AP Course Design and Philosophy

In one high school year-long course, AP Calculus AB provides a semester or more of university-level calculus. As students of a university-level course, calculus students spend a minimum of three hours out of class on study and preparation for every hour in class. Students work at a rigorous and rapid pace, which demands work outside of class via homework, projects, and tutorials. Calculus AB begins with *calculus topics*; a spiral review of precalculus concepts such as algebra and trigonometry will occur as needed. The course emphasizes a multi-representational approach to mathematics with problems and solutions expressed graphically, numerically, analytically, and verbally.

By the end of this course, students will ─

1) discover the beauty of calculus – not only the mechanics, but also the manifold applications and the natural relationship between differentiation and integration.
2) prepare for college-level courses by improving study habits, engaging in rigorous mathematics, and participating in a community of life-long learners with common goals.
3) pass the AP Calculus AB Exam with a score of three or higher and earn college mathematics credit for the first semester of the calculus series.

Course Planner [C2]

Unit	Unit Description
Unit 1: Limits and Their Properties[SC1] Students analyze graphs, limits of functions, asymptotes, and unbounded behavior and continuity as properties of functions. 2 Weeks	• Investigate tangent lines and the length of a curve • Describe limits and the limit process • Use graphs and tables of data to determine limits • Use properties of limits • Evaluate limits algebraically • Compare relative magnitudes of functions and their rates of change • Determine continuity and one-sided limits • Investigate continuity of functions geometrically • Investigate and apply the intermediate value theorem • Explore and determine infinite limits • Use limits to determine the asymptotes of a function

Unit 2: Differentiation [SC2]
Part 1: Rules of Differentiation
Students analyze derivatives, the derivative at a point, the derivative as a function, second derivatives, and computation of derivatives.
 	
Part 2: Differentiation of Transcendental Functions
Students analyze derivatives, the derivative at a point, the derivative as a function, second derivatives, and computation of derivatives for transcendental functions.
 	
6 Weeks Total
	Part 1: Rules of Differentiation (3 weeks)
	• Determine the derivative graphically, numerically, and analytically
	• Approximate rates of change from graphs and tables of data
	• Calculate the derivative as the limit of the average rate of change, an instantaneous rate of change, the limit of the difference quotient, and the slope of a curve at a point
	• Translate verbal descriptions of the derivative into equations and vice versa
	• Identify the relationship between differentiability and continuity
	• Identify functions that have a vertical tangent at a point
	• Identify functions that have a point at which there is no tangent
	• Calculate the derivative of a function, including power functions, using basic rules of differentiation
	• Calculate the derivatives using rules of differentiation for sums, differences, products, quotients, and composite functions (chain rule)
	
	Part 2: Differentiation of Transcendental Functions (3 weeks)
	• Differentiate –
	  ○ trigonometric functions
	  ○ natural logarithmic functions
	  ○ exponential functions
	  ○ inverse functions
	  ○ inverse trigonometric functions
	  ○ implicit relations

Unit	Unit Description
Unit 3: Applications of Differentiation [SC2] **Part 1: Characteristics of Graphs** Students investigate the characteristics of graphs and relate those specific characteristics to their derivatives. **Part 2: Rates of Change** Students apply the derivative to optimization, motion, and related rates. 5 Weeks Total	Part 1: Characteristics of Graphs (2 weeks) • Determine the tangent line to a curve and evaluate linear approximations • Locate extrema on an interval and apply the extreme value theorem • Apply Rolle's theorem and the mean value theorem • Apply the first derivative test • Using the first derivative test to determine increasing and decreasing functions • Describe concavity and points of inflection • Compare concavity with its relationship to the first and second derivatives • Apply the second derivative test • Evaluate limits at infinity • Summarize curve sketching using geometric and analytic information to predict the behavior of a function • Sketch the graphs of f, f', and f'' Part 2: Rates of Change (3 weeks) • Solve optimization problems including both relative and absolute extrema • Compute the derivative to solve optimization, motion, and related rates problems including position, velocity, acceleration, and rectilinear motion
Unit 4: Integration[SC3] **Part 1: Antiderivatives** Students interpret properties of definite integrals and techniques of integration. 2 Weeks (continued in Cycle 4)	Part 1: Antiderivatives (2 weeks) • Investigate and compute antiderivatives and indefinite integrals • Apply properties of the definite integral to determine the area under the curve
Final Exams Review Week **Final Exams Week**	**First Semester Exam**

Unit	Unit Description
Unit 4 (continued): Integration[SC3] **Part 2: Accumulation** Students investigate the fundamental theorem of calculus, techniques of antidifferentiation, and numerical approximations of definite integrals. 2 Weeks	Part 2: Accumulation (2 weeks) • Compute the area under a curve • Describe the meaning of the definite integral • Calculate the definite integral as a limit of Riemann sums • Calculate a trapezoidal sum or a Riemann sum, including left, right, and midpoint • Use the Riemann sum or trapezoidal sum to approximate definite integrals of functions that are represented analytically, graphically, and tabularly • Apply the first and second fundamental theorems of calculus to determine the area under a curve and the initial/end value of a function • Use the first fundamental theorem of calculus to evaluate definite integrals • Integrate by substitution • Use the mean value theorem for integrals to compute the average value of a function

Unit 5: Logarithmic, Exponential, and Other Transcendental Functions[SC3]
Students apply techniques of integration to transcendental functions.
 	
2 Weeks
	• Integrate –
	  ○ natural logarithmic functions
	  ○ exponential functions
	  ○ inverse trigonometric functions
	• Solve differential equations using separation of variables
	• Apply differential equations to model exponential growth
	• Sketch slope fields and particular solutions of differential equations

Unit 6: Applications of Integration[SC3] Students apply integration to describe motion on a line and to solve differential equations. 2 Weeks	• Determine the integral as an accumulator of rates of change • Calculate the area of a region between two curves in a plane • Calculate the volume of a solid with known cross sections • Calculate the volume of solids of revolution • Apply integration in physical, biological, and economic contexts • Apply integration in problems involving motion in a straight line to compute distance or displacement from given initial conditions.

Unit	Unit Description
Unit 7: Culminating Comparisons and Applications **Part 1: Rates and Accumulation** Students apply differentiation and integration to rates and accumulation.	Part 1: Rates and Accumulation (1 week) • Compare differentiation and integration techniques that may be used to solve real-world situations • Develop written justifications within responses to open-ended problems using correct units • Justify reasonableness of answers • Interpret and analyze initial values
Part 2: Data Analysis Students apply analytical skills to data using differentiation and integration.	Part 2: Data Analysis (1 week) • Determine behavior of continuous function given discrete data points • Deconstruct information for more efficient problem-solving
Part 3: Motion Students analyze rates, direction, speed vs. velocity, and displacement vs. total distance.	Part 3: Motion (1 Week) • Interpret rates, direction, speed vs. velocity, displacement vs. total distance, and other methods of analysis.
Part 4: Graphical Analysis Students analyze first and second derivatives using multiple representations.	Part 4: Graphical Analysis (1 week) • Move between original function and first and second derivatives • Identify functions and multi-representations of functions (functions expressed as derivatives or integrals) • Analyze graphical representations of functions
Part 5: Differential Equations Students compare and contrast directions fields and analyze real-world situations.	Part 5: Differential Equations (1 week) • Model real-world situations • Compare and contrast directional fields and analytical methods
Part 6: Area and Volume Students apply differentiation and integration to motion, graphs, differential equations, area, and volume.	Part 6: Area and Volume (1 week) • Connect among geometrical representations in order to determine the most efficient strategy to solve the problem • Perform a discrete estimation of the area under a continuous function
Part 7: Essential Theorems Students synthesize differentiation and integration concepts with a focus on comparisons among the key theorems, such as intermediate value theorem, mean value theorem, first and second fundamental theorem, and mean value theorem for integrals.	Part 7: Essential Theorems (1 week) • Connect L'Hôpital's rule to limits • Apply intermediate value theorem to determine the values contained in a function • Compare mean value theorem to mean value theorem of integrals • Apply fundamental theorem of calculus I and II to real-world scenarios • Practice Exam 1

Unit	Unit Description
7 Weeks Total	
Unit 8: Data-Driven Target Lessons Students study concepts identified from practice exam 1 and practice exam 2 data. 3 Weeks Total	• According to data from Practice Exam I, students enhance writing skills to free-response questions by analyzing exam items and justify answers and process. • Practice Exam 2
	AP EXAM
Unit 9: Bridge to Calculus II[C4] 1 Week	• Debrief 2015 free-response questions • Integrate functions by parts • Calculate trigonometric integrals • Integrate functions using partial fractions
Second Semester Exam Review	**Second Semester Review and Exam**

Teaching Strategies

Learning with Purpose

Each unit begins with a discussion of a real application to provide relevance. This assists in providing motivation for mastery of unit targets.

Graphing Technology [SC4, SC5, SC6, SC10, SC11, SC12]

Students use the graphing calculator on assignments, to analyze functions, represent concepts numerically, graphically, analytically, and verbally, and justify solutions. Additionally, the student will use the graphing calculator to conduct explorations, graph functions within arbitrary windows, solve equations numerically, analyze and interpret results, justify and explain results of graphs and equations. For example, students explore local linearity and tangent approximations by zooming in on a function and its tangent near a given point of tangency. Although approximately 70% of the AP Calculus AB exam does not require graphing technology, and two out of four sections on the exam prohibits its use, the exploration and analysis of multiple representations of concepts with graphing technology provides increasing opportunity for student understanding.

Oral and Written Explanation of Solutions [SC4, SC5, SC6, SC7, SC8, SC9]

Student study groups provide opportunities for them to communicate solutions to one another. Each group presents a written and oral solution to an assigned problem. Students justify their solution to the class by answering direct questions from their peers and instructor. This strategy is very effective when learning optimization, motion, and related rates applications. In addition, to meet the emphasis on writing on the AP exam, students practice writing daily through reflections, justification, and interpretation of mathematical results, and analysis of real-world situations.

Students explain concept(s) in each assignment. Test questions include narrative items that require students to formulate and express ideas in writing and justify all answers verbally. Released AP items are analyzed. Key words and terms are identified to determine which concept is being assessed, and students provide a narrative description of the solution process in complete sentences.

Adopted Resource:

- Larso, R., and Edwards, B.H. (2015). *Calculus: Early Transcendental Functions (AP® Edition)*, 6th Edition. Cengage

Supporting Resources:

- Larson, R., Hostetler, R.P., and Edwards, B.H. (2006). *Calculus with Analytic Geometry*, 8th ed. Boston: Houghton Mifflin.
- Hockett, S. and Bock, D. (2010). *Barron's AP Calculus Review*, 10th ed. Barron's Education Series.
- Lackey, R., Minor, Z., Folwaczny, L., and Strozewski, A. (2012) *5 Steps to A 5, AP Calculus AB/BC.* New York: McGraw-Hill.

Online Resources:

- AP Central: Calculus AB
  - AP Central Practice Supplements
  - AP Calculus Released Multiple-Choice Questions 1998, 2003, 2008
  - AP Calculus Released Free-Response Questions 1998-2015