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Derivatives Project

The constant rule $\frac{d}{dx}[c] = 0$ *Example* $f(x) = 10$ $\frac{d}{dx}[10] = 0$ $f'(x) = 0$
The power rule $\frac{d}{dx}[x^n] = nx^{n-1}$ *Example* $f(x) = 3x^6$ $\frac{d}{dx}f(x) = (6)(3)x^{6-1}$ $\frac{d}{dx}f(x) = 18x^5$ $f'(x) = 18x^5$
The product rule $\frac{d}{dx}[uv] = uv' + vu'$ *Example* $f(x) = x^4(x^3 + 2x^2 + 5)$ $\frac{d}{dx}f(x) = \left[x^4 \frac{d}{dx}(x^3 + 2x^2 + 5)\right] + \left[x^3 + 2x^2 + 5 \frac{d}{dx}(x^4)\right]$ $\frac{d}{dx}f(x) = (x^4)(3x^2 + 4x) + (x^3 + 2x^2 + 5)(4x^3)$ $\frac{d}{dx}f(x) = 3x^6 + 4x^5 + 4x^6 + 8x^5 + 20x^3$ $\frac{d}{dx}f(x) = 7x^6 + 12x^5 + 20x^3$ $f'(x) = 7x^6 + 12x^5 + 20x^3$
The quotient rule $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$ *Example* $f(x) = \left[\frac{4x + 2}{x^2 - 1}\right]$

Kinza Rizwan

$$\frac{d}{dx}f(x) = \frac{\left[(x^2-1)\frac{d}{dx}(4x+2)\right] - \left[(4x+2)\frac{d}{dx}(x^2-1)\right]}{(x^2-1)^2}$$

$$\frac{d}{dx}f(x) = \frac{(x^2-1)(4)-(4x+2)(2x)}{(x^2-1)^2}$$

$$\frac{d}{dx}f(x) = \frac{4x^2-4-8x^2-4x}{(x^2-1)^2}$$

$$\frac{d}{dx}f(x) = \frac{-4x^2-4x-4}{(x^2-1)^2}$$

$$f'(x) = \frac{-4x^2-4x-4}{(x^2-1)^2}$$

The Chain rule

$$\frac{d}{dx}[f\left(g(x)\right) = f'(g(x)) \cdot g'(x)$$

Example

$$h(x) = (2x^2+3x^4)^2$$

$$f(x) = x^2 \qquad \frac{d}{dx}f(x) = 2x$$

$$g(x) = 2x^2+3x^4 \qquad \frac{d}{dx}g(x) = 4x+12x^3$$

$$\frac{d}{dx}h(x) = [2(2x^2+3x^4)] \cdot [4x+12x^3]$$

$$\frac{d}{dx}h(x) = [4x^2+6x^4] \cdot [4x+12x^3]$$

$$\frac{d}{dx}h(x) = 16x^3+48x^5+24x^5+72x^7$$

$$\frac{d}{dx}h(x) = 16x^3+72x^5+72x^7$$

$$f'(x) = 16x^3+72x^5+72x^7$$

The derivative of sine

$$\frac{d}{dx}[\sin u] = \cos u\,\frac{du}{dx}$$

Example

$$f(x) = \sin 7x^2$$

$$\frac{d}{dx}f(x) = \cos(7x^2)\frac{d}{dx}7x^2$$

$$\frac{d}{dx}f(x) = [\cos(7x^2)][14x]$$

$$\frac{d}{dx}f(x) = 14x\cos(7x^2)$$

$$f'(x) = 14x\cos(7x^2)$$

Kinza Rizwan

The derivative of cosine $$\frac{d}{dx}[\cos u] = -\sin u\frac{du}{dx}$$ *Example* $$f(x) = \cos(2x^3 + 1)$$ $$\frac{d}{dx}f(x) = -\sin(2x^3 + 1)\frac{d}{dx}(2x^3 + 1)$$ $$\frac{d}{dx}f(x) = [-\sin(2x^3 + 1)][6x^2]$$ $$\frac{d}{dx}f(x) = [6x^2][-\sin(2x^3 + 1)]$$ $$f'(x) = [6x^2][-\sin(2x^3 + 1)]$$
The derivative of tangent $$\frac{d}{dx}[\tan u] = sec^2 u\frac{du}{dx}$$ *Example* $$f(x) = \tan(4x + 1)$$ $$\frac{d}{dx}f(x) = sec^2(4x + 1)\frac{d}{dx}(4x + 1)$$ $$\frac{d}{dx}f(x) = [sec^2(4x + 1)][4]$$ $$\frac{d}{dx}f(x) = 4sec^2(4x + 1)$$ $$f'(x) = 4sec^2(4x + 1)$$
The derivative of cotangent $$\frac{d}{dx}[\cot u] = -\csc u\frac{du}{dx}$$ *Example* $$f(x) = \cot(9x - 2x^4)$$ $$\frac{d}{dx}f(x) = -\csc(9x - 2x^4)\frac{d}{dx}(9x - 2x^4)$$ $$\frac{d}{dx}f(x) = [-\csc(9x - 2x^4)][9 - 8x^3]$$ $$\frac{d}{dx}f(x) = [9 - 8x^3][-\csc(9x - 2x^4)]$$ $$f'(x) = [9 - 8x^3][-\csc(9x - 2x^4)]$$
The derivative of secant $$\frac{d}{dx}[\sec u] = \sec u \tan u\ \frac{du}{dx}$$ *Example* $$f(x) = \sec(2x^2 - 3x)$$

$$\frac{d}{dx}f(x) = \sec(2x^2 - 3x)\tan(2x^2 - 3x)\frac{d}{dx}(2x^2 - 3x)$$ $$\frac{d}{dx}f(x) = [\sec(2x^2 - 3x)\tan(2x^2 - 3x)][4x - 3]$$ $$\frac{d}{dx}f(x) = [4x - 3][\sec(2x^2 - 3x)\tan(2x^2 - 3x)]$$ $$f'(x) = [4x - 3][\sec(2x^2 - 3x)\tan(2x^2 - 3x)]$$
The derivative of cosecant $$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$ *Example* $$f(x) = \csc 4\sqrt{x}$$ $$\frac{d}{dx}f(x) = -\csc 4\sqrt{x}\ \cot 4\sqrt{x}\ \frac{d}{dx}4\sqrt{x}$$ $$\frac{d}{dx}f(x) = \left[-\csc 4\sqrt{x}\ \cot 4\sqrt{x}\right]\left[2x^{-\frac{1}{2}}\right]$$ $$\frac{d}{dx}f(x) = \left[-\csc 4\sqrt{x}\ \cot 4\sqrt{x}\right]\left[\frac{2}{\sqrt{x}}\right]$$ $$\frac{d}{dx}f(x) = \left[\frac{2}{\sqrt{x}}\right]\left[-\csc 4\sqrt{x}\ \cot 4\sqrt{x}\right]$$ $$f'(x) = \left[\frac{2}{\sqrt{x}}\right]\left[-\csc 4\sqrt{x}\ \cot 4\sqrt{x}\right]$$
The derivative of natural log $$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$$ *Example* $$f(x) = \ln(25x + x^2)$$ $$\frac{d}{dx}f(x) = \frac{1}{25x + x^2}\frac{d}{dx}(25x + x^2)$$ $$\frac{d}{dx}f(x) = \frac{1}{25x + x^2}(25 + 2x)$$ $$\frac{d}{dx}f(x) = \frac{25 + 2x}{25x + x^2}$$ $$f'(x) = \frac{25 + 2x}{25x + x^2}$$
The derivative of logarithm $$\frac{d}{dx}[log_a u] = \frac{1}{u \ln a}\frac{du}{dx}$$

Example

$$f(x) = log_3(2x^2 + 3x + 1)$$

$$\frac{d}{dx}f(x) = \frac{1}{(2x^2 + 3x + 1)\ln 3}\frac{d}{dx}(2x^2 + 3x + 1)$$

$$\frac{d}{dx}f(x) = \frac{1}{(2x^2 + 3x + 1)\ln 3}(4x + 3)$$

$$\frac{d}{dx}f(x) = \frac{4x + 3}{(2x^2 + 3x + 1)\ln 3}$$

$$f'(x) = \frac{4x + 3}{(2x^2 + 3x + 1)\ln 3}$$

The derivative of 'e' raised to a variable

$$\frac{d}{dx}[e^u] = e^u\frac{du}{dx}$$

Example

$$f(x) = e^{-4x^2-3x}$$

$$\frac{d}{dx}f(x) = [e^{-4x^2-3x}]\frac{d}{dx}(-4x^2 - 3x)$$

$$\frac{d}{dx}f(x) = [e^{-4x^2-3x}][-8x - 3]$$

$$\frac{d}{dx}f(x) = [-8x - 3][e^{-4x^2-3x}]$$

$$f'(x) = [-8x - 3][e^{-4x^2-3x}]$$

The derivative of a constant raised to a variable

$$\frac{d}{dx}[a^u] = a^u \ln a\frac{du}{dx}$$

Example

$$f(x) = 3^{-4x^2}$$

$$\frac{d}{dx}f(x) = 3^{-4x^2}\ln 3\frac{d}{dx}(-4x^2)$$

$$\frac{d}{dx}f(x) = \left[3^{-4x^2}\ln 3\right][-8x]$$

$$\frac{d}{dx}f(x) = -8x\left[3^{-4x^2}\ln 3\right]$$

$$f'(x) = -8x\left[3^{-4x^2}\ln 3\right]$$

The derivative of inverse sine

$$\frac{d}{dx}[\sin^{-1}u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

Example

$$f(x) = \sin^{-1}2x$$

$$\frac{d}{dx}f(x) = \frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)$$

$$\frac{d}{dx}f(x) = \frac{1}{\sqrt{(1-2x)(1+2x)}}(2)$$

$$\frac{d}{dx}f(x) = \left[\frac{2}{\sqrt{(1-2x)(1+2x)}}\right]^2$$

$$\frac{d}{dx}f(x) = \frac{4}{(1-2x)(1+2x)}$$

$$f'(x) = \frac{4}{(1-2x)(1+2x)}$$

<u>The derivative of inverse cosine</u>

$$\frac{d}{dx}[\cos^{-1} u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

<u>Example</u>

$$f(x) = \cos^{-1} 3x^{-2}$$

$$\frac{d}{dx}f(x) = -\frac{1}{\sqrt{1-(3x^{-2})^2}}\frac{d}{dx}(3x^{-2})$$

$$\frac{d}{dx}f(x) = -\frac{1}{\sqrt{(1-3x^{-2})(1+3x^{-2})}}(-6x^{-3})$$

$$\frac{d}{dx}f(x) = \left[-\frac{-6x^{-3}}{\sqrt{(1-3x^{-2})(1+3x^{-2})}}\right]^2$$

$$\frac{d}{dx}f(x) = \frac{36x^{-6}}{(1-3x^{-2})(1+3x^{-2})}$$

$$f'(x) = \frac{36x^{-6}}{(1-3x^{-2})(1+3x^{-2})}$$

<u>The derivative of inverse tangent</u>

$$\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2}\frac{du}{dx}$$

<u>Example</u>

$$f(x) = \tan^{-1}(5x^3)$$

$$\frac{d}{dx}f(x) = \frac{1}{1+(5x^3)^2}\frac{d}{dx}(5x^3)$$

$$\frac{d}{dx}f(x) = \frac{1}{1+25x^6}(15x^2)$$

$$\frac{d}{dx}f(x) = \frac{15x^2}{1+25x^6}$$

$$f'(x) = \frac{15x^2}{1+25x^6}$$

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The derivative of inverse cotangent $$\frac{d}{dx}[\cot^{-1} u] = -\frac{1}{1+u^2}\frac{du}{dx}$$ *Example* $$f(x) = \cot^{-1}(x^5)$$ $$\frac{d}{dx}f(x) = -\frac{1}{1+(x^5)^2}\frac{d}{dx}(x^5)$$ $$\frac{d}{dx}f(x) = -\frac{1}{1+x^{10}}(5x^4)$$ $$\frac{d}{dx}f(x) = -\frac{5x^4}{1+x^{10}}$$ $$f'(x) = -\frac{5x^4}{1+x^{10}}$$
The derivative of inverse secant $$\frac{d}{dx}[\sec^{-1} u] = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$$ *Example* $$f(x) = (4x^5)$$ $$\frac{d}{dx}f(x) = \frac{1}{\lvert 4x^5\rvert\sqrt{(4x^5)^2-1}}\frac{d}{dx}(4x^5)$$ $$\frac{d}{dx}f(x) = \frac{1}{\lvert 4x^5\rvert\sqrt{(4x^5-1)(4x^5+1)}}(20x^4)$$ $$\frac{d}{dx}f(x) = \frac{20x^4}{\lvert 4x^5\rvert\sqrt{(4x^5-1)(4x^5+1)}}$$
The derivative of inverse cosecant $$\frac{d}{dx}[\csc^{-1} u] = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$$ *Example* $$f(x) = \csc^{-1} 2x$$ $$\frac{d}{dx}f(x) == -\frac{1}{\lvert 2x\rvert\sqrt{(2x)^2-1}}\frac{d}{dx}(2x)$$ $$\frac{d}{dx}f(x) = -\frac{2}{\lvert 2x\rvert\sqrt{(2x)^2-1}}$$ $$\frac{d}{dx}f(x) = -\frac{2}{\lvert 2x\rvert\sqrt{(2x-1)(2x+1)}}$$ $$f'(x) = -\frac{2}{\lvert 2x\rvert\sqrt{(2x-1)(2x+1)}}$$

Kinza Rizwan

The derivative of inverse function

$$(f^{-1})'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$$

Example

$f(x) = x^2$ $\quad f^{-1}(x) = \sqrt{x}$ $\quad f'(x) = 2x$

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'^{\left(f^{-1}(x)\right)}}$$

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{2 \cdot \sqrt{x}}$$