Introduction to Logic Statements

When we define and explain things in geometry, we use declarative sentences. For example, "Perpendicular lines intersects at a 90 degree angle" is a declarative sentence. It is also a sentence that can be classified in one, and only one, of two ways: true or false. Most geometric sentences have this special quality, and are known as statements. In the following lessons we'll take a look at logic statements**. Logic is the general study of systems of conditional statements;** in the following lessons we'll just study the most basic forms of logic pertaining to geometry.

Conditional statements are combinations of two statements in an if-then structure. For example, **"If lines intersect at a 90 degree angle, then they are perpendicular"** is a **conditional statement**. The parts of a conditional statement can be interchanged to make systematic changes to the meaning of the original conditional statement. Based on the truth value (there are only two truth values, either true or false) of a conditional statement, **we can deduce the truth value of its converse, contrapositive, and inverse.** These three types of conditional statements are all related to the original conditional statement in a different way.

GEOMETRY: LOGIC STATEMENTS

Variations on Conditional Statements

The three most common ways to change a conditional statement are by taking its **inverse, its converse, or it contrapositive**. In each case, either the hypothesis and the conclusion switch places, or a statement is replaced by its negation.

The Inverse

The inverse of a conditional statement is arrived at by replacing the hypothesis and the conclusion with their negations. If a statement reads, "The vertex of an inscribed angle is on a circle", then the inverse of this statement is "The vertex of an angle that is not an inscribed angle is not on a circle." Both the hypothesis and the conclusion were negated. **If the original statement reads "if *j* , then *k* ", the inverse reads, "if not *j* , then not *k* ."**

The truth value of the inverse of a statement is undetermined. That is, some statements may have the same truth value as their inverse, and some may not. For example, "A four-sided polygon is a quadrilateral" and its inverse, "A polygon with greater or less than four sides is not a quadrilateral," are both true (the truth value of each is T). In the example in the paragraph above about inscribed angles, however, the original statement and its inverse do not have the same truth value. The original statement is true, but the inverse is false: it *is* possible for an angle to have its vertex on a circle and still not be an inscribed angle.

The Converse

The converse of a statement is formed by switching the hypothesis and the conclusion. The converse of "If two lines don't intersect, then they are parallel" is "If two lines are parallel, then they don't intersect." The converse of "if *p* , then *q* " is "if *q* , then *p* ."

The truth value of the converse of a statement is not always the same as the original statement. For example, the converse of "All tigers are mammals" is "All mammals are tigers." This is certainly not true.

The converse of a definition, however, must always be true. If this is not the case, then the definition is not valid. For example, we know the definition of an equilateral triangle well: "if all three sides of a triangle are equal, then the triangle is equilateral." The converse of this definition is true also: "If a triangle is equilateral, then all three of its sides are equal." What if we performed this test on a faulty definition? If we incorrectly stated the definition of a tangent line as: "A tangent line is a line that intersects a circle", the statement would be true. But it's converse, "A line that intersects a circle is a tangent line" is false; the converse could describe a secant line as well as a tangent line. The converse is therefore a very helpful tool in determining the validity of a definition.

The Contrapositive

The contrapositive of a statement is formed when the hypothesis and the conclusion are interchanged, and both are replaced by their negation. In other words, the contrapositive of a statement is the same as the inverse of that statement's converse, or the converse of its inverse.

Take the statement, **"Long books are fun to read." Its contrapositive is "Books that aren't fun to read aren't** long." **The statement "if *p* , then *q* " becomes "if not *q* , then not *p* ."**

The contrapositive of a statement always has the same truth value as the original statement. Therefore, the contrapositive of a definition is always true. For example, the statement "A triangle is a three-sided polygon" is true. Its contrapositive, "A polygon with greater or less than three sides is not a triangle" is also true.