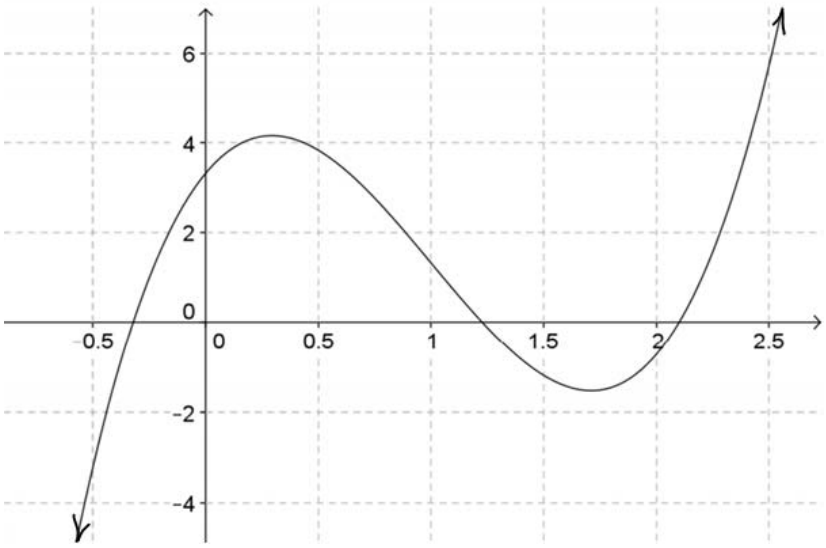
10. Give a comma separated list of the items below that are true. In each case, $f(x)$ is defined for all values of x.

A. If f is differentiable and invertible, and $ax+by=c$ is a tangent line to the graph of f, then $bx+ay=c$ is a tangent line to the graph of f^{-1}.

B. If $f'(x)>0$ when $x\neq 0$, then f is increasing.

C. The graph of $f'(x)$ is shown below and $f(0)=1$. $f(x)$ is never negative.



(Note: This graph only pertains to part C.)

D. If $f'(x)>0$ for all $x>2$, then $\lim_{x\to\infty} f(x)=\infty$.

E. If $\lim_{h\to 0}\dfrac{f(1+h)-f(1-h)}{2h}=0$ then $f'(1)=0$.

F. There is a continuous function $f(x)$ so that $\lim_{x\to\infty} f(x)=0$ and $\sum_{n=1}^{\infty} f(n)(-1)^n$ diverges.

G. The series $\sum_{n=1}^{\infty}\dfrac{n^3-7n^2+4n-7}{2n^4+6n^3-5n+3}(-1)^n$ converges.

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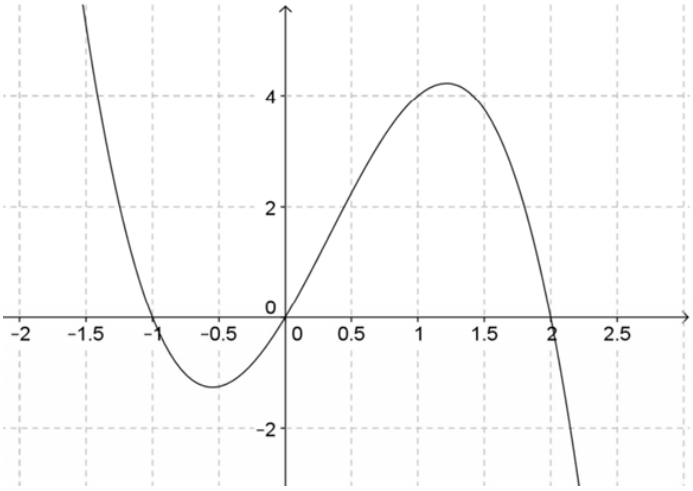
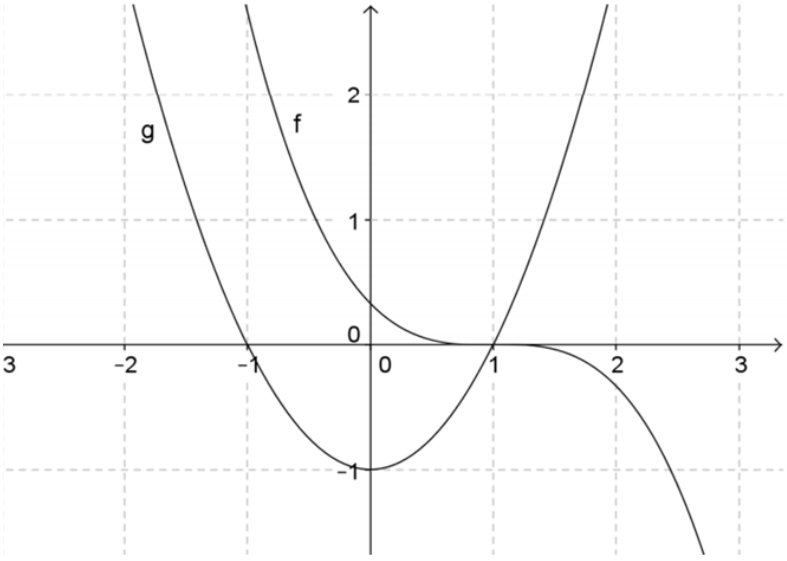
11. The graph of $f'(x)$ is shown below. Give the value of x where the absolute maximum value of f occurs.



12. The graphs of $f(x)$ and $g(x)$ are given below. Give the vertical asymptote(s) of $H(x)=\dfrac{f(x)}{g(x)}$.



13. Evaluate: $\lim_{x\to\infty}\left[\left(2x^4+3x^3\right)^{1/4}-\left(2x^4-x^3\right)^{1/4}+2x\arctan(x)\sin\left(\dfrac{1}{x}\right)\right]=$

14. Evaluate: $\sum_{n=1}^{\infty}\dfrac{n^2-2}{2^n}(-1)^n=$

15. Give the area of the polar region in the fourth quadrant that is bounded between the graphs of $r=2-2\cos(\theta)$ and $r=2\cos(\theta)$.

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16. The function $f(x)$ solves the initial value problem $\frac{dy}{dx} = 2y(3-y),\ y(0)=1.$

Evaluate: $\lim_{x\to\infty} f(x) =$

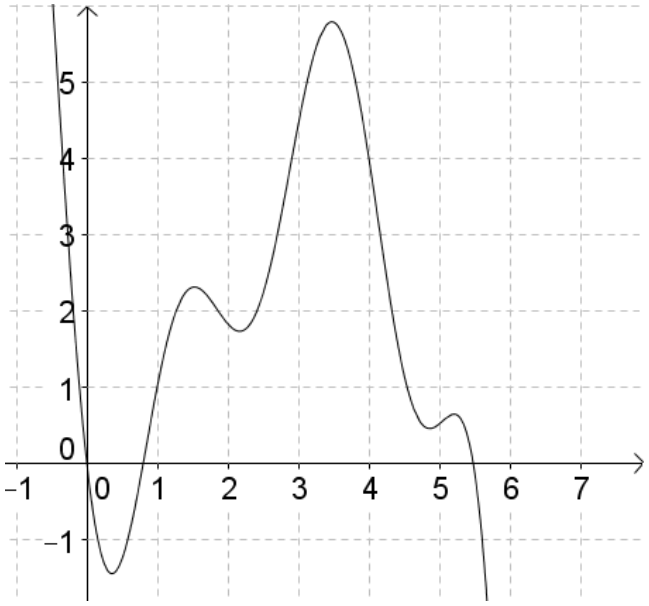
17. a and b are real numbers and $f(x) = \begin{cases} ax^2 + 3\ln(x-2), & x > 3 \\ 2x - \frac{3b}{x-4}, & x \le 3 \end{cases}$ is a differentiable function. Evaluate: $a + b =$

18. f and g are differentiable functions, and $\frac{1}{f(x+y)} + g(xy) = 3$. Also, $f(2) = \frac{1}{2}$, $g(1) = 1$, $f'(2) = 3$ and $g'(1) = 2$. Give an approximate value of x where the graph of $\frac{1}{f(x+y)} + g(xy) = 3$ crosses the x-axis.

19. $\int_{2x}^{1} f(u^2+1)du = e^x - 3x + 2$. Evaluate: $f(5) + f(1) =$

20. A 500 gallon tank is full of pure water at time $t = 0$, and a mixture of salt water that contains 2 grams of salt per gallon is poured into the tank at the rate of 5 gallons per minute. Assume the water in the tank and the mixture poured into the tank mix instantly, and the excess spills out of the tank. Give the total amount of salt in the tank one hour later.

21. The graph of $f(x)$ is shown below. Give the number of values that satisfy the conclusion of the mean value theorem for derivatives on the interval [1,4].



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22. Solve the inequality $\frac{1}{3-x}+\frac{3x}{x^2-4}\le 0$.

23. Let $P(x)$ be the 4th degree Taylor polynomial centered at 0 for $f(x)=\sin(3x)-\exp(x^2)+\cos(x^2)+3x^3-1$. Evaluate: $P(1)=$

24. Suppose a is a real number and $f(x)=x^2+a$. Let L_f be the lower Riemann sum of f on the interval [-1,1] with respect to the partition $\left\{-1,-\frac{1}{2},0,\frac{1}{2},1\right\}$. Find the value of a so that $L_f=3$.

25. $f(x)=\begin{cases} x\sin\left(\frac{\pi}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$. Give the absolute maximum value of $f(x)$ on the interval [0,4].

26. Evaluate: $\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^{3n}=$

27. Give the radius of the right circular cone of maximum volume that is inscribed in a sphere of radius 1.