School: _

Directions: Provide answers to the following questions. The following are examples of acceptable answers.

$$\frac{1}{2}\ln(37)-1$$
, $3\sin(2)+\cos(3)$, $-\frac{1}{2}$, 0.75, $\sqrt{1+\sin(13)+\cos(1)}$, There is no solution.

- 1. The domain of $f(x) = \arcsin(\sqrt{2x-1}-1)$ is an interval of the form [a,b]. a+b=
- 2. A ball is dropped from a height of 5 feet. It bounces upwards after striking the ground, but it loses energy from impact so that it only rises to 2.5 feet before heading downward again. Again, it bounces upwards after striking the ground, but it loses energy from impact so that it only rises 1.25 feet before heading downward again. Suppose this pattern continues forever. Give the total vertical distance traveled by the ball.
- 3. Simplify $\tan\left(\arcsin\left(\cos\left(\arctan\left(\frac{1}{a+1}\right)\right)\right)\right)$.
- 4. Solve the equation $\log_2 x + \log_4 x + \log_{16} x + \log_{32} x = 13$.
- 5. Give the largest value of b so that the system of equations

$$y - ax^2 = 3$$
$$x - ay^2 = 3$$

has 2 solutions whenever 0 < a < b.

6. $f(x) = \frac{x}{x-1}$ and $g(x) = \frac{x+1}{x}$. Give the *y* intercept for the graph of $(g \circ f)(x)$.

7. Give the horizontal asymptote for the graph of $h(x) = \ln\left(\frac{2x}{x-1}\right) + e^{-5x} + e^{-3x} + 2$.

- 8. Give the number of solutions to $2^{\sin^2(x)} = 4^{\cos(x)}$ for $-4\pi \le x \le 4\pi$.
- 9. A line with negative slope passes through the point (2,3), and the portion of this line in the first quadrant is the hypotenuse of a right triangle. The base of this triangle is parallel to the *x*-axis, and has length *a*. Give a formula for the area of the triangle in terms of *a*.
- 10. A polynomial with real coefficients has the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, and roots 2, 3 and

1-2i. Give the value of $\frac{e}{a}$.

11.
$$\sin^2(1^o) + \sin^2(2^o) + \dots + \sin^2(359^o) + \sin^2(360^o) =$$

- 12. a, b > 0, and the fundamental period of the function $f(x) = a \sin(ax-1) + b \cos(bx+2)$ is 3. Give the larger of the values a and b.
- 13. Find the point in the first quadrant which is the intersection of the curve given in polar coordinates by

 $r = 2\cos(\theta)$ with the curve given in Cartesian coordinates by $y = \frac{1}{2}(x^2 + y^2)$. Record your answer in

Cartesian coordinates.

- 14. $f(2x-1) = 2x-3x^2$. Simplify f(3x+2).
- 15. Simplify $\cos(x)\sin\left(\frac{\pi}{2}-x\right)-\sin(x)\cos\left(\frac{\pi}{2}-x\right)$.
- 16. Suppose a, b, c are real numbers. Give the largest interval (a, c) so that the equation

 $2x^2 - bxy + 3y^2 + 2x - 5y + 6 = 0$ represents an ellipse or a circle when a < b < c.

- 17. Give an equation in Cartesian coordinates for the curve given parametrically by $x = 2\cos(t) - 3$, $y = 2 - 3\sin(t)$ for $0 \le t \le 2\pi$.
- 18. Let u = 2i + j and v = i + 5j. S is the set of all points that are terminal ends of a vector of the form $\alpha u + \beta v$, where $\alpha, \beta \in [0,1]$, and the initial end of the vector is located at (1,2). Give the area of *S*.
- 19. The vertical and horizontal positions of a particle at time t (in seconds) are given by x = 10t feet and $y = -16t^2 + bt + 20$ feet for $t \ge 0$, as long as $y \ge 0$. In this setting, the value of y represents the vertical distance of the particle from the ground, and y = 0 when the particle strikes the ground. The velocity of this particle is given by the vector 10i + (-16t + b)j, and its speed is the magnitude of the velocity vector. Give the speed of the particle when it reaches the point (20,24). Please include appropriate units in your answer.
- 20. Suppose N is a positive integer. Simplify $1 \sum_{n=1}^{N} \frac{2}{n(n+1)}$.
- 21. Give the largest integer c which is an integer multiple of 3, and $\frac{1}{2}x + 5\sin(6x) = 0$ has at least c

solutions.

22. Give the value h shown below.



23. Give the value of y associated with the solution to the system $\begin{pmatrix} -2e^{-2x} + 3e^y = 17\\ 3e^{-2x} - 5e^y = -45 \end{pmatrix}$.

- 24. $\sum_{n=1}^{100} \left(\cos\left(\frac{n\pi}{100}\right) + 1 \right) \cos\left(\frac{n\pi}{100}\right) =$
- 25. Give the positive solution to the equation $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots + \frac{1}{x^{100}} = 3$, accurate to 4 digits after the

decimal point.

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