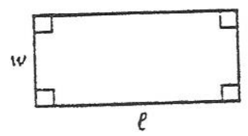
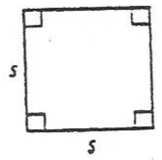
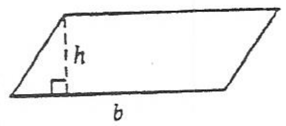
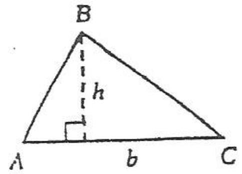
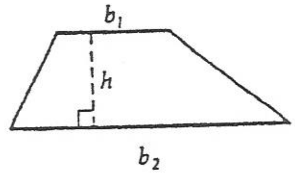
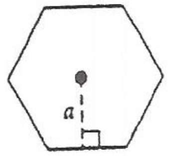
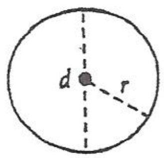
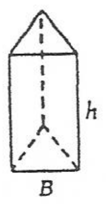
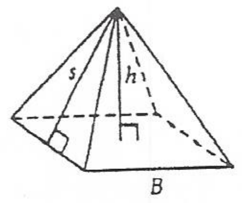
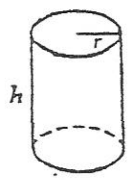
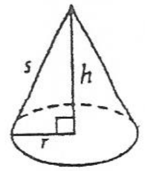
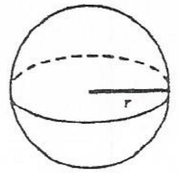
GEOMETRIC FORMULAS

 **Rectangle** Area: $A = \ell w$ Perimeter: $p = 2\ell + 2w$	 **Square** Area: $A = s^2$ Perimeter: $p = 4s$	 **Parallelogram** Area: $A = bh$
 **Triangle** Area: $A = \frac{1}{2}bh$ $m\angle A + m\angle B + m\angle C = 180°$	 **Trapezoid** Area: $A = \frac{1}{2}h(b_1 + b_2)$	 **Regular Polygon** Area: $A = \frac{1}{2}ap$
 **Circle** Area: $A = \pi r^2$ Circumference: $C = \pi d = 2\pi r$	 **Right Prism** Volume: $V = Bh$ Lateral Area: $LA = ph$ Surface Area: $SA = ph + 2B$	 **Regular Pyramid** Volume: $V = \frac{1}{3}Bh$ Lateral Area: $LA = \frac{1}{2}ps$ Surface Area: $SA = \frac{1}{2}ps + B$
 **Right Cylinder** Volume: $V = \pi r^2 h$ Lateral Area: $LA = 2\pi rh$ Surface Area: $SA = 2\pi rh + 2\pi r^2$	 **Right Cone** Volume: $V = \frac{1}{3}\pi r^2 h$ Lateral Area: $LA = \pi rs$ Surface Area: $SA = \pi rs + \pi r^2$	 **Sphere** Volume: $V = \frac{4}{3}\pi r^3$ Surface Area: $SA = 4\pi r^2$

$\sec\theta = \frac{r}{x} = \frac{hyp}{adj}$

$\cot\theta = \frac{x}{y} = \frac{adj}{opp}$

$\tan\theta = \frac{y}{x} = \frac{opp}{adj}$

$\csc\theta = \frac{r}{y} = \frac{hyp}{opp}$

$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$

$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$

distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

30-60-90 rt. Δ

shorter leg = $\frac{hyp}{2}$

longer leg = $\sqrt{3}$ (shorter leg)

45-45-90 rt. Δ

hyp = $\sqrt{2}$ (leg)

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

FORMULAS FROM GEOMETRY

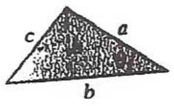
Triangle

$h = a \sin \theta$

Area $= \frac{1}{2}bh$

(Law of Cosines)

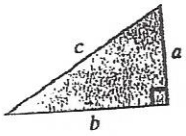
$c^2 = a^2 + b^2 - 2ab \cos \theta$



Right Triangle

(Pythagorean Theorem)

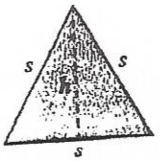
$c^2 = a^2 + b^2$



Equilateral Triangle

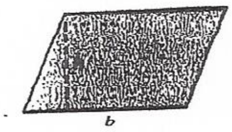
$h = \frac{\sqrt{3}s}{2}$

Area $= \frac{\sqrt{3}s^2}{4}$



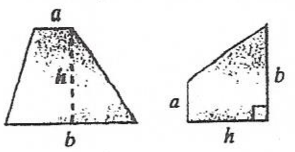
Parallelogram

Area $= bh$



Trapezoid

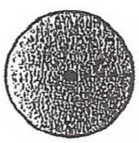
Area $= \frac{h}{2}(a + b)$



Circle

Area $= \pi r^2$

Circumference $= 2\pi r$

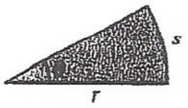


Sector of Circle

(θ in radians)

Area $= \frac{\theta r^2}{2}$

$s = r\theta$



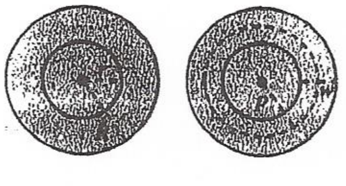
Circular Ring

(p = average radius,

w = width of ring)

Area $= \pi(R^2 - r^2)$

$= 2\pi pw$



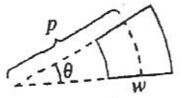
Sector of Circular Ring

(p = average radius,

w = width of ring,

θ in radians)

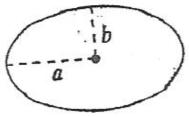
Area $= \theta pw$



Ellipse

Area $= \pi ab$

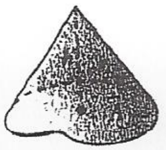
Circumference $\approx 2\pi\sqrt{\frac{a^2 + b^2}{2}}$



Cone

(A = area of base)

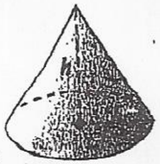
Volume $= \frac{Ah}{3}$



Right Circular Cone

Volume $= \frac{\pi r^2 h}{3}$

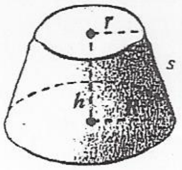
Lateral Surface Area $= \pi r\sqrt{r^2 + h^2}$



Frustum of Right Circular Cone

Volume $= \frac{\pi(r^2 + rR + R^2)h}{3}$

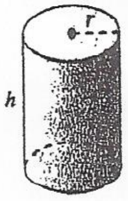
Lateral Surface Area $= \pi s(R + r)$



Right Circular Cylinder

Volume $= \pi r^2 h$

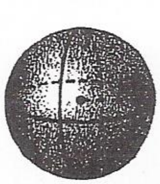
Lateral Surface Area $= 2\pi rh$



Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

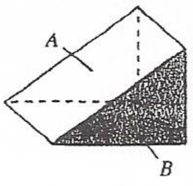


Wedge

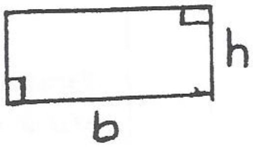
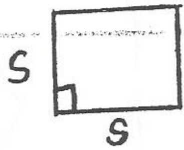
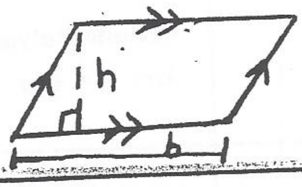
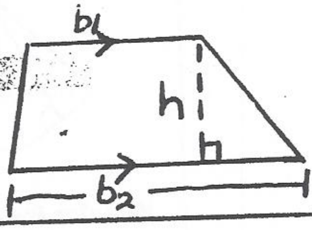
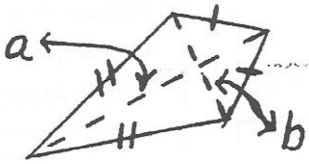
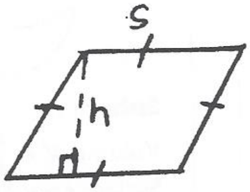
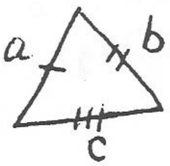
(A = area of upper face,

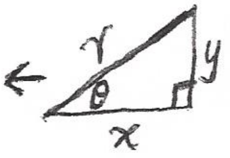
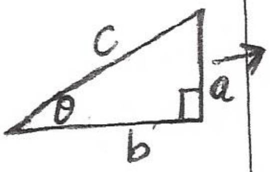
B = area of base)

$A = B \sec \theta$



Areas Of Quads

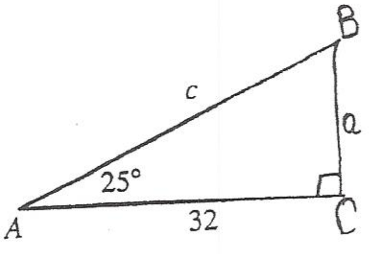
NAME	Drawing	FORMULAS	Description
Rectangle		$A=bh$	
Square		$A=S^2$	
Parallelogram		$A=bh$	
Trapezoid		$A=\frac{1}{2}(b_1+b_2)h$	
Kite		$A=\frac{ab}{2}$	
Rhombus		$A=Sh$ or $A=\frac{ab}{2}$	
* HERON'S FORMULA *	 $S=\frac{1}{2}(a+b+c)$	$A=\sqrt{s(s-a)(s-b)(s-c)}$	

Trigonometric Functions	Right Triangle Ratios ($0° < \theta < 90°$)
$\sin\theta = \frac{y}{r}$	$\sin\theta = \frac{a}{c} = \frac{\text{length of leg opposite }\theta}{\text{length of hypotenuse}}$
$\cos\theta = \frac{x}{r}$ 	$\cos\theta = \frac{b}{c} = \frac{\text{length of leg adjacent to }\theta}{\text{length of hypotenuse}}$
$\tan\theta = \frac{y}{x}$	$\tan\theta = \frac{a}{b} = \frac{\text{length of leg opposite }\theta}{\text{length of leg adjacent to }\theta}$
$\csc\theta = \frac{r}{y}$ 	$\csc\theta = \frac{c}{a} = \frac{\text{length of hypotenuse}}{\text{length of leg opposite }\theta}$
$\sec\theta = \frac{r}{x}$	$\sec\theta = \frac{c}{b} = \frac{\text{length of hypotenuse}}{\text{length of leg adjacent to }\theta}$
$\cot\theta = \frac{x}{y}$	$\cot\theta = \frac{b}{a} = \frac{\text{length of leg adjacent to }\theta}{\text{length of leg opposite }\theta}$

Capital letters are usually used to represent the angles of triangles, or their measures. Lowercase letters refer to the sides opposite their respective angles, or to their measures. The right triangle ratios can be used to *solve* a right triangle, that is, to find the unknown measures of the sides and angles.

EXAMPLE 1 **Solve right triangle ABC if $b = 32$, $\angle A = 25°$, and $\angle C = 90°$. Fin and c to the nearest unit.**

To find a, use tan 25°.



$\tan 25° = \frac{a}{32}$ *$\tan A = \frac{a}{b}$*

$a = 32 \tan 25°$ *Calculation-ready form*

$a = 15$ *To the nearest unit*

To find c, use cos 25°.

$\cos 25° = \frac{32}{c}$ *$\cos A = \frac{b}{c}$*

$c = \frac{32}{\cos 25°} = 35$ *To the nearest unit*

Since angles A and B are complementary, $\angle B = 90° - 25° = 65°$.

A **significant digit** is any nonzero digit or any zero that serves a purpose other than to locate the decimal point. Consider the following examples:

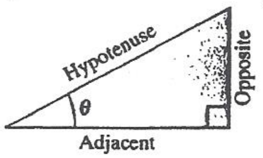
0.00304	Three significant digits
29.40	Four significant digits

*Keep For Pre-Calculus

TRIGONOMETRY

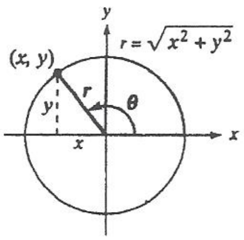
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.



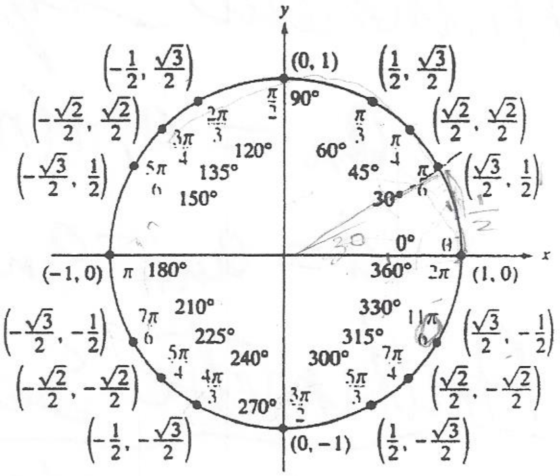
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Unit Circle



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$
$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$
$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$
$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$
$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$
$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

FINITE DIFFERENCE RULE

1. QUADRATIC EXPRESSION (QUADRATIC PATTERNS)

$$an^2 + bn + c$$

TERM	VALUE	D1	D2
0	c=		
1	6	a + b=	a=(D2)/2
2	28		a=(D2)/2
3	65		a=(D2)/2
4	117		a=(D2)/2
5	184		a=(D2)/2
6	266		a=(D2)/2
n			

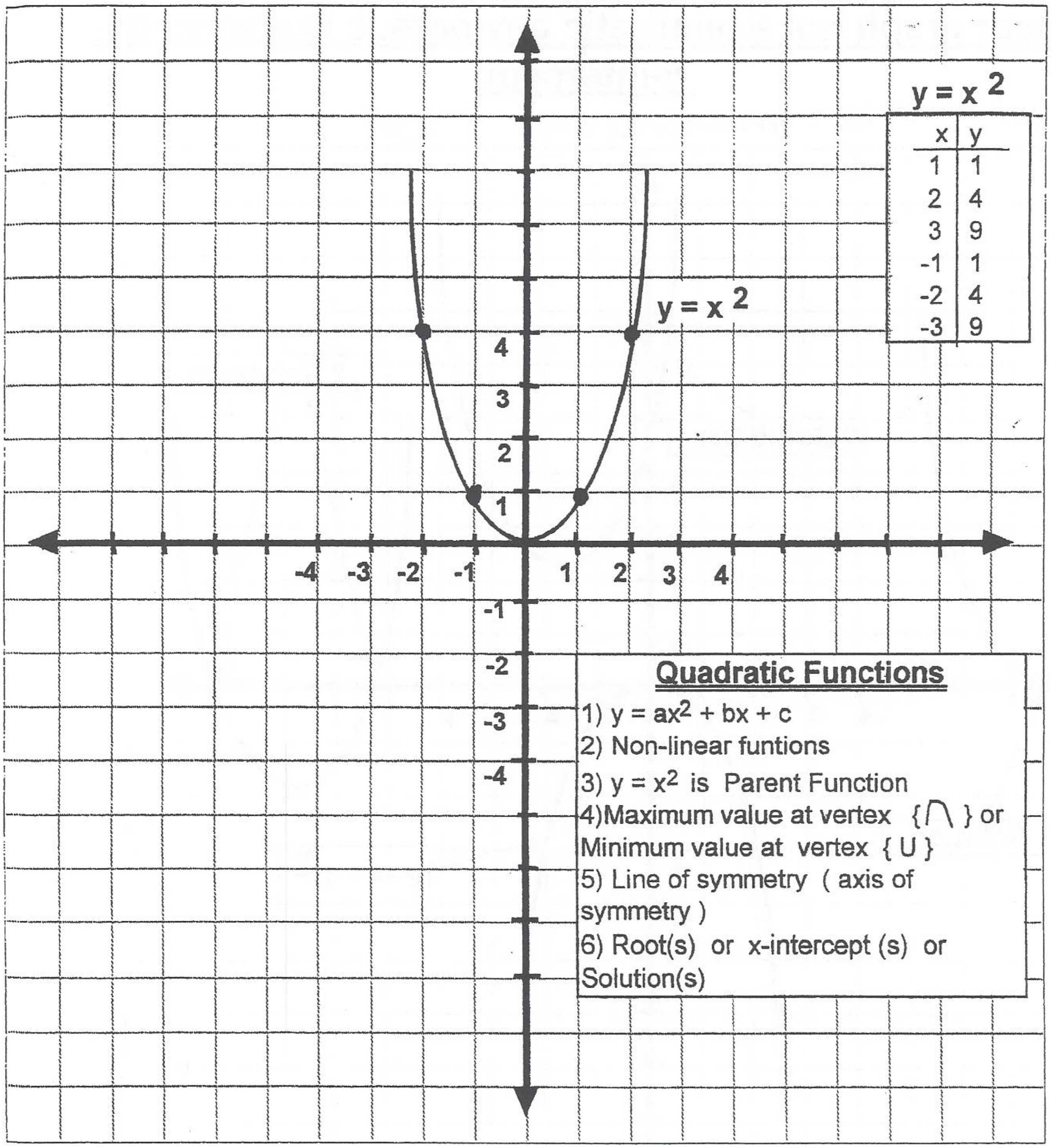
2.LINEAR EXPRESSION (LINEAR PATTERNS)

$$an + b$$

TERM	VALUE	D1	
0	b =		
1	-1	a= D1	
2	-6	a= D1	
3	-11	a= D1	
4	-16	a= D1	
5	-21	a= D1	
6	-26	a= D1	
n			

;QUADRATIC PARENT FUNCTION :

NAME: ________________________________ ; **PERIOD:** ______ .



$y = x^2$

x	y
1	1
2	4
3	9
-1	1
-2	4
-3	9

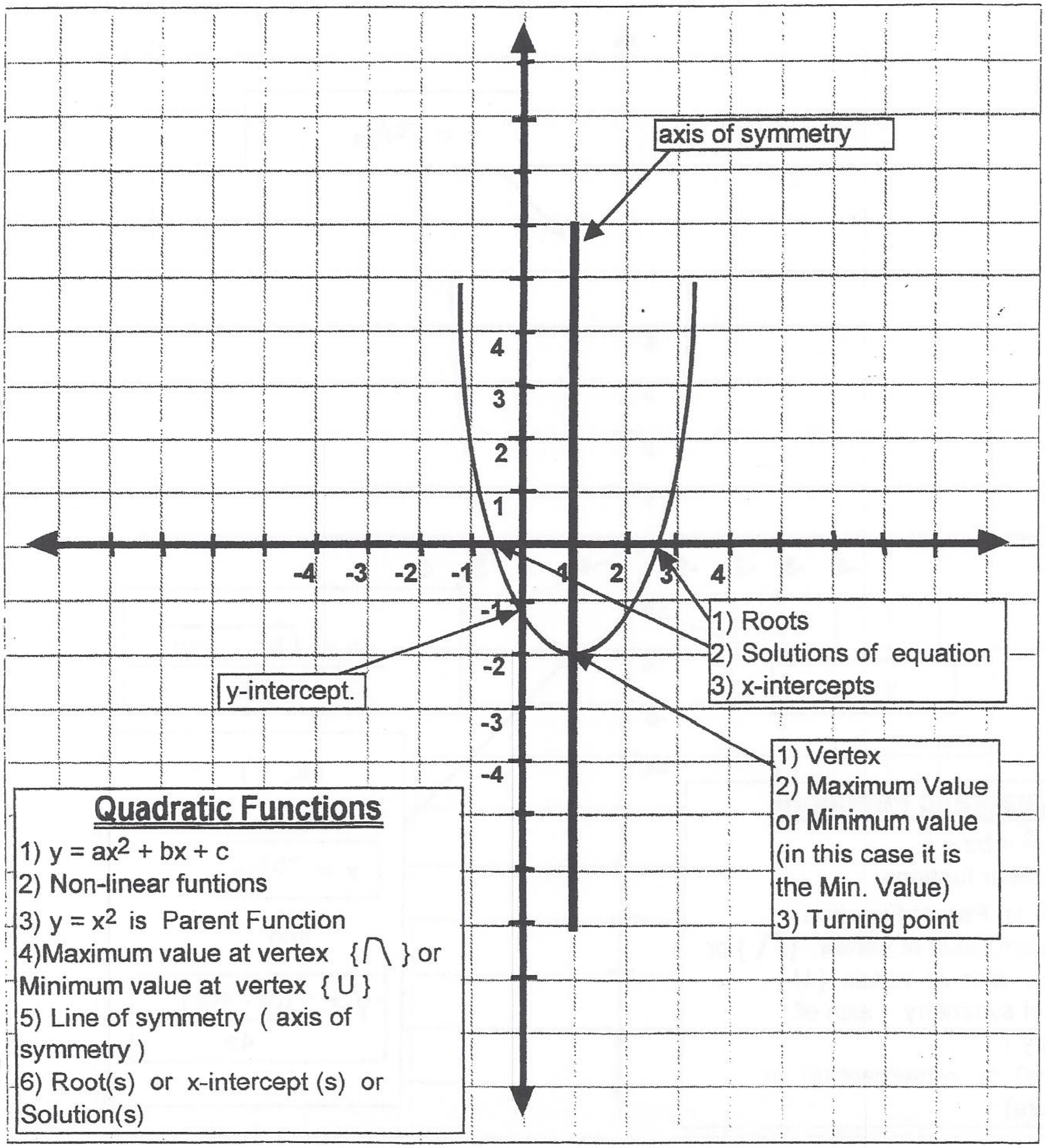
Quadratic Functions

1) $y = ax^2 + bx + c$
2) Non-linear funtions
3) $y = x^2$ is Parent Function
4)Maximum value at vertex {∩ } or Minimum value at vertex { U }
5) Line of symmetry (axis of symmetry)
6) Root(s) or x-intercept (s) or Solution(s)

;QUADRATIC FUNCTION GRAPH COMPONENTS:

NAME: $y = ax^2 + bx + c$;

If $a < 0$, then you have a Maximum Value graph. If $a > 0$, Then you have Minimum Value graph. If $a = 0$, then you have a linear graph not a quadratic anymore. c is always the y-intercept.



Quadratic Functions

1) $y = ax^2 + bx + c$
2) Non-linear funtions
3) $y = x^2$ is Parent Function
4)Maximum value at vertex {∩} or Minimum value at vertex { U }
5) Line of symmetry (axis of symmetry)
6) Root(s) or x-intercept (s) or Solution(s)

;QUADRATIC GRAPH COMPONENTS EQUATIONS:

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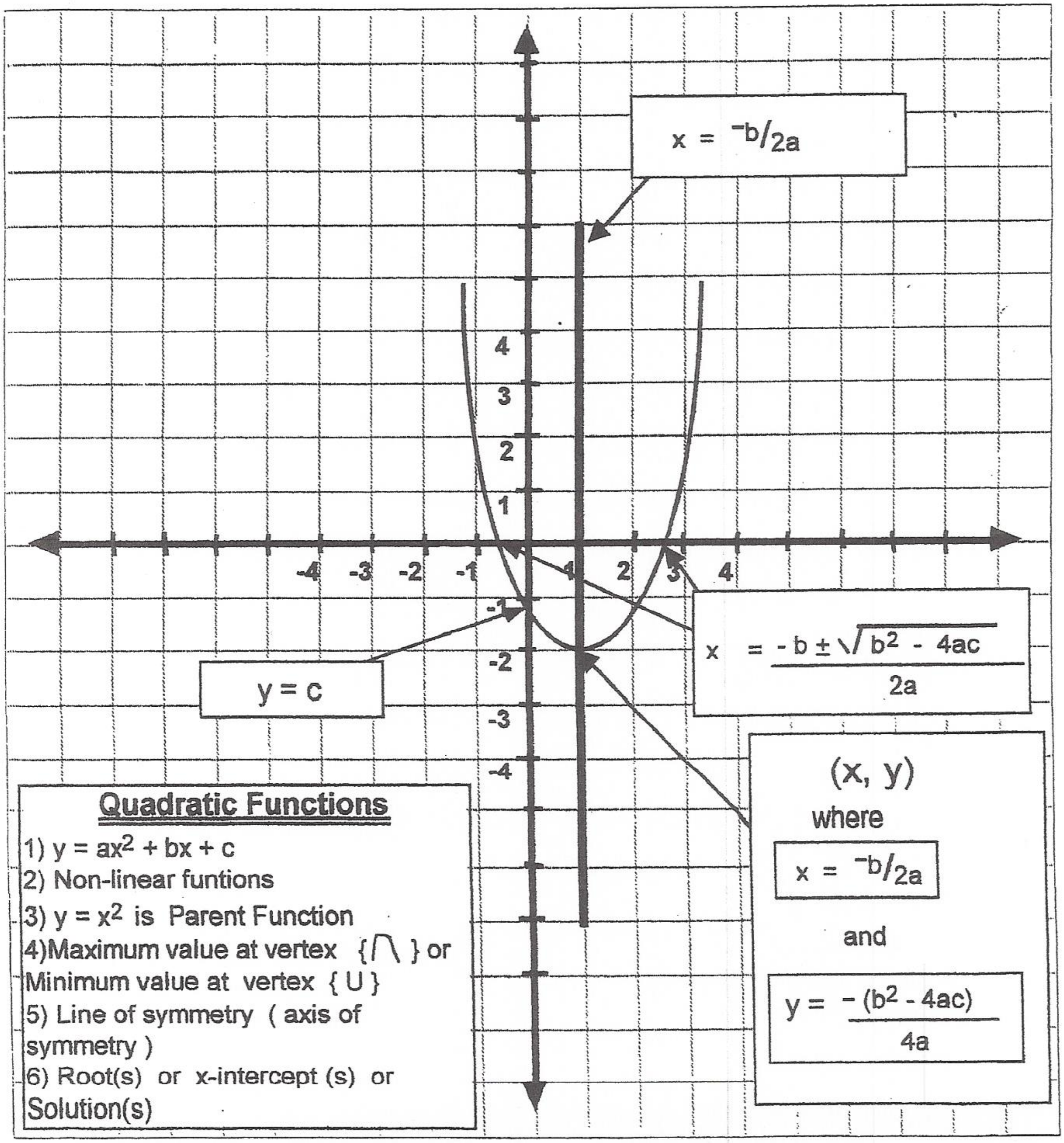


$x = {}^{-}b/_{2a}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = c$

(x, y)

where

$x = {}^{-}b/_{2a}$

and

$y = \dfrac{-(b^2 - 4ac)}{4a}$

Quadratic Functions

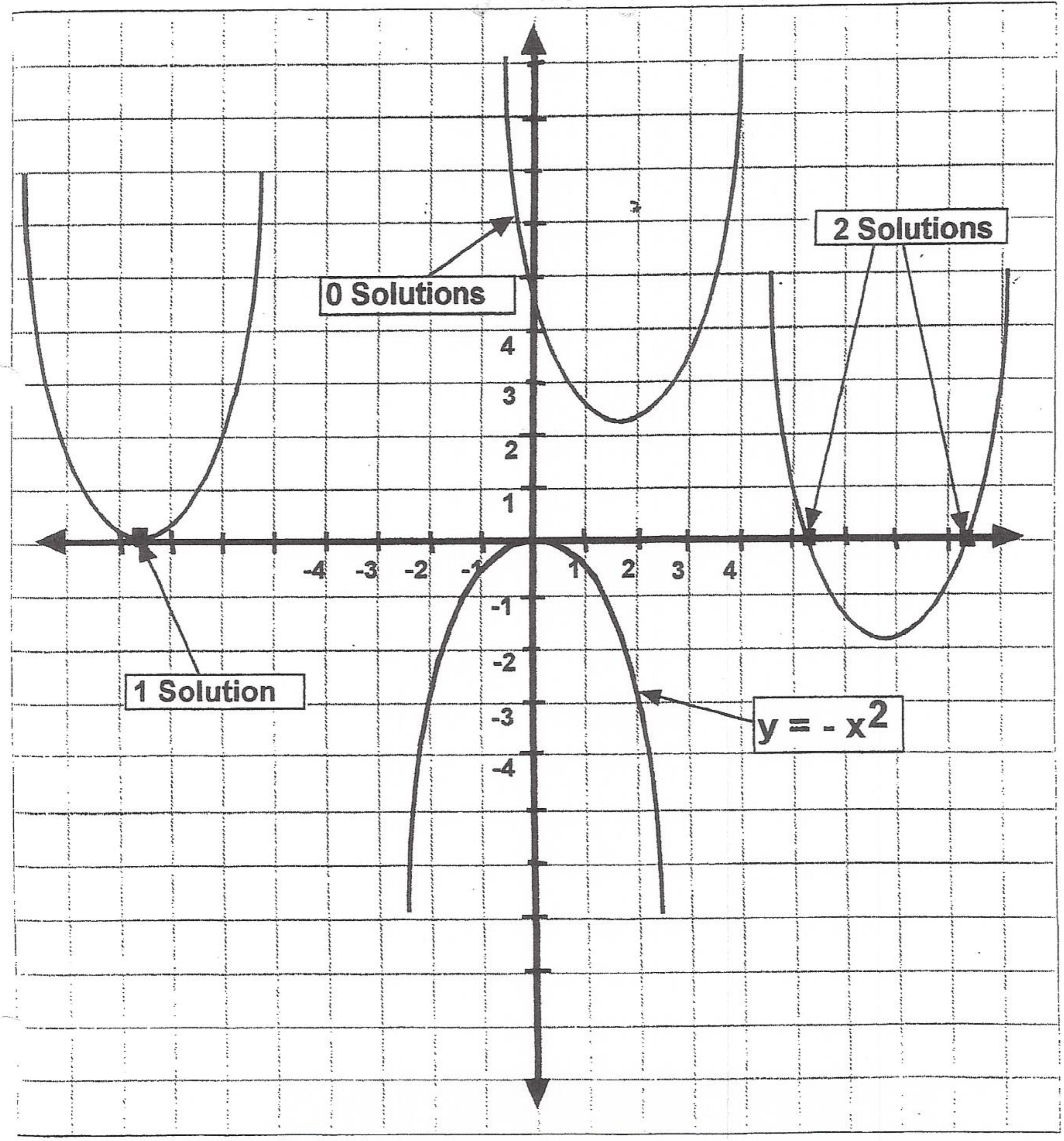
1) $y = ax^2 + bx + c$
2) Non-linear funtions
3) $y = x^2$ is Parent Function
4)Maximum value at vertex {∩ } or Minimum value at vertex { U }
5) Line of symmetry (axis of symmetry)
6) Root(s) or x-intercept (s) or Solution(s)

;QUADRATIC FUNCTION GRAPH PROPERTIES:

NAME: $y = ax^2 + bx + c$;

If a < 0, then you have a Maximum Value graph. If a >0, Then you have Minimum Value graph. If a = 0, then you have a linear graph not a quadratic anymore. c is always the y-intercept.

0 Solutions = imaginary roots



ALGEBRA

Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. If $p(a) = 0$, then a is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a *factor* of the polynomial.

Fundamental Theorem of Algebra

An nth degree polynomial has n (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of p are $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$.

Special Factors

$x^2 - a^2 = (x - a)(x + a)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$$

Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every *rational zero* of p is of the form $x = r/s$, where r is a factor of a_0 and s is a factor of a_n.

Factoring by Grouping

$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$

Arithmetic Operations

$ab + ac = a(b + c)$

$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

$\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\dfrac{a}{b}\right)\left(\dfrac{d}{c}\right) = \dfrac{ad}{bc}$

$\dfrac{\left(\frac{a}{b}\right)}{c} = \dfrac{a}{bc}$

$\dfrac{a}{\left(\frac{b}{c}\right)} = \dfrac{ac}{b}$

$a\left(\dfrac{b}{c}\right) = \dfrac{ab}{c}$

$\dfrac{a - b}{c - d} = \dfrac{b - a}{d - c}$

$\dfrac{ab + ac}{a} = b + c$

Exponents and Radicals

$a^0 = 1, \quad a \ne 0$

$(ab)^x = a^x b^x$

$a^x a^y = a^{x+y}$

$\sqrt{a} = a^{1/2}$

$\dfrac{a^x}{a^y} = a^{x-y}$

$\sqrt[n]{a} = a^{1/n}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

$\sqrt[n]{a^m} = a^{m/n}$

$a^{-x} = \dfrac{1}{a^x}$

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

$(a^x)^y = a^{xy}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Notes: Exponentiation

Definitions:

An **exponential function** is a function whose general equation is $y = a \cdot b^x$ where *a* and *b* stand for constants, b is positive and *x* and *y* are independent and dependent variables.

Exponentiation for positive integer exponent: (when *n* is positive) x^n means the product of *n* *x*'s.

In the expression x^n,

x is called the *base,*
n is called the *exponent*
x^n is called a *power.*

Example 1: $-x^3 = -(x)(x)(x)$
$(-x)^3 = (-x)(-x)(-x)$

Example 2: $4x^4 = 4(x)(x)(x)(x)$

$(4x)^4 = (4x)(4x)(4x)(4x)$

Rule: If the *base* contains more than one symbol, then the *base* must be placed in parentheses.

Properties of Exponentiation:

1. *Product of two powers with equal bases:*

$$x^a \cdot x^b = x^{a+b}$$

2. *Quotient of two powers with equal bases:*

$$\frac{x^a}{x^b} = x^{a-b}$$

3. *Power of a power:*

$$(x^a)^b = x^{a * b}$$

Log Rules

~~X~~

Exponent

$$\underset{\text{Standard Form}}{a} = \underset{\text{Base}}{b}^{\overset{\text{Exponent}}{c}} \Longrightarrow$$

standard

$$\log_{\underset{\text{base}}{b}} \overset{\text{standard}}{a} = c \leftarrow \text{Exponent}$$

① $a = b^c \Longrightarrow \log_b a = c$

② $\log_b MN \Longrightarrow \log_b M + \log_b N$

③ $\log_b \frac{M}{N} \Longrightarrow \log_b M - \log_b N$

④ $\log_b N^a \Longrightarrow a \log_b N$

⑤ $\log_{b^*} a = \dfrac{\log a}{\log b}$ $\Big\}$

eg: $\log_5 3 = \dfrac{\log 3}{\log 5}$ use calculator Now.

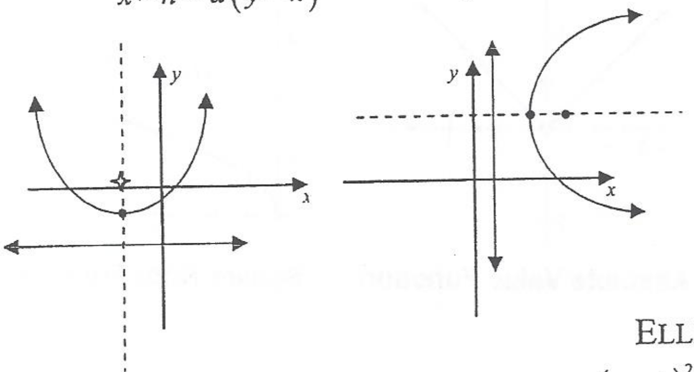
CONICS

PARABOLA

$$y-k=a\left(x-h\right)^2$$

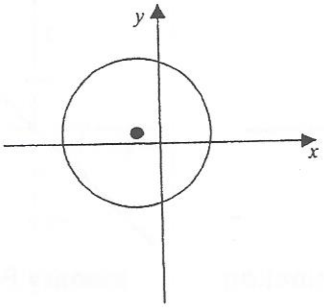
$$x-h=a\left(y-k\right)^2$$

$$a=\frac{1}{4p}$$



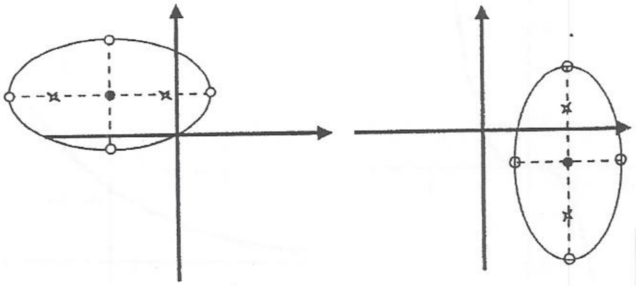
CIRCLE

$$\left(x-h\right)^2+\left(y-k\right)^2=r^2$$



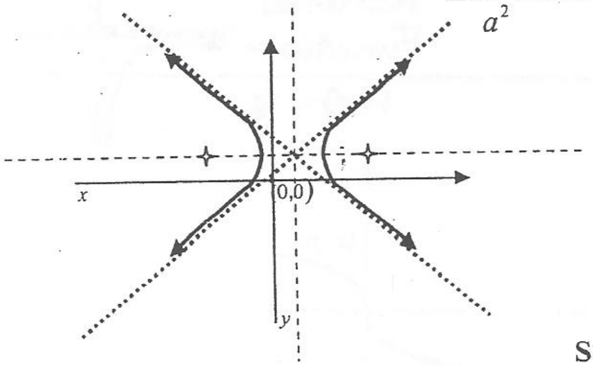
ELLIPSE

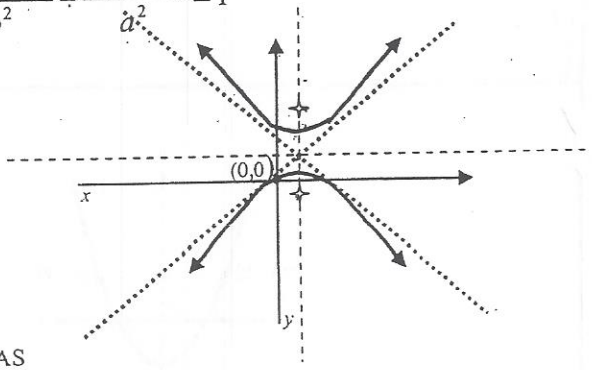
$$\frac{\left(x-h\right)^2}{a^2}+\frac{\left(y-k\right)^2}{b^2}=1$$



HYPERBOLA

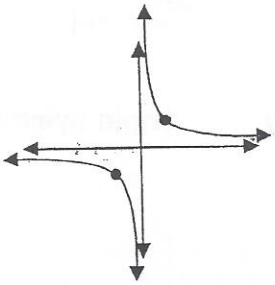
$$\frac{\left(x-h\right)^2}{a^2}-\frac{\left(y-k\right)^2}{b^2}=1 \text{ or } \frac{\left(y-k\right)^2}{b^2}-\frac{\left(x-h\right)^2}{a^2}=1$$

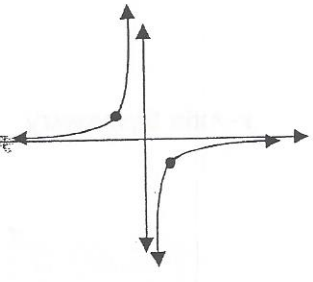




SPECIAL HYPERBOLAS

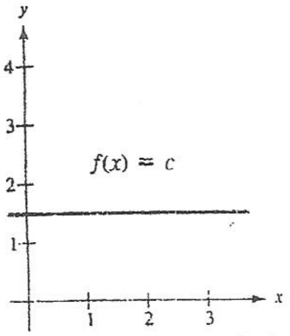
$xy=k$ or $xy=-k$



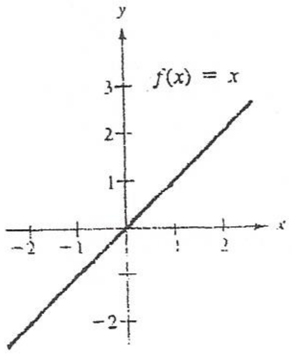


GRAPHS OF COMMON FUNCTIONS

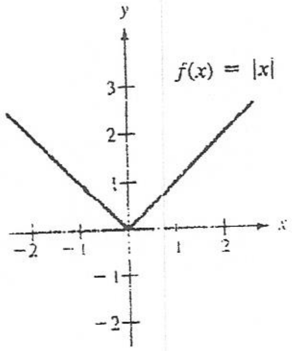
PARENT FUNCTIONS



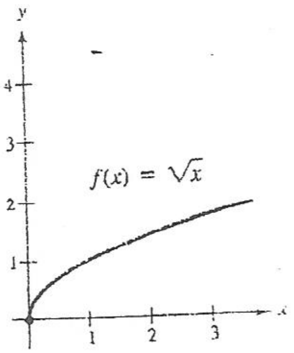
Constant Function



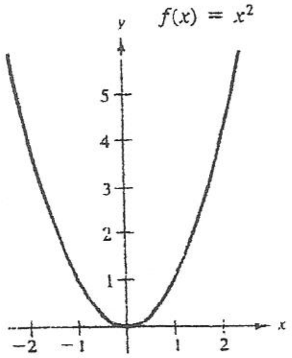
Identity Function



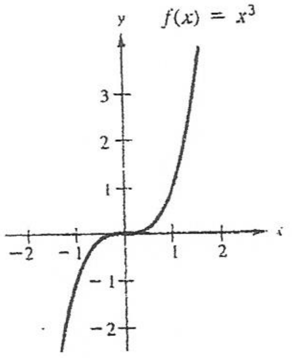
Absolute Value Function



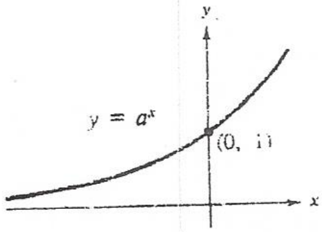
Square Root Function



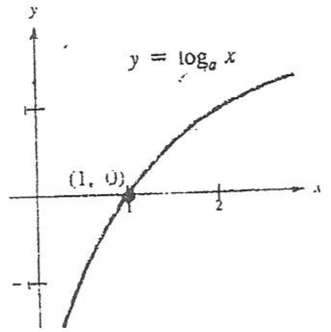
Squaring Function



Cubing Function



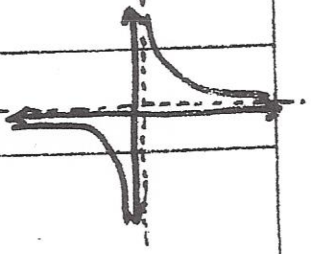
Exponential Function



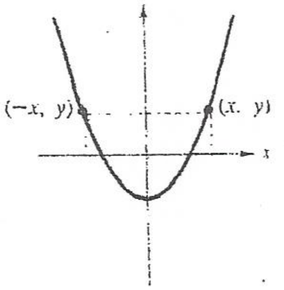
Logarithmic Function

Rational Function

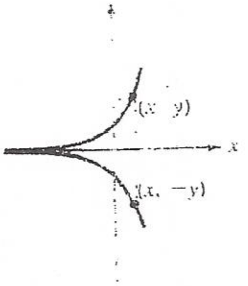
$f(x) = \frac{1}{x}$



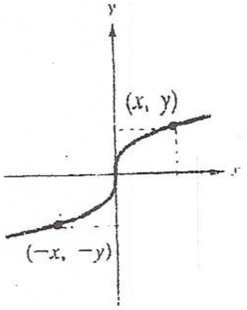
SYMMETRY



***y*-Axis Symmetry**



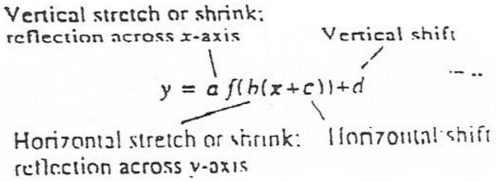
***x*-Axis Symmetry**



Origin Symmetry

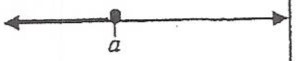
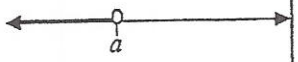
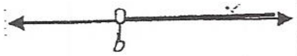
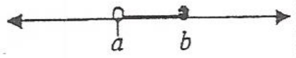
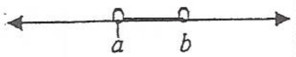
Transformations for $y = f(x)$

$y = f(x) + k$		vertical shift k units up
$y = f(x) - k$		vertical shift k units down
$y = f(x - h)$		horizontal shift h units right
$y = f(x + h)$		horizontal shift h units left
$y = -f(x)$		reflected in the x-axis
$y = f(-x)$		reflected in the y-axis
$x = f(y)$		reflected in the line $y = x$
$y = \|f(x)\|$		unchanged when $f(x) > 0$; reflected in the x-axis when $f(x) < 0$
$y = f(\|x\|)$		unchanged when $x > 0$; reflected in the y-axis when $x < 0$
$y = af(x), \|a\| > 1$	multiply "y" values by "a"	stretched vertically
$y = af(x),\ 0 < \|a\| < 1$		shrunk vertically
$y = f(bx), \|b\| > 1$	multiply "x" values by $\frac{1}{b}$	shrunk horizontally
$y = f(bx),\ 0 < \|b\| < 1$		stretched horizontally



Interval notation is used in Algebra II for many topics.

1. Symbols:
   - a. ∞ infinity
   - b. $-\infty$ negative infinity
   - c. [or] the value is included
   - d. (or) the value is not included
   - e. $\cup$ union or joining together two sets of numbers
2. Infinity <u>always</u> using soft brackets.

Interval Notation	*Set Builder Notation*	*Inequality Notation*	*Graph of Inequality*
$(-\infty, a]$	$\{x \mid x \leq a\}$	$x \leq a$	
$(-\infty, a)$	$\{x \mid x < a\}$	$x < a$	
$[b, \infty)$	$\{x \mid x \geq b\}$	$x \geq b$	
(b, ∞)	$\{x \mid x > b\}$	$x > b$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	$a \leq x \leq b$	
$[a, b)$	$\{x \mid a \leq x < b\}$	$a \leq x < b$	
$(a, b]$	$\{x \mid a < x \leq b\}$	$a < x \leq b$	
(a, b)	$\{x \mid a < x < b\}$	$a < x < b$	
$(-\infty, a] \cup (b, \infty)$	$\{x \mid x \leq a \text{ or } x > b\}$	$x \leq a, x > b$