Pre_Calc_Pages 437 to 439

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac*, and compare the actual hours of daylight to the results found in part (c).

Discussion and Writing

31. Explain how the amplitude and period of a sinusoidal graph are used to establish the scale on each coordinate axis.

32. Find an application in your major field that leads to a sinusoidal graph. Write a paper about your findings.

Chapter Review

Things to Know

Definitions

Angle in standard position (p. 356) — Vertex is at the origin; initial side is along the positive x-axis

1 Degree (1°) (p. 357) — $1° = \frac{1}{360}$ revolution

1 Radian (p. 360) — The measure of a central angle of a circle whose rays subtend an arc whose length is the radius of the circle

Trigonometric functions (pp. 371–372) — $P = (x, y)$ is the point on the unit circle corresponding to $\theta = t$ radians.

$$\sin t = \sin\theta = y \qquad \cos t = \cos\theta = x \qquad \tan t = \tan\theta = \frac{y}{x},\ x \neq 0$$

$$\csc t = \csc\theta = \frac{1}{y},\ y \neq 0 \qquad \sec t = \sec\theta = \frac{1}{x},\ x \neq 0 \qquad \cot t = \cot\theta = \frac{x}{y},\ y \neq 0$$

Trigonometric functions using a circle of radius r (pp. 382–383) — For an angle θ in standard position $P = (x, y)$ is the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x},\ x \neq 0$$

$$\csc\theta = \frac{r}{y},\ y \neq 0 \qquad \sec\theta = \frac{r}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

Periodic function (p. 391) — $f(\theta + p) = f(\theta)$, for all θ, $p > 0$, where the smallest such p is the fundamental period

Formulas

1 revolution $= 360°$ (p. 358)

$= 2\pi$ radians (p. 361)

$s = r\,\theta$ (p. 360) — θ is measured in radians; s is the length of arc subtended by the central angle θ of the circle of radius r; A is the area of the sector.

$A = \frac{1}{2}r^2\,\theta$ (p. 364)

$v = r\omega$ (p. 365) — v is the linear speed along the circle of radius r; ω is the angular speed (measured in radians per unit time).

TABLE OF VALUES

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	$0°$	0	1	0	Not defined	1	Not defined
$\frac{\pi}{6}$	$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	$90°$	1	0	Not defined	1	Not defined	0
π	$180°$	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	$270°$	-1	0	Not defined	-1	Not defined	0

Fundamental Identities (p. 394)

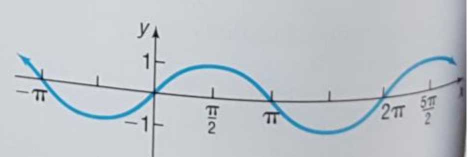
$$\tan\theta = \frac{\sin\theta}{\cos\theta},\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta},\quad \sec\theta = \frac{1}{\cos\theta},\quad \cot\theta = \frac{1}{\tan\theta}$$

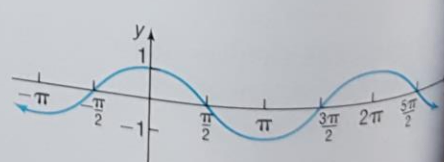
$$\sin^2\theta + \cos^2\theta = 1,\quad \tan^2\theta + 1 = \sec^2\theta,\quad 1 + \cot^2\theta = \csc^2\theta$$

Properties of the Trigonometric Functions

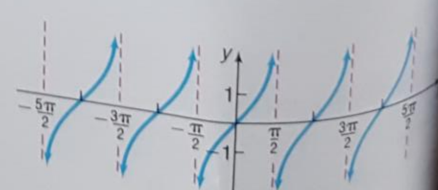
$y = \sin x$ (p. 404)
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi\ (360°)$
Odd function



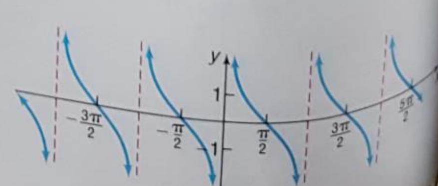
$y = \cos x$ (p. 406)
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi\ (360°)$
Even function



$y = \tan x$ (p. 420)
Domain: $-\infty < x < \infty$, except odd multiples of $\frac{\pi}{2}\ (90°)$
Range: $-\infty < y < \infty$
Periodic: period $= \pi\ (180°)$
Odd function

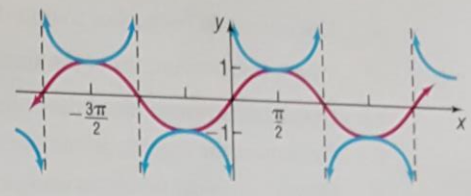


$y = \cot x$ (p. 422)
Domain: $-\infty < x < \infty$, except integer multiples of $\pi\ (180°)$
Range: $-\infty < y < \infty$
Periodic: period $= \pi\ (180°)$
Odd function



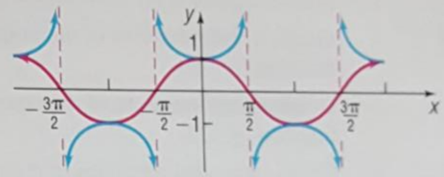
$y = \csc x$ (p. 422)

Domain: $-\infty < x < \infty$, except integer multiples of π (180°)
Range: $|y| \geq 1$
Periodic: period $= 2\pi$ (360°)
Odd function



$y = \sec x$ (p. 423)

Domain: $-\infty < x < \infty$, except odd multiples of $\frac{\pi}{2}$ (90°)
Range: $|y| \geq 1$
Periodic: period $= 2\pi$ (360°)
Even function



inusoidal graphs (pp. 409 and 426)

$y = A\sin(\omega x), \quad \omega > 0$ Period $= \dfrac{2\pi}{\omega}$

$y = A\cos(\omega x), \quad \omega > 0$ Amplitude $= |A|$

$y = A\sin(\omega x - \phi) = A\sin\left[\omega\left(x - \dfrac{\phi}{\omega}\right)\right]$ Phase shift $= \dfrac{\phi}{\omega}$

$y = A\cos(\omega x - \phi) = A\cos\left[\omega\left(x - \dfrac{\phi}{\omega}\right)\right]$