**Calculus**
**Worksheet on Implicit Differentiation**

Work these on notebook paper. Show all work, and circle your answers.

On problems 1 – 3, find $\frac{dy}{dx}$.

1. $x^3 + xy + y^3 = 1$

2. $y - x\sin y = 3$

3. $x + \tan(xy) = 0$

4. If $y = xy + x^2 + 1$, find $\frac{dy}{dx}$ when $x = -1$.

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, given $y^2 + 2y = 2x + 1$.

6. If $x^3 + y^3 = 8$, show that the second derivative of $y$ with respect to $x$ is $-\frac{16x}{y^5}$.

7. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

   a. Find $\frac{dy}{dx}$ in terms of $y$.

   b. Write an equation for each vertical tangent to the curve.

   c. Find $\frac{d^2y}{dx^2}$ in terms of $y$.

8. Consider the curve $y^2 = 4 + x$ and the chord AB joining points $A(-4, 0)$ and $B(0, 2)$ on the curve. Find the $x$- and $y$-coordinates of the point on the curve where the tangent line is parallel to chord AB.

9. The function $f$ is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point $(x, y)$ on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$. Find $\frac{d^2y}{dx^2}$, and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

# L'Hopital's Rule

Given that $f$ and $g$ are differentiable functions on an open interval $(a, b)$ containing $c$ (except possibly at $c$ itself), assume that $g'(x) \neq 0$ for all $x$ in the interval (except possibly at $c$ itself).

If $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)}$ produces the indeterminate form $\dfrac{0}{0}$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies when the limit produces any one of the indeterminate forms $\dfrac{\infty}{\infty}, \dfrac{-\infty}{\infty}, \dfrac{\infty}{-\infty},$ or $\dfrac{-\infty}{-\infty}$. Other indeterminate forms are $\infty - \infty$, $0 \bullet \infty$, $0^0$, $1^\infty$, and $\infty^0$. Determinate forms include $\infty + \infty$, $-\infty - \infty$, $0^\infty$, $0^{-\infty}$ since

$\infty + \infty \to \infty$; $-\infty - \infty \to -\infty$; $0^\infty \to 0$; $0^{-\infty} \to \infty$

For free response questions involving L'Hopital's rule, it is important for students to show that substitution yields the indeterminate form before applying the rule to evaluate the limit.

From the BC Calculus Course Description:

- Students should know L'Hopital's rule, including its use in determining limits and convergence of improper integrals and series.

Examples:

1. $\lim\limits_{x \to 3} \dfrac{x-3}{x^2 - 9}$

2. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{x - \frac{\pi}{2}}$

3. $\lim\limits_{x \to 2} \dfrac{\sqrt{4-x} - \sqrt{x}}{x-2}$