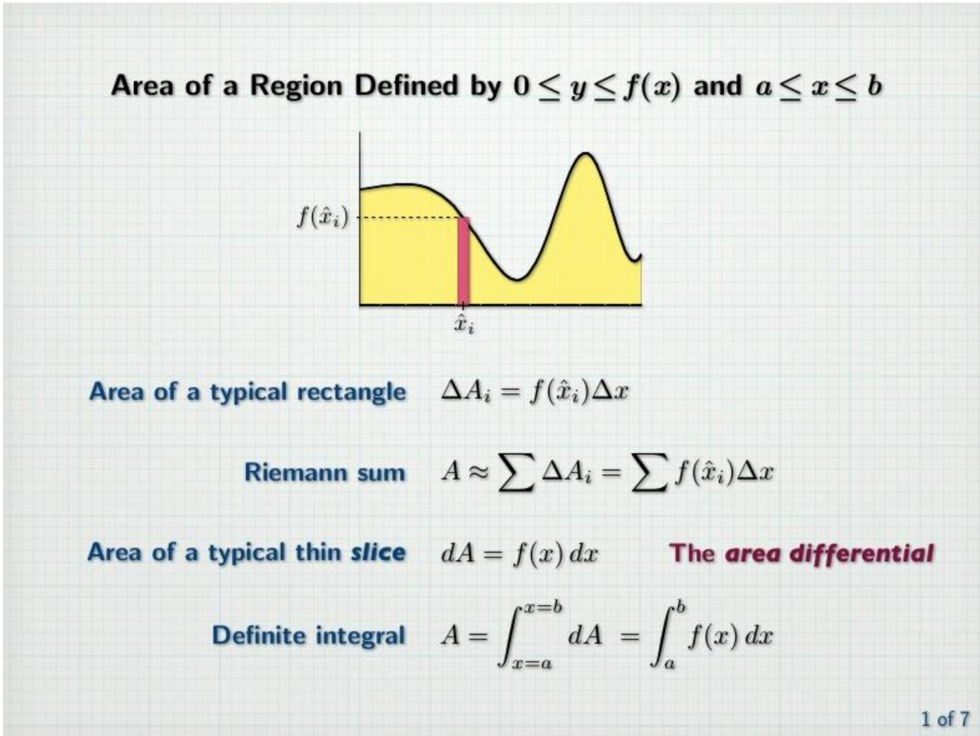
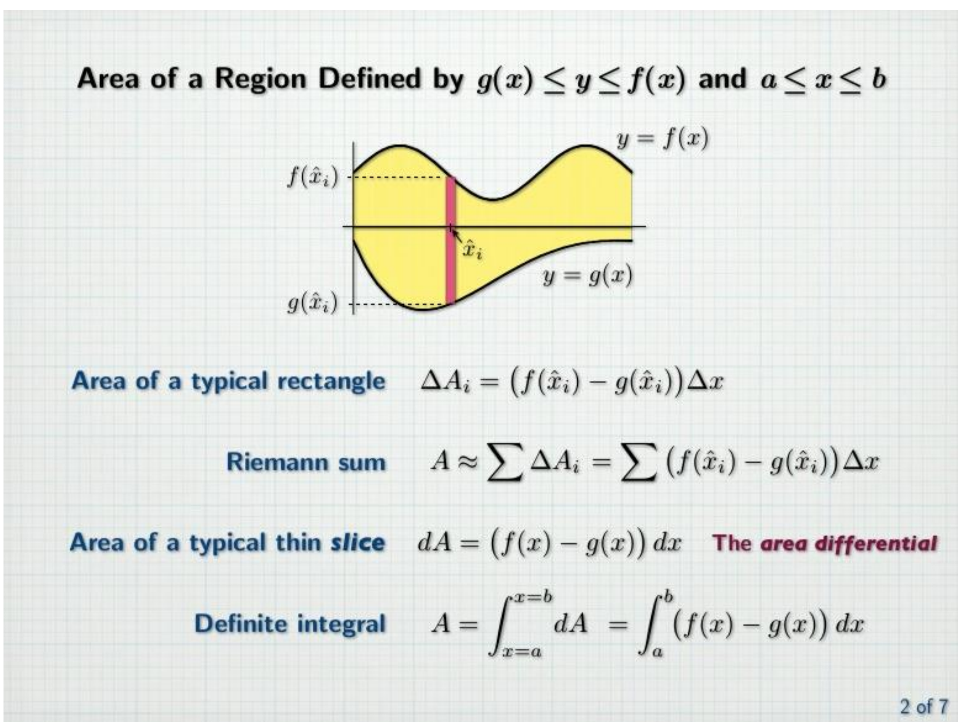
Find the Area between two curves: (Video #26, is 19 minutes)

http://online.math.uh.edu/HoustonACT/videocalculus/SV3/26-areas.mov

Area under the curve:

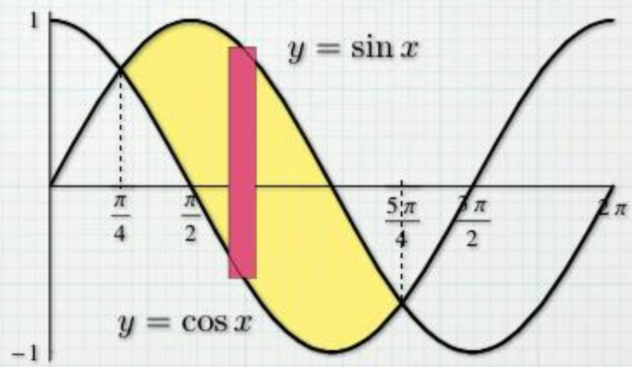


Area between two curves:



Solve this problem (Page 3)

Example Find the area of the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ between $x = \pi/4$ and $x = 5\pi/4$.



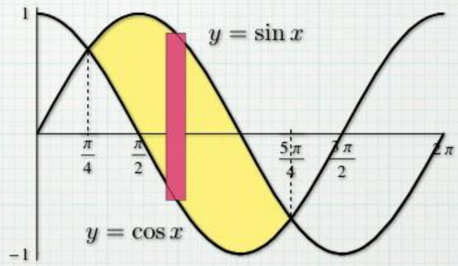
$$\Delta A_i = (\sin \hat{x}_i - \cos \hat{x}_i)\Delta x$$

$$dA = (\sin x - \cos x)\, dx$$

$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$

Answer to page 3

Example Find the area of the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ between $x = \pi/4$ and $x = 5\pi/4$.



$$\Delta A_i = (\sin \hat{x}_i - \cos \hat{x}_i)\Delta x$$

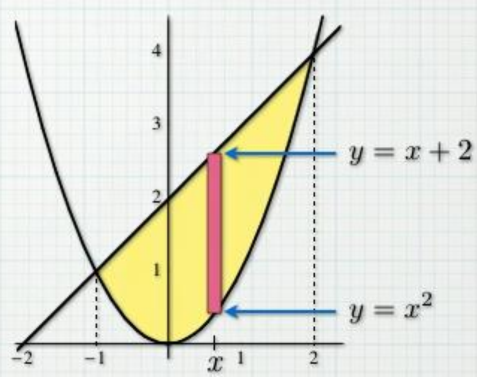
$$dA = (\sin x - \cos x)\, dx$$

$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx = (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

Answer to Page 4

Example Find the area of the region bounded by the graphs of $y = x^2$ and $y = x + 2$.



Points of intersection

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x+1)(x-2) = 0$$
$$x = -1,\ 2$$

$$A = \int_{-1}^{2} (x + 2 - x^2)\, dx$$

$$dA = (x + 2 - x^2)\, dx$$

$$= \left(\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3\right)\Big|_{-1}^{2}$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = 8 - \frac{9}{3} - \frac{1}{2} = \frac{9}{2}$$

Solve this problem (Page 4)

Example Find the area of the region bounded by the graphs of $y = x^2$ and $y = x + 2$.

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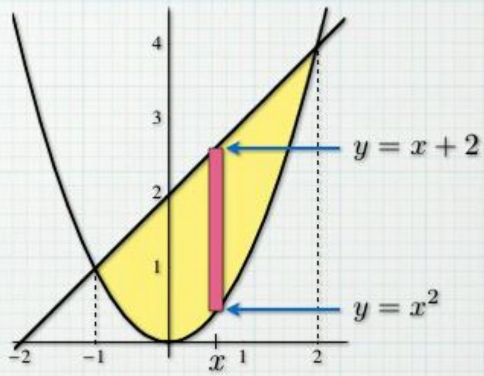
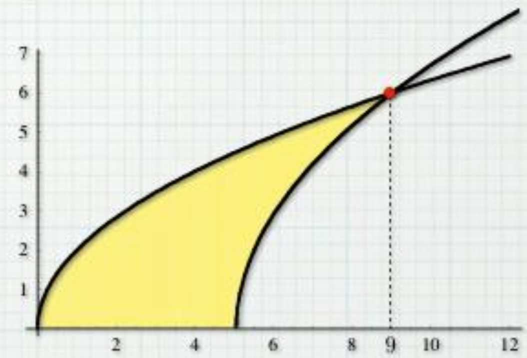


$$dA = (x + 2 - x^2)\, dx$$

Lets talk about dx (Vertical Rectangle) and dy (Horizontal Rectangle) options

Example Find the area of the region bounded by the graphs of

$$y = 2\sqrt{x}, \quad y = 0, \text{ and } y = 3\sqrt{x-5}.$$

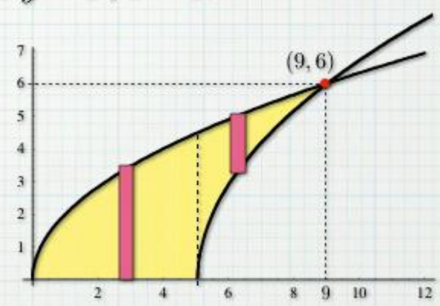


$$\begin{aligned} 2\sqrt{x} &= 3\sqrt{x-5} \\ 4x &= 9(x-5) \\ 45 &= 5x \\ x &= 9 \end{aligned}$$

Using dx (Vertical Rectangle):

Example Find the area of the region bounded by the graphs of

$$y = 2\sqrt{x}, \quad y = 0, \text{ and } y = 3\sqrt{x-5}.$$

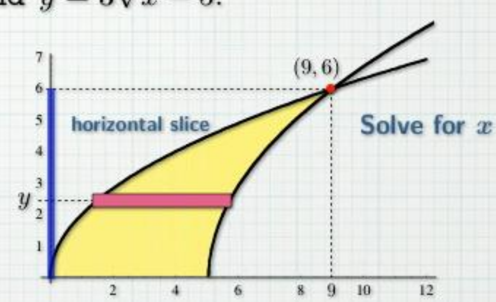


$$A = \int_0^5 2\sqrt{x}\,dx + \int_5^9 \left(2\sqrt{x} - 3\sqrt{x-5}\right) dx$$

Using dy (Horizontal Rectangle)

Example Find the area of the region bounded by the graphs of

$$y = 2\sqrt{x}, \quad y = 0, \text{ and } y = 3\sqrt{x-5}.$$



$$\begin{aligned} dA &= \left(\tfrac{1}{9}\,y^2 + 5 - \tfrac{1}{4}\,y^2\right) dy \\ &= \left(5 - \tfrac{5}{36}\,y^2\right) dy \\ &= \tfrac{5}{36}\left(36 - y^2\right) dy \end{aligned}$$

$$A = \frac{5}{36}\int_0^6 \left(36 - y^2\right) dy$$

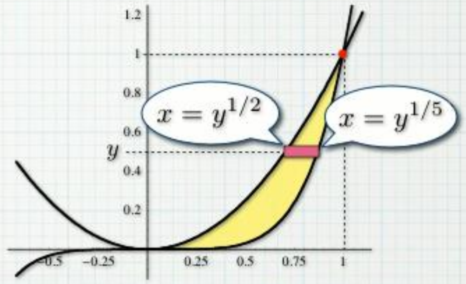
A = 20 Sq Units for figures above:

Solve the problem below without a calculator: Page 6

Example Find the area of the region bounded by the graphs of $y = x^2$ and $y = x^5$ two ways.

Answer to page 6

Example Find the area of the region bounded by the graphs of $y = x^2$ and $y = x^5$ two ways.



Vertical slices:
Integration with respect to x

$$dA = (x^2 - x^5)\,dx$$

$$A = \int_0^1 (x^2 - x^5)\,dx = \left(\tfrac{1}{3}\,x^3 - \tfrac{1}{6}\,x^6\right)\Big|_0^1 = \tfrac{1}{3} - \tfrac{1}{6} = \tfrac{1}{6}$$

Horizontal slices: Integration with respect to y

$$dA = (y^{1/5} - y^{1/2})\,dy$$

$$A = \int_0^1 (y^{1/5} - y^{1/2})\,dy = \left(\tfrac{5}{6}\,y^{6/5} - \tfrac{2}{3}\,y^{3/2}\right)\Big|_0^1 = \tfrac{5}{6} - \tfrac{2}{3} = \tfrac{1}{6}$$

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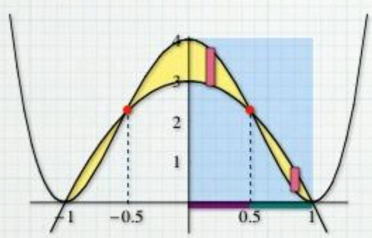
Answer to page 7:

Example Find the area of the region bounded by the graphs of $y = 3\,(1 - x^2)$ and $y = 4\,(1 - x^2)^2$.



Intersections

$$3\,(1 - x^2) = 4\,(1 - x^2)^2$$
$$3 = 4\,(1 - x^2)$$
$$3 = 4 - 4x^2$$
$$4x^2 = 1$$
$$x = \pm\tfrac{1}{2}$$

$$\frac{1}{2}A = \int_0^{1/2} \left(4\,(1-x^2)^2 - 3\,(1-x^2)\right)dx + \int_{1/2}^{1} \left(3\,(1-x^2) - 4\,(1-x^2)^2\right)dx$$

$$= \int_0^{1/2} (4x^4 - 5x^2 + 1)\,dx + \int_{1/2}^{1} (-4x^4 + 5x^2 - 1)\,dx$$

$$= \left(\tfrac{4}{5}\,x^5 - \tfrac{5}{3}\,x^3 + x\right)\Big|_0^{1/2} + \left(-\tfrac{4}{5}\,x^5 + \tfrac{5}{3}\,x^3 - x\right)\Big|_{1/2}^{1}$$

$$= \left(\tfrac{4}{5}\,\tfrac{1}{32} - \tfrac{5}{3}\,\tfrac{1}{8} + \tfrac{1}{2}\right) - 0 + \left(-\tfrac{4}{5} + \tfrac{5}{3} - 1\right) - \left(-\tfrac{4}{5}\,\tfrac{1}{32} + \tfrac{5}{3}\,\tfrac{1}{8} - \tfrac{1}{2}\right)$$

$$= \tfrac{8}{160} - \tfrac{10}{24} - \tfrac{4}{5} + \tfrac{5}{3} = \tfrac{1}{20} - \tfrac{5}{12} - \tfrac{4}{5} + \tfrac{5}{3} = \tfrac{3 - 25 - 48 + 100}{60} = \tfrac{30}{60} = \tfrac{1}{2}$$

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Do not forget A = 2(1/2) = 1 Sq Unit, because the work above was right hand side.

Solve the problem below without a calculator: Page 7

Example Find the area of the region bounded by the graphs of $y = 3(1-x^2)$ and $y = 4(1-x^2)^2$.