

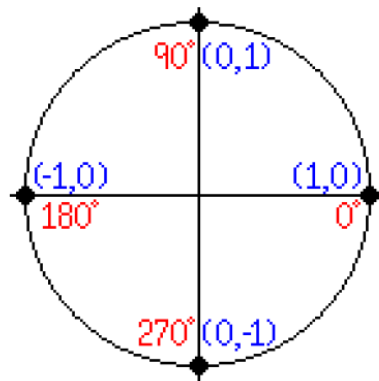
Quadrants: Video

<https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/intro-to-radians-trig/v/rotation-by-radians-and-quadrants>

Trigonometry notes link


http://www.montereyinstitute.org/courses/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U19_L1_T3_text_final.html

Quadrant angles



What are the QUADRANTAL angles? ^

Quadrantal Angle. An **angle** with terminal side on the x-axis or y-axis. That is, the **angles** 0° , 90° , 180° , 270° , 360° , 450° , ... as well as -90° , -180° , -270° , -360° , ...

Mathwords: Quadrantal Angle  Norton
www.mathwords.com/q/quadrantal_angle.htm

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What is the use of unit circle? ^

The **unit circle** is **used** to understand sines and cosines of angles found in right triangles. The **unit circle** has a center at the origin (0,0) and a radius of one **unit**. Angles are measured starting from the positive x-axis in quadrant I and continue around the **unit circle**.

Unit Circle Trigonometry

Learning Objective(s)

- Understand unit circle, reference angle, terminal side, standard position.
- Find the exact trigonometric function values for angles that measure 30° , 45° , and 60° using the unit circle.
- Find the exact trigonometric function values of any angle whose reference angle measures 30° , 45° , or 60° .
- Determine the quadrants where sine, cosine, and tangent are positive and negative.

Introduction

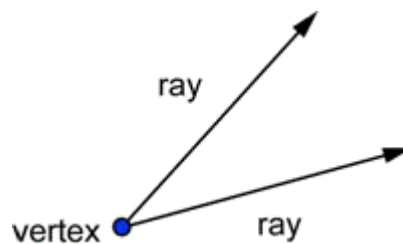
Mathematicians create definitions because they have a use in solving certain kinds of problems. For example, the **six trigonometric functions** were originally defined in terms of right triangles because that was useful in solving real-world problems that involved right triangles, such as finding angles of elevation. The domain, or set of input values, of these functions is the set of angles between 0° and 90° . You will now learn *new* definitions for these functions in which the domain is the set of *all* angles. The new functions will have the same values as the original functions when the input is an acute angle. In a right triangle you can only have acute angles, but you will see the definition extended to include other angles.

One use for these new functions is that they can be used to find unknown side lengths and angle measures in any kind of triangle. These new functions can be used in many situations that have nothing to do with triangles at all.

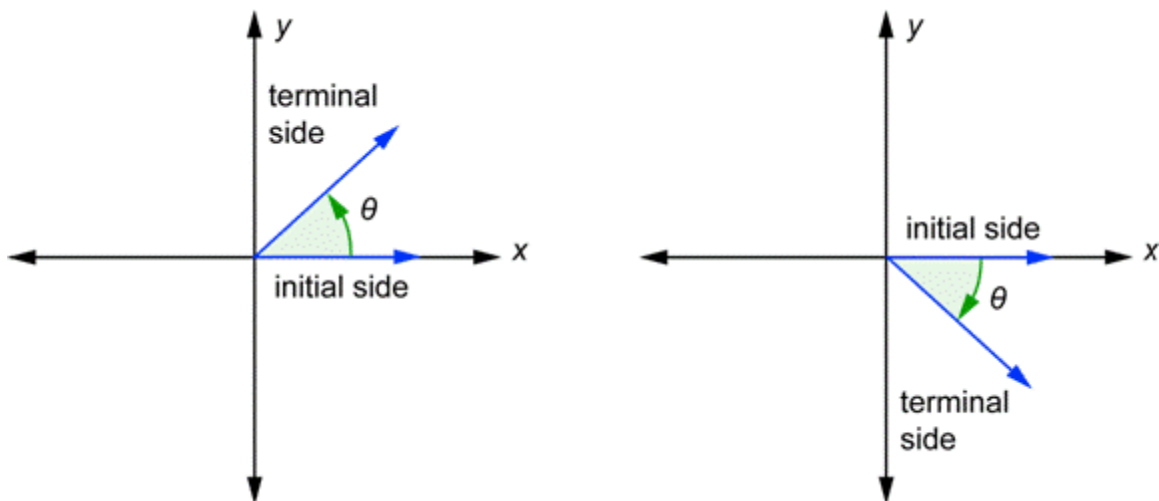
Before looking at the new definitions, you need to become familiar with the standard way that mathematicians draw and label angles.

General Angles

From geometry, you know that an angle is formed by two rays. The rays meet at a point called a vertex.



In trigonometry, angles are placed on coordinate axes. The vertex is always placed at the origin and one ray is always placed on the positive x-axis. This ray is called the **initial side** of the angle. The other ray is called the **terminal side** of the angle. This positioning of an angle is called **standard position**. The Greek letter theta (θ) is often used to represent an angle measure. Two angles in standard position are shown below.



When an angle is drawn in standard position, it has a direction. Notice that there are little curved arrows in the above drawing. The one on the left goes **counterclockwise and is defined to be a positive angle**. The one on the right goes **clockwise and is defined to be a negative angle**. If you used a protractor to measure the angles, you would get 50° in both cases. We refer to the first one as a **50°** angle, and we refer to the second one as a **-50°** angle.

Why would you even have negative angles? As with all definitions, it is a matter of convenience. A spaceship in a circular orbit around Earth's equator could be traveling in either of two directions. So you could say that it traveled through a -50° angle to indicate that it went in the opposite direction of a spaceship that went through a 50° angle. Why is counterclockwise positive? This is just a convention—something that mathematicians have agreed on—because one way has to be positive and the other way negative.

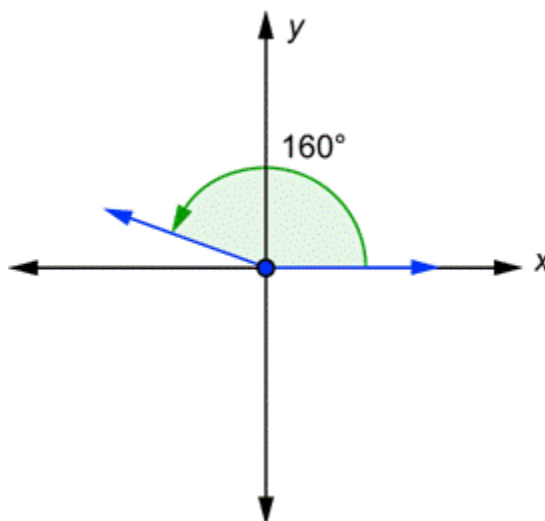
To see how positive angles result from counterclockwise rotation and negative angles result from clockwise rotation, try the interactive exercise below. Either enter an angle measure in the box labeled "Angle" and hit enter or use the slider to move the terminal side of angle θ through the quadrants.

Example

Problem Draw a 160° angle in standard position.

The angle is positive, so you start at the x-axis and go 160° counterclockwise.

Answer

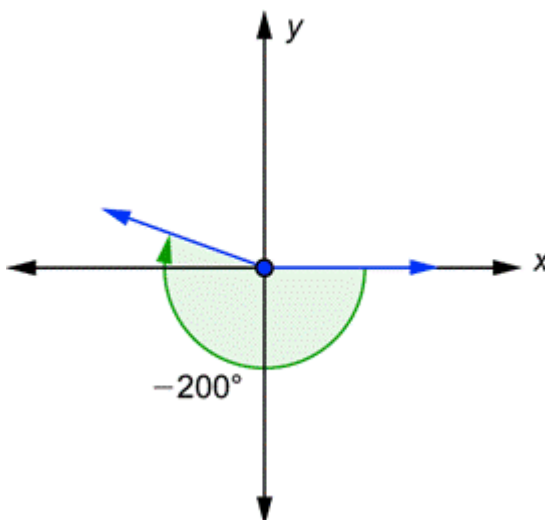


Example

Problem Draw a -200° angle in standard position.

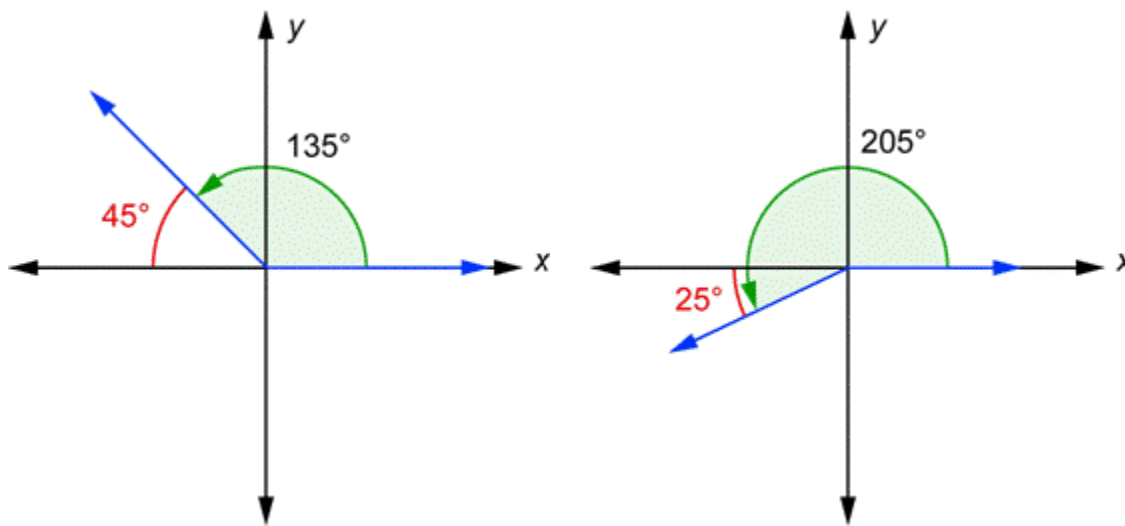
The angle is negative, so you start at the x-axis and go 200° clockwise. Remember that 180° is a straight line. That will bring you to the negative x-axis, and then you have to go 20° farther.

Answer



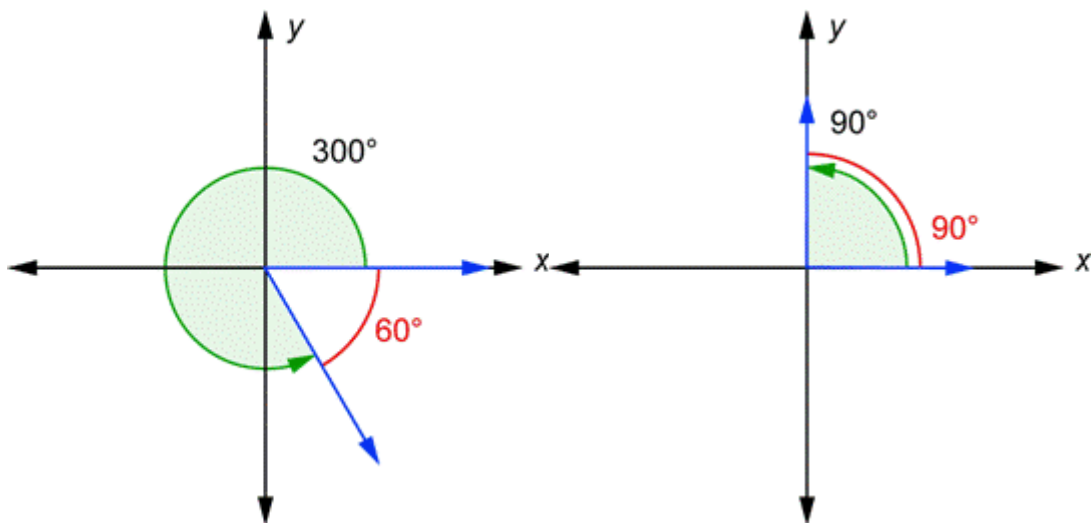
Notice that the terminal sides in the two examples above are the same, but they represent different angles. Such pairs of angles are said to be **coterminal angles**.

For each angle drawn in standard position, there is a related angle known as a **reference angle**. This is the angle **formed by the terminal side and the x-axis**. The reference angle is always considered to be positive, and has a value anywhere from 0° to 90° . Two angles are shown below in standard position.

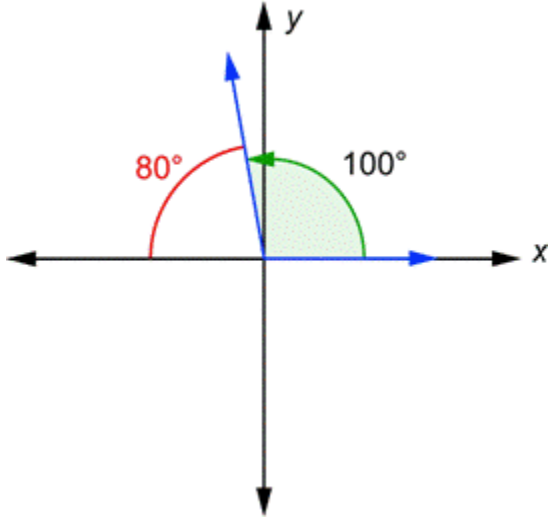


You can see that the **terminal side of the 135° angle and the x-axis form a 45° angle** (this is because the two angles must add up to 180°). This 45° angle, shown in red, is the reference angle for 135° . The terminal side of the 205° angle and the x-axis form a 25° angle. It is 25° because $205^\circ = 180^\circ + 25^\circ$. This 25° angle, shown in red, is the reference angle for 205° .

Here are two more angles in standard position.



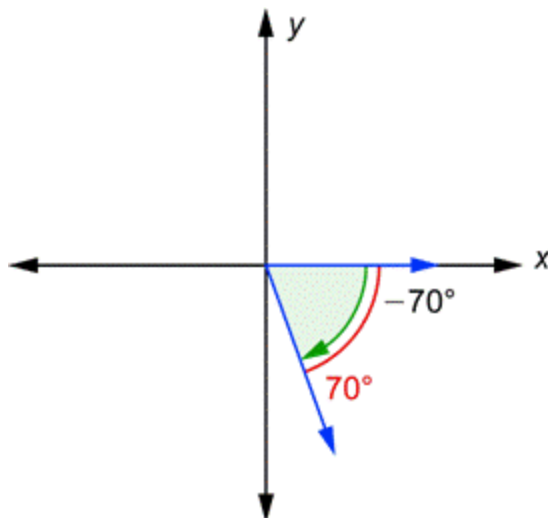
The terminal side of the 300° angle and the x-axis form a 60° angle (this is because the two angles must add up to 360°). This 60° angle, shown in red, is the reference angle for 300° . The terminal side of the 90° angle and the x-axis form a 90° angle. The reference angle is the same as the original angle in this case. In fact, any angle from 0° to 90° is the same as its reference angle.

Example	
Problem	What is the reference angle for 100° ?
<div></div> <div>The terminal side is in Quadrant II. The original angle and the reference angle together form a straight line along the x-axis, so their sum is 180°. Therefore, the reference angle is 80°.</div>	
Answer	The reference angle for 100° is 80° .

Example

Problem

What is the reference angle for -70° ?



The terminal side and the x-axis form the “same” angle as the original. A reference angle is always a positive number, so the reference angle here is 70° , shown in red.

Answer

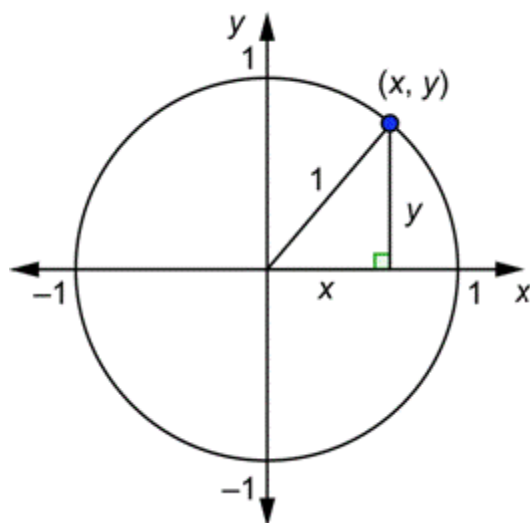
The reference angle for -70° is 70° .

What is the reference angle for 310° ?

- A) 40°
- B) 50°
- C) -40°
- D) -50°

[Show/Hide Answer](#)

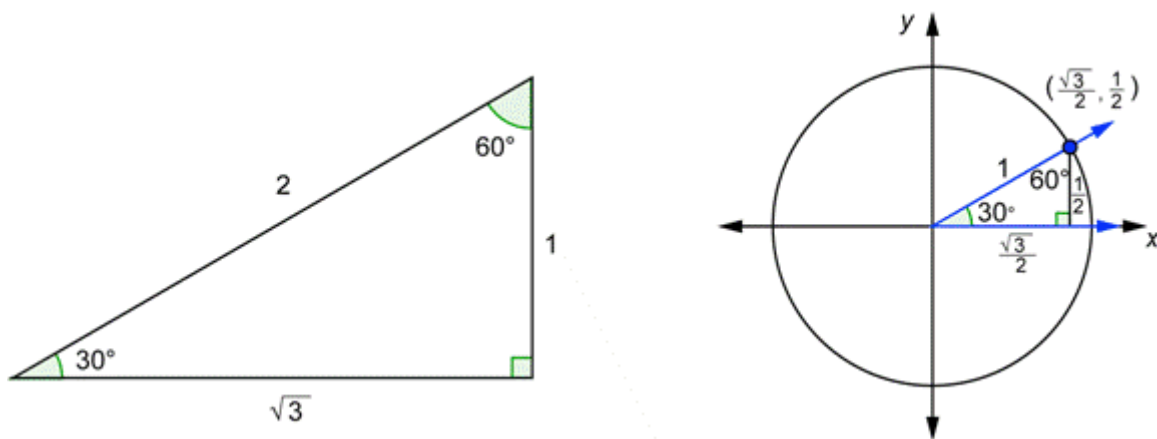
A unit circle is a circle that is centered at the origin and has radius 1, as shown below.



If (x, y) are the coordinates of a point on the circle, then you can see from the right triangle in the drawing and the Pythagorean Theorem that $x^2 + y^2 = 1$. This is the equation of the unit circle.

The General Definition of the Trigonometric Functions

The 30° - 60° - 90° triangle is seen below on the left. Next to that is a 30° angle drawn in standard position together with a unit circle.



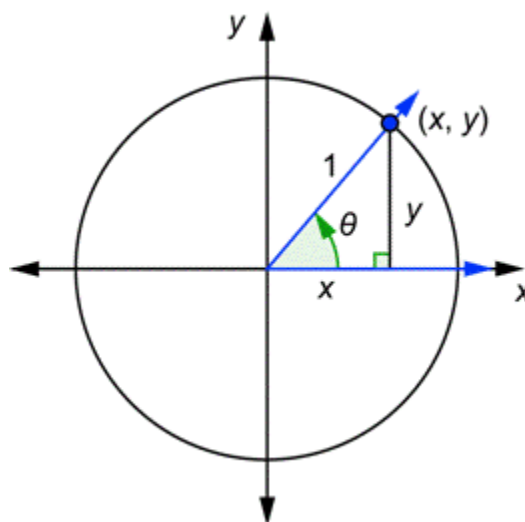
The two triangles have the same angles, so they are similar. Therefore, corresponding sides are proportional. The hypotenuse on the right has length 1 (because it is a radius). Since this is half of the hypotenuse on the left, all of the sides on the right are half of the corresponding sides on the left. For example, the side adjacent to the 30 degree angle on the left is $\sqrt{3}$; therefore the corresponding side on the triangle on the right has to be half that, or $\frac{\sqrt{3}}{2}$.

Look at the right triangle on the left. Using the definitions of sine and cosine:

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \text{ and } \sin 30^\circ = \frac{1}{2}$$

Now look at the point where the terminal side intersects the unit circle. The x -coordinate is equal to $\cos 30^\circ$, and the y -coordinate is equal to $\sin 30^\circ$. This is not a coincidence. Let's look at a more general case.

Suppose you draw any acute angle θ in standard position together with a unit circle, as seen below.



The terminal side of the angle intersects the unit circle at the point (x, y) . Let's write the definitions of **the six trigonometric functions** and then rewrite them by referring to the triangle above and using the variables x and y .

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{1} = y$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{y}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{1} = x$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{x}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

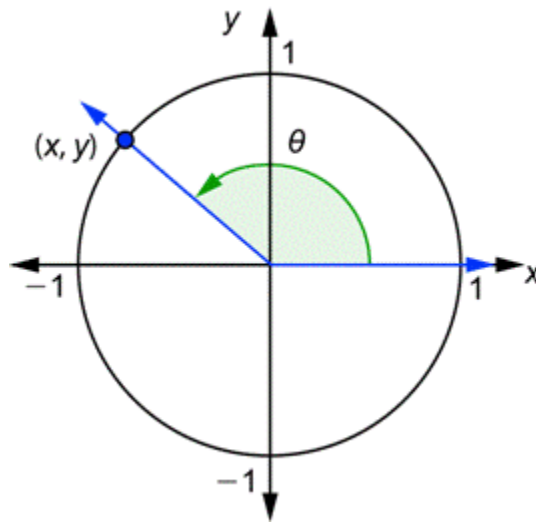
$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$$

The first equation and the one below it, with the middle steps cut out, tell you:

$$\sin \theta = y \text{ and } \cos \theta = x$$

Now you can see that the y -coordinate of this point is always equal to the sine of the angle, and the x -coordinate of this point is always equal to the cosine of the angle.

Now consider *any* angle θ in standard position together with a unit circle. The terminal side will intersect the circle at some point (x, y) . Depending on the angle, that point could be in the first, second, third, or fourth quadrant.

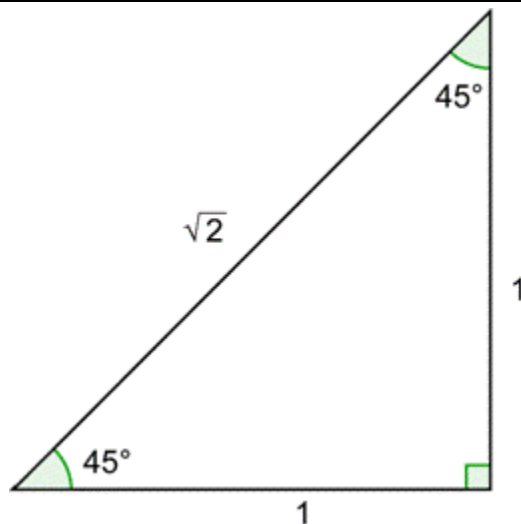


The sine of the angle θ is equal to the y -coordinate of this point and the cosine of the angle θ is equal to the x -coordinate of this point. In fact, the six trigonometric functions for any angle θ are now defined by the six equations listed above.

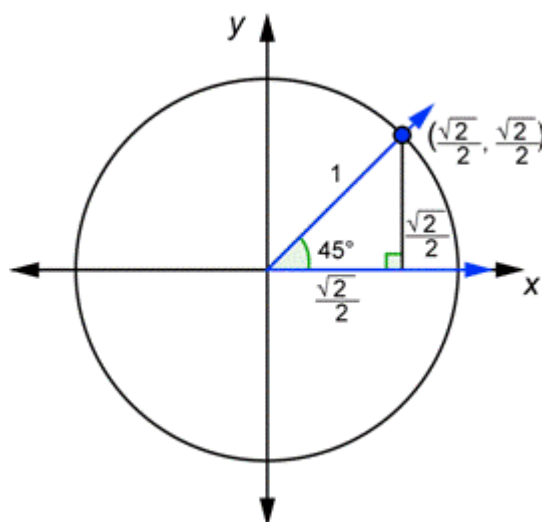
The next few examples will help you confirm that when θ is an acute angle, these new definitions give you the same results as the original definitions.

Example

Problem Draw a 45° angle in standard position together with a unit circle. Confirm that the x- and y-coordinates of the point of intersection of the terminal side and the circle are equal to $\cos 45^\circ$ and $\sin 45^\circ$.



Start with the 45° - 45° - 90° triangle. Then draw the 45° angle in standard position.



This unit circle triangle is similar to the 45° - 45° - 90° triangle. Each side length can be obtained by dividing the lengths of the 45° - 45° - 90° triangle by $\sqrt{2}$. So each leg on the unit circle triangle is:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

From the coordinates on the unit circle: $x = \frac{\sqrt{2}}{2}$

From the triangle: $\cos 45^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

From the coordinates on the unit circle: $y = \frac{\sqrt{2}}{2}$

Look at the x- and y-coordinates of the point on the unit circle, then use the triangle to find $\cos 45^\circ$ and $\sin 45^\circ$.

From the triangle: $\sin 45^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

The x-coordinate is equal to $\cos 45^\circ$, and the y-coordinate is equal to $\sin 45^\circ$.

Answer

Yes, $x = \cos 45^\circ$ and $y = \sin 45^\circ$.

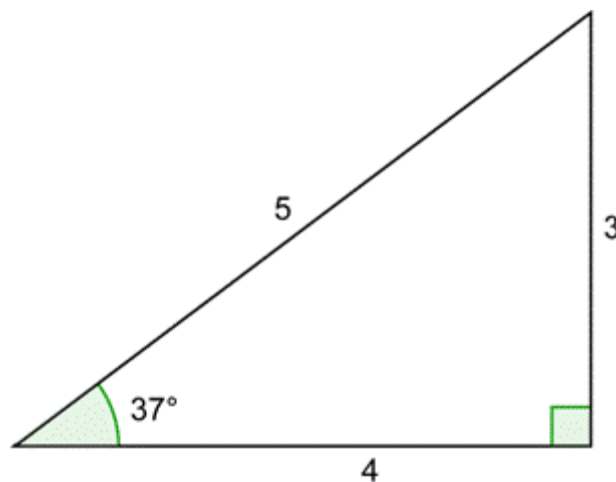
In the next two examples, the angle labels of 37° and 53° are actually very close approximations. The main idea of the examples (that those fractions involving x and y are equal to the various trigonometric functions) still holds true.

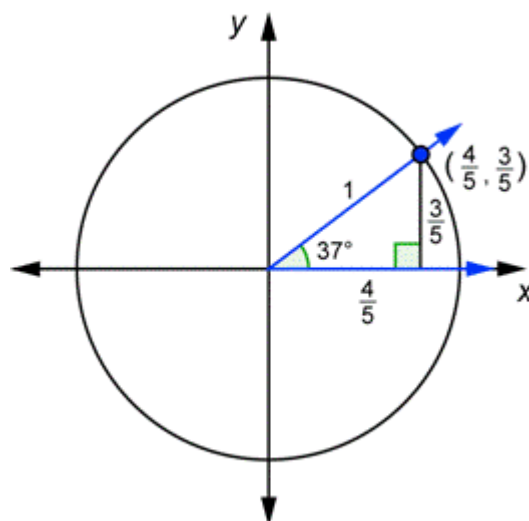
Example

Problem

Draw a 37° angle in standard position together with a unit circle. Use the triangle below to find the x - any y -coordinates of the point of intersection of the terminal side and the circle.

Compute $\frac{y}{x}$ and $\frac{x}{y}$. Confirm that they are equal to $\tan 37^\circ$ and $\cot 37^\circ$.





Draw the 37° angle in standard position. The unit circle triangle is similar to the 3-4-5 right triangle. Because this hypotenuse equals the original hypotenuse divided by 5, you can find the leg lengths by dividing the original leg lengths by 5.

$$x = \frac{4}{5}, y = \frac{3}{5}$$

Find the x - and y -coordinates.

$$\frac{y}{x} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$$

Compute the ratios. Compare the results to what you would get for $\tan 37^\circ$ and

$$\tan 37^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

$\cot 37^\circ$ using the original triangle. They are the same!

$$\frac{x}{y} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{5} \cdot \frac{5}{3} = \frac{4}{3}$$

$$\cot 37^\circ = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

Answer

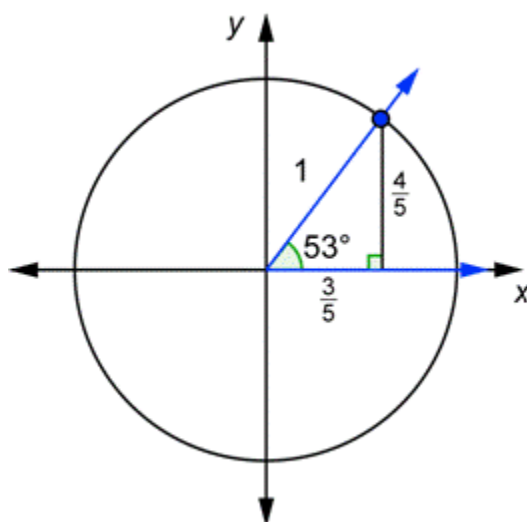
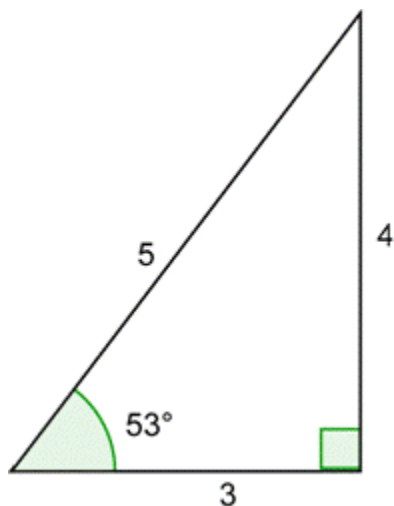
So yes, $\frac{y}{x} = \tan 37^\circ$ and $\frac{x}{y} = \cot 37^\circ$.

Example

Problem

Draw a 53° angle in standard position together with a unit circle. Use the triangle below to find the x - and y -coordinates of the point of intersection of the terminal side and the circle.

Compute $\frac{1}{x}$ and $\frac{1}{y}$. Confirm that they are equal to $\sec 53^\circ$ and $\csc 53^\circ$.



Draw the 53° angle in standard position. The unit circle triangle is similar to the 3-4-5 right triangle. Because this hypotenuse equals the original hypotenuse divided by 5, you can find the leg lengths by dividing the original leg lengths by 5.

$$x = \frac{3}{5}, y = \frac{4}{5}$$

Find the x - and y -coordinates.

$$\frac{1}{x} = \frac{1}{3/5} = \frac{5}{3}$$

$$\sec 53^\circ = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{3}$$

Compute the ratios. Compare the results to what you would get for $\sec 53^\circ$ and $\csc 53^\circ$ using the original triangle. They

$$\frac{1}{y} = \frac{1}{4/5} = \frac{5}{4}$$

are the same!

$$\csc 53^\circ = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{4}$$

Answer

Yes, $\frac{1}{x} = \sec 53^\circ$ and $\frac{1}{y} = \csc 53^\circ$.

The first three of our new definitions lead us to one more important identity:

$$\sin \theta = y, \cos \theta = x, \tan \theta = \frac{y}{x}$$

We can replace y by $\sin \theta$ and x by $\cos \theta$ in $\tan \theta = \frac{y}{x}$ to get the trigonometry identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

$$\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

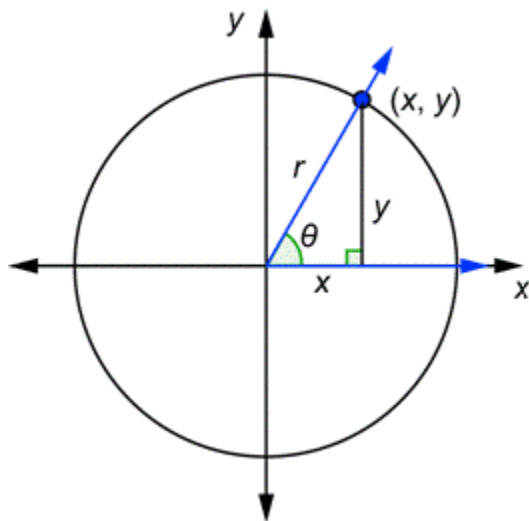
Since **cotangent is the reciprocal of tangent**, this gives you another trigonometric identity.

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Remember, **an identity is true for every possible value of the variable**. So no matter what angle you are using, the values of tangent and cotangent are given by these quotients.

Although some textbooks give slightly different general definitions of the trigonometric functions, the important thing to know is that they end up giving you the same values as the definitions already given you.

For example, start with a circle of radius r (in place of radius 1) and an angle θ in standard position. The terminal side will intersect the circle at some point (x, y) . That point could be in any quadrant, but we show one in the first quadrant.



Now write down the original definitions and then rewrite them using the variables x , y , and r .

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{r}{x}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

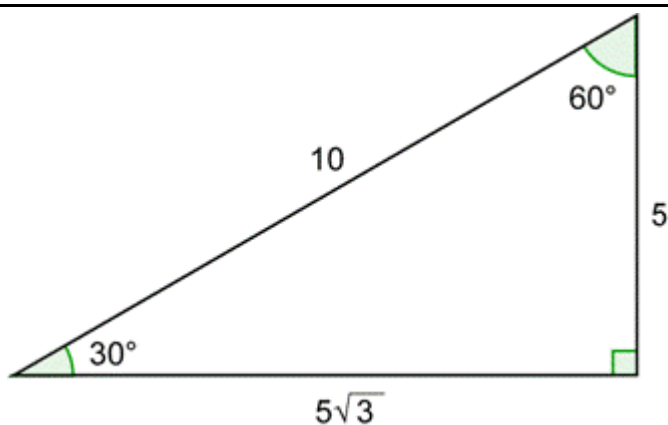
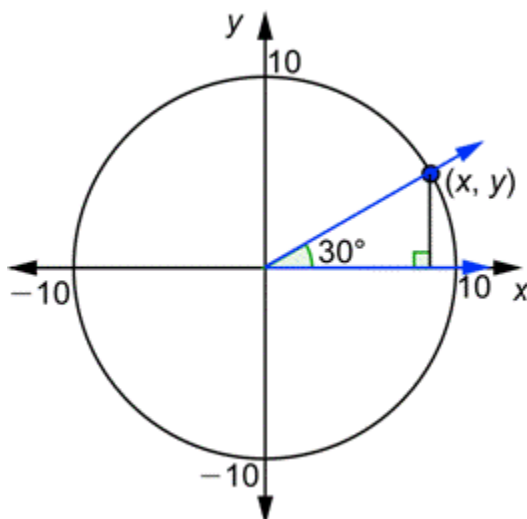
$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$$

These six fractions are used as the general definitions of the trigonometric functions for any angle θ , in any quadrant.

Example

Problem

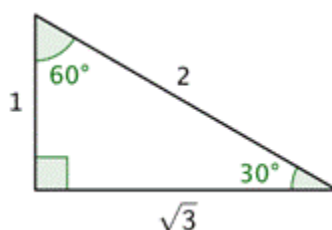
Compute $\frac{y}{x}$ using the diagram below. Confirm that this is the same as the value of $\tan 30^\circ$.



$$y = 5$$

$$x = 5\sqrt{3}$$

$$\tan \theta = \frac{y}{x} = \frac{5}{5\sqrt{3}} = \frac{5}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{5 \cdot 3} = \frac{\sqrt{3}}{3}$$



The above diagram contains a 30° - 60° - 90° triangle. The hypotenuse equals the radius, so it is 10. The side opposite 30° is half of 10, or 5. The adjacent side is $\sqrt{3}$ times the opposite side, or $5\sqrt{3}$.

The sides of the triangle give you the values of x and y in the first diagram.

Substitute these into the definition.

Here is our standard 30° - 60° - 90° triangle.

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Compute $\tan 30^\circ$ using the right triangle definition.

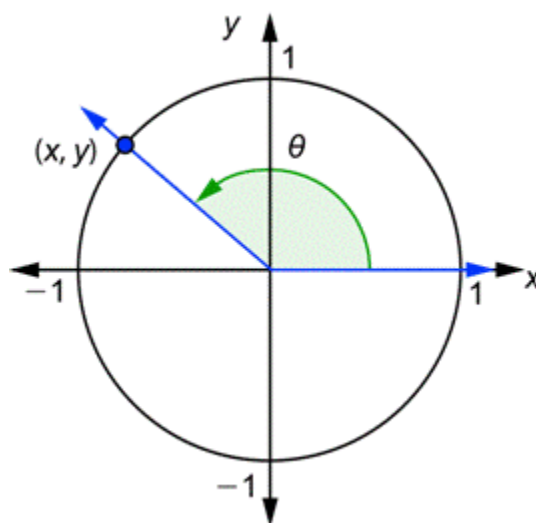
Answer

So yes, $\frac{y}{x} = \frac{\sqrt{3}}{3} = \tan 30^\circ$.

Using the General Definitions of the Trigonometric Functions

You have been given new or “general” definitions of the six trigonometric functions and have seen that, when you compute these functions using acute angles, the result is the same as the result you would get from using the original definitions. Now you will learn how to apply these definitions to angles that are not acute and to negative angles.

Given any angle θ , draw it in standard position together with a unit circle. The terminal side will intersect the circle at some point (x, y) , as shown below.

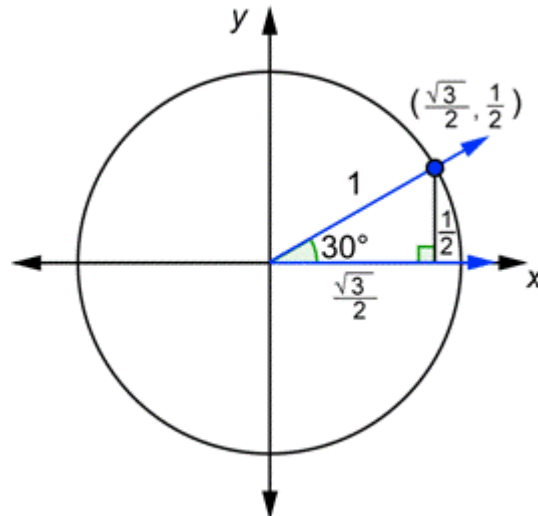


Here again are the general definitions of the six trigonometric functions using a unit circle.

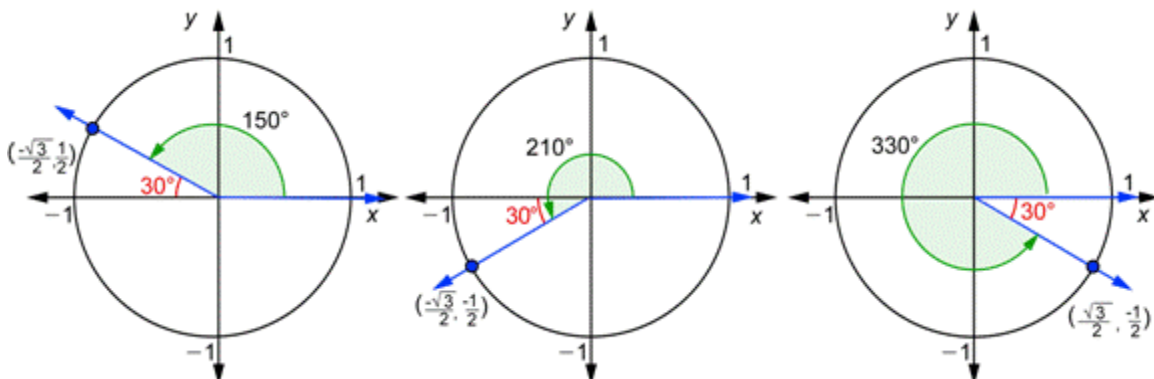


$$\begin{aligned}\sin \theta &= y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

Now let's use these definitions with the angles **30°, 150°, 210°, and 330°**. You have already done this for 30°. Here is that drawing:



The angles 150°, 210°, and 330° have something in common. Every one of them has a reference angle of 30°, as you can see from the drawings below. Because of this, it is easy to find the coordinates of the points where the terminal sides intersect the unit circle using the drawing above.



For the angle 150° , this intersection point is the mirror image of $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ over the y -axis, so the coordinates for 150° are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

For the angle 210° , this point is the mirror image of $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ over the x -axis, so the coordinates for 210° are $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

For the angle 330° , this point is the mirror image of $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ over the x -axis, so the coordinates for 330° are $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

You can now find the values of all six trigonometric functions for 150° , 210° , and 330° . Let's pick a few trigonometric functions and evaluate them using these angles. For example, using the leftmost diagram above and the definition of cosine:

$$\begin{aligned}\cos \theta &= x \\ \cos 150^\circ &= \frac{-\sqrt{3}}{2}\end{aligned}$$

Using the middle diagram and the definition of cotangent:

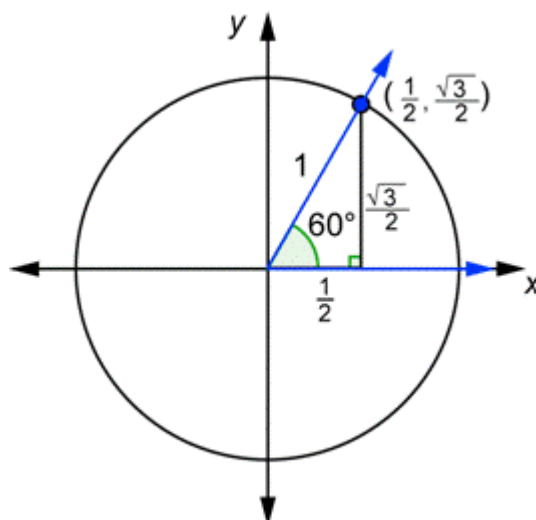
$$\begin{aligned}\cot \theta &= \frac{x}{y} \\ \cot 210^\circ &= \frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{-\sqrt{3}}{2} \cdot \frac{2}{-1} = \frac{-\sqrt{3}}{-1} = \sqrt{3}\end{aligned}$$

Using the rightmost diagram and the definition of cosecant:

$$\csc \theta = \frac{1}{y}$$

$$\csc 330^\circ = \frac{1}{-1/2} = -2$$

If you take the drawing above with the 30° angle in standard position, and turn the triangle so that the shorter leg is on the x-axis, you get a drawing of a 60° angle in standard position, as seen below.

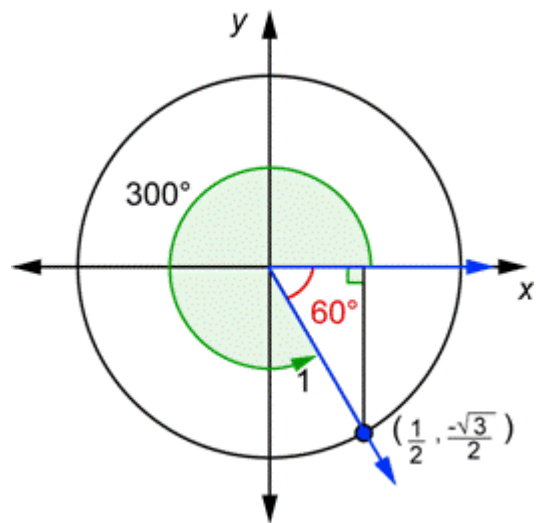


You can use the information in this diagram to find the values of the six trigonometric functions for any angle that has a reference angle of 60° .

Example

Problem

Determine $\cos(300^\circ)$ **and** $\sec(300^\circ)$.



Draw 300° in standard position and find the reference angle. Find the x-coordinate of the point (x, y) where the terminal side intersects the unit circle. Because $\cos 60^\circ = \frac{1}{2}$, we know $x = \frac{1}{2}$.

$$\cos \theta = x$$

$$\cos 300^\circ = \frac{1}{2}$$

Use the definition of cosine. Substitute the value of the x-coordinate that you found above.

$$\sec \theta = \frac{1}{x}$$

$$\sec 300^\circ = \frac{1}{1/2} = 2$$

Use the definition of secant. Substitute the value of the x-coordinate that you found above. Note that, just as with acute angles, secant and cosine are reciprocals.

Answer

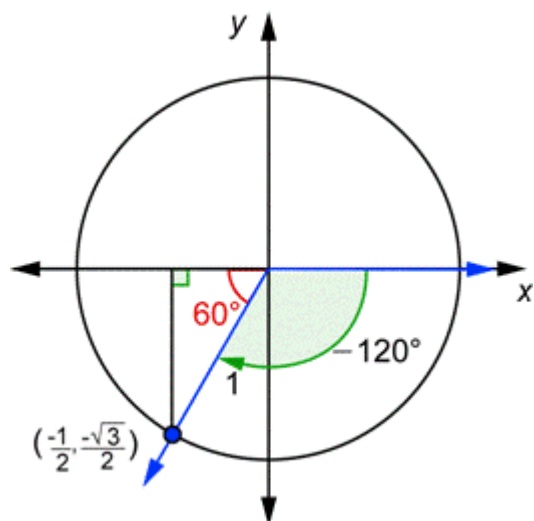
$$\cos 300^\circ = \frac{1}{2}, \sec 300^\circ = 2$$

The procedure is the same even if the angle is negative. Remember that a negative angle is simply one whose direction is clockwise.

Example

Problem

Find the values of $\sin(-120^\circ)$ and $\csc(-120^\circ)$.



Draw -120° in standard position and find the reference angle. Find the y -coordinate of the point (x, y) where the terminal side intersects the unit circle.

Because

$\sin 60^\circ = \frac{\sqrt{3}}{2}$ and we are in the third quadrant, we

know $y = -\frac{\sqrt{3}}{2}$.

Use the definition of sine. Substitute the value of the y -coordinate that you found above.

Use the definition of cosecant. Substitute the value of the y -coordinate that you found above. Rationalize the denominator. Note that, just as with acute angles, cosecant and sine are reciprocals.

$$\sin \theta = y$$

$$\sin(-120^\circ) = -\frac{\sqrt{3}}{2}$$

$$\csc \theta = \frac{1}{y}$$

$$\csc(-120^\circ) = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Answer

$$\sin(-120^\circ) = -\frac{\sqrt{3}}{2}, \quad \csc(-120^\circ) = -\frac{2\sqrt{3}}{3}$$

Look at the results from the last two examples and observe the following:

In each case, the value of the trigonometric function was either the same as the value of that function for the reference angle (60°), or the negative of the value of that function for the reference angle. Why did this

happen? The computations for 60° were done using the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The computations for 300° and -120° were done using the points $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. The x-coordinates have the same absolute value. The y-coordinates also have the same absolute value. When you substitute into the expressions x , $\frac{1}{x}$, y , and $\frac{1}{y}$, the result will be the same, or have a negative sign.

You will get a similar result with other angles. So the procedure for finding the value of a trigonometric function simplifies to the following:

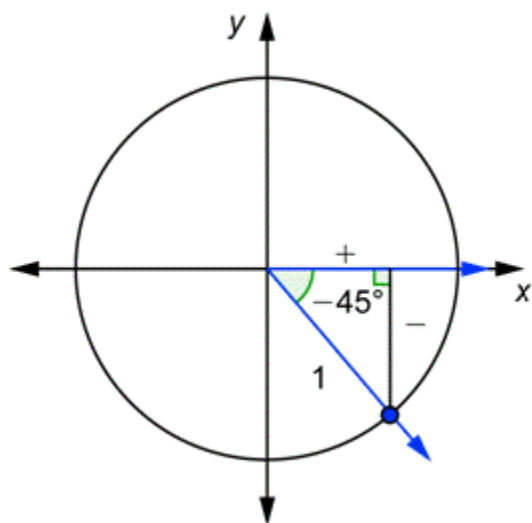
- Determine the reference angle.
- Calculate the trigonometric function value of the reference angle.
- Determine if the value of the function is positive or negative.

Let's try this procedure in the following example.

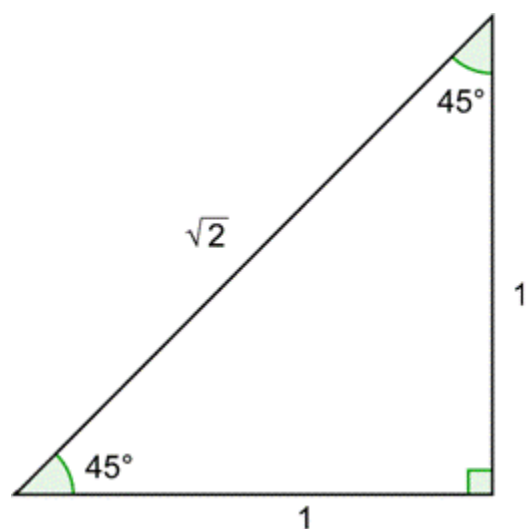
Example

Problem

Compute $\tan(-45^\circ)$ and $\cot(-45^\circ)$.



Draw the angle in standard position. The reference angle is 45° .



Use the $45^\circ - 45^\circ - 90^\circ$ triangle. The value of $\tan 45^\circ$ is 1.

$$\tan(-45^\circ) = \frac{y}{x} = \frac{(-)}{(+)} = (-)$$

$$\tan(-45^\circ) = -\tan 45^\circ = -1$$

In the first diagram, we put a $+$ sign to indicate that x is positive, and a $-$ sign to indicate that y is negative. Use this to determine the sign of $\tan(-45^\circ)$.

Since the result was negative, the value of $\tan(-45^\circ)$ is negative.

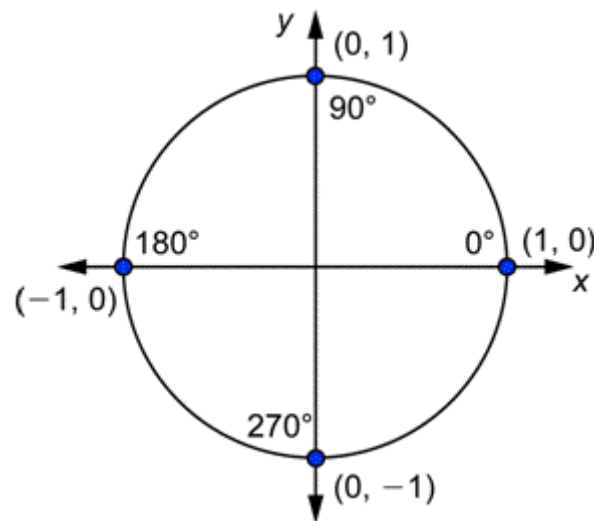
$$\cot(-45^\circ) = \frac{1}{\tan(-45^\circ)} = \frac{1}{-1} = -1$$

You can go through a similar procedure with cotangent or use the fact that it is the reciprocal of tangent.

Answer

$$\tan(-45^\circ) = -1, \cot(-45^\circ) = -1$$

The angles whose measures are a multiple of 90° have terminal sides on the axes. This can be confusing, because the terminal side is not in one quadrant, but rather on a border between quadrants. So let's look at these angles separately. The drawing below shows the points of intersection of the terminal sides of 0° , 90° , 180° , and 270° with the unit circle. **(Quadrant Angles)**



You can use this drawing and the definitions to find the trigonometric functions for 0° , 90° , 180° , and 270° . For example:

$$\begin{aligned}\sin \theta &= y \\ \sin 0^\circ &= 0 \\ \sin 90^\circ &= 1 \\ \sin 180^\circ &= 0 \\ \sin 270^\circ &= -1\end{aligned}$$

For all six functions, you substitute the values of x and y as you did earlier. However, what happens if you try to

compute $\tan 90^\circ$ using the definition $\tan \theta = \frac{y}{x}$?

$$\tan \theta = \frac{y}{x}$$

$$\tan 90^\circ = \frac{1}{0}$$

You cannot divide by 0, so $\tan 90^\circ$ is simply undefined. Similarly $\csc 180^\circ$ is undefined, because if you try to apply the definition, you will end up dividing by 0. The same is true any time one of the definitions leads to division by 0: the trigonometric function is undefined for that angle.

What are the values of $\cos 90^\circ$ and $\cot 180^\circ$?

- A) $\cos 90^\circ = 0$ and $\cot 180^\circ = 0$
- B) $\cos 90^\circ = 0$ and $\cot 180^\circ = \text{undefined}$
- C) $\cos 90^\circ = 1$ and $\cot 180^\circ = 0$
- D) $\cos 90^\circ = 1$ and $\cot 180^\circ = \text{undefined}$

[Show/Hide Answer](#)

You can use the following charts to help you remember the values of the trigonometric functions for the reference angles 0° , 30° , 45° , 60° , and 90° for sine and cosine. Once you have these, you can get the value of

tangent from the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$, and the values of the other three trigonometric functions using reciprocals.

Make a table as follows:

	0°	30°	45°	60°	90°
sine					
cosine					

As an initial step, put the numbers 0, 1, 2, 3, and 4 in the “sine” row and 4, 3, 2, 1, and 0 in the “cosine” row. Do this in pencil. You are going to replace these numbers!

	0°	30°	45°	60°	90°
sine	0	1	2	3	4
cosine	4	3	2	1	0

Now replace the numbers 0 through 4 by taking their square roots and dividing by 2. The rows now contain the correct, but unsimplified, values for sine and cosine.

	0°	30°	45°	60°	90°
sine	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cosine	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

You can simplify $\sqrt{0}$ to 0, $\sqrt{1}$ to 1, and $\sqrt{4}$ to 2, and then divide by 2. This will give you the final table with the correct values of sine and cosine at these angles.

	0°	30°	45°	60°	90°
sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Determining the Sign of the Trigonometric Functions

First you learned the definitions for the trigonometric functions of an acute angle. Then you learned the general definitions of these functions, which can be used for any angle, and the method for applying them. Finally, you learned a simpler procedure for finding the values of trigonometric functions:

- Determine the reference angle.
- Calculate the trigonometric function value of the reference angle.
- Determine if the value of the function is positive or negative.

Now you'll learn an easy way to remember where the trigonometric functions are positive and where they are negative.

The sine function: since $\sin \theta = y$, **sine is positive** when $y > 0$. This occurs in **Quadrants I and II**.

The cosine function: since $\cos \theta = x$, **cosine is positive** when $x > 0$. This occurs in **Quadrants I and IV**.

The tangent function: since $\tan \theta = \frac{y}{x}$, **tangent is positive** when x and y are both positive or both negative. This occurs in **Quadrants I and III**.

We can summarize this information by quadrant:

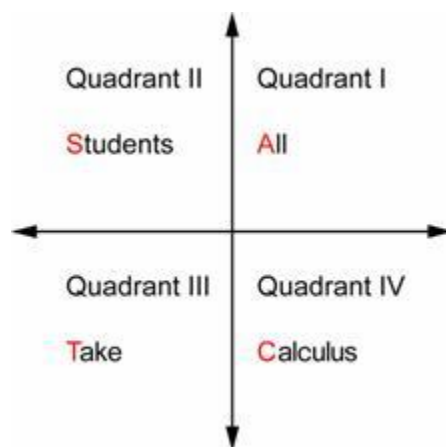
Quadrant I: sine, cosine, and tangent are positive.

Quadrant II: sine is positive (cosine and tangent are negative).

Quadrant III: tangent is positive (sine and cosine are negative).

Quadrant IV: cosine is positive (sine and tangent are negative).

Let **A** stand for all (three functions, sine, cosine, and tangent), **S** stand for sine, **T** stand for tangent, and **C** stand for cosine. Now you can use these single letters to remember in which quadrant sine, cosine, and tangent are positive. The final step is to replace each letter by a word to give you a phrase that's easy to remember: "All Students Take Calculus." Going counterclockwise, place these words in the four quadrants.



Now if you look in Quadrant II, for example, you see the word **S**tudents. The **S** tells you that **s**ine is positive (while cosine and tangent are negative).

Example	
Problem	What signs are $\tan 200^\circ$ and $\cos 200^\circ$?
<p>Since $200^\circ = 180^\circ + 20^\circ$, 200° is in Quadrant III.</p> <p>The word "Take" represents the fact that tangent is positive, so $\tan 200^\circ$ is positive.</p> <p>Sine and cosine are negative in Quadrant III, so $\cos 200^\circ$ is negative.</p>	
Answer	$\tan 200^\circ$ is positive and $\cos 200^\circ$ is negative.

Example	
Problem	In which quadrant must an angle lie if its sine is positive and its tangent is negative?
<p>The words “All” and “Students” tell us that sine is positive in Quadrants I and II.</p> <p>Tangent is positive in Quadrant I, but negative in Quadrant II.</p>	
Answer	Quadrant II

This device applies to the functions sine, cosine, and tangent. The other three trigonometric functions are reciprocals of these three. Recall the basic fact that the reciprocal of a positive number is positive, and the reciprocal of a negative number is negative. This implies that sine and cosecant have the same sign, cosine and secant have the same sign, and tangent and cotangent have the same sign. So if you want to know the sign of cosecant, secant, or cotangent, find the sign of sine, cosine, or tangent, respectively.

What signs are $\cos(-25^\circ)$ and $\sec(-25^\circ)$?

- A) They are both positive.
- B) They are both negative.
- C) $\cos(-25^\circ) > 0$ and $\sec(-25^\circ) < 0$
- D) $\cos(-25^\circ) < 0$ and $\sec(-25^\circ) > 0$

Summary

The trigonometric functions were originally defined for acute angles. There are general definitions of these functions, which apply to angles of any size, including negative angles. Values of trigonometric functions are computed by finding the reference angle, determining the value of the trigonometric function of the reference angle, and then determining if the value of the function is positive or negative. A useful way to remember this last step is “**All Students Take Calculus.**”