<u>Define the number “e”</u>

<u>Euler’s number</u>

The **number *e*** is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n \qquad (2)$$

approaches as $n \to \infty$. In calculus, this is expressed using limit notation as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Table 6

n	$\frac{1}{n}$	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^n$
1	1	2	2
2	0.5	1.5	2.25
5	0.2	1.2	2.48832
10	0.1	1.1	2.59374246
100	0.01	1.01	2.704813829
1,000	0.001	1.001	2.716923932
10,000	0.0001	1.0001	2.718145927
100,000	0.00001	1.00001	2.718268237
1,000,000	0.000001	1.000001	2.718280469
1,000,000,000	10^{-9}	$1 + 10^{-9}$	2.718281827

e (Euler's Number)

The number ***e*** is a famous irrational number, and is one of the most important numbers in mathematics.

The first few digits are:

2.7182818284590452353602874713527 (and more ...)

*It is often called **Euler's number** after Leonhard Euler.*

And Euler is spoken like "Oiler".

e is the base of the Natural Logarithms (invented by John Napier).

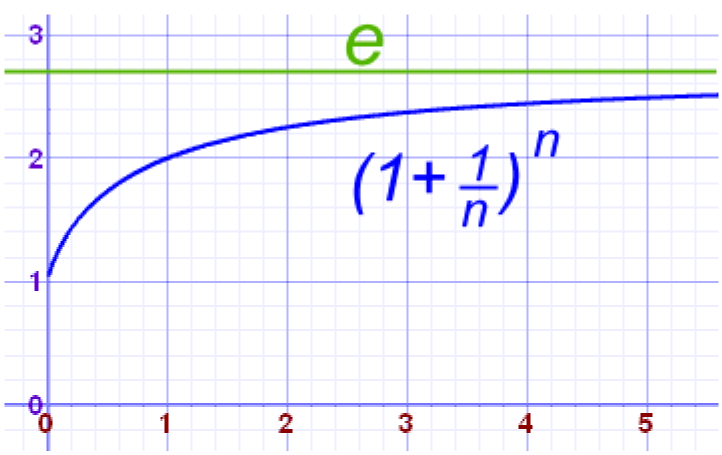
e is found in many interesting areas, so it is worth learning about.

Calculating

There are many ways of calculating the value of ***e***, but none of them ever give an exact answer, because ***e*** is irrational (not the ratio of two integers).

But it **is** known to over 1 trillion digits of accuracy!

For example, the value of $(1 + 1/n)^n$ approaches ***e*** as n gets bigger and bigger:



n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

Another Calculation

The value of ***e*** is also equal to 1/0! + 1/1! + 1/2! + 1/3! + 1/4! + 1/5! + 1/6! + 1/7! + ... (etc)

(Note: "!" means factorial)

The first few terms add up to: 1 + 1 + 1/2 + 1/6 + 1/24 + 1/120 = 2.718055556

And you can try that yourself at Sigma Calculator.

Remembering

Or you can remember the curious pattern that after the "2.7" the number "1828" appears TWICE:

2.7 1828 1828

And following THAT are the angles 45°, 90°, 45° in a Right-Angled Isosceles (two equal angles) Triangle:

2.7 1828 1828 45 90 45

(An instant way to seem really smart!)

Advanced: Use of *e* in Compound Interest

Often the number ***e*** appears in unexpected places.

For example, ***e*** is used in Continuous Compounding (for loans and investments):

$$e^r - 1$$

Formula for *Continuous* Compounding

Why does that happen?

Well, the formula for Periodic Compounding is:

$$FV = PV\,(1+r/n)^n$$

where **FV** = Future Value
PV = Present Value
r = annual interest rate (as a decimal)
n = number of periods

But what happens when the number of periods heads to infinity?

The answer lies in the similarity between:

$(1+r/n)^n$	and	$(1 + 1/n)^n$
Compounding Formula		***e*** (as n approaches infinity)

By substituting **x = n/r** :

- **r/n** becomes **1/x** and
- **n** becomes **xr**

And so:

$(1+r/n)^n$ becomes $(1+(1/x))^{xr}$

Which is **just like** the formula for ***e*** *(as n approaches infinity)*, with an extra **r** as an exponent.

So, as **x** goes to **infinity**, then $(1+(1/x))^{xr}$ goes to e^r

And that is why ***e*** makes an appearance in interest calculations!

Transcendental

e is also a transcendental number.