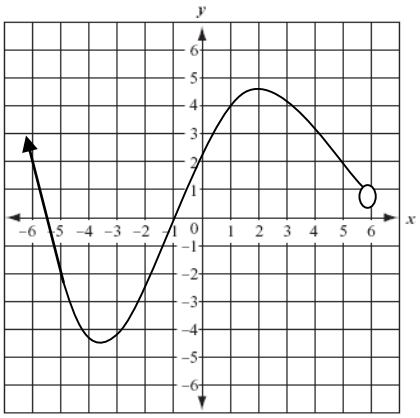
Precal BC Summer Assignment

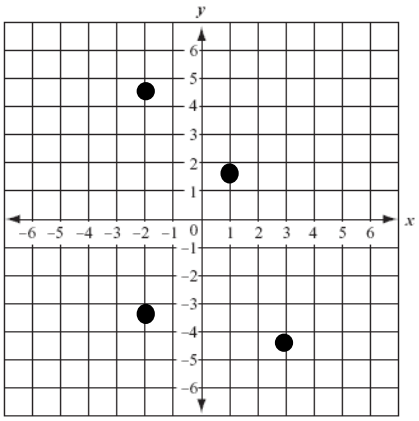
Work these problems on notebook paper. **All work must be shown**. Unless specified, you should **not** use a graphing calculator on these problems.

The following are graphs f(x) and g(x). Answer the following questions about them.

f(x)



g(x)



1. Are f(x) and g(x) functions? Are $f^{-1}(x)$ and $g^{-1}(x)$ functions?
2. Are they continuous or discrete? State the domain in interval notation and range in set notation of each.
3. On a separate graph, draw –f(x – 4) + 1 and g(x + 3) – 5.
4. Find f(g(3)), g(f(-2)), and x when f(g(x)) = 5.
5. Draw the inverse of each.
6. For all numbers a and b, let the operation ☻ be defined by: a ☻ b = b^2 - ab. Find (1 ☻ -2) ☻ 3

The following tables are used for problems 7-11

x	1	2.5	4	5.5	7
f(x)	3.5	5	0.5	3	6.5

x	0.5	2.5	4.5	5.5	6.5
g(x)	3	4.5	7	5.5	1

7. g(5.5) – f(5.5)
8. f(2.5) * g(2.5)
9. f(g(6.5))
10. g(f(4))
11. Find the inverses of the above data.

Let f(x) = $-\frac{12}{x}$, g(x) = $3x^3+1x^2-3x-1$, and h(x) = $6x^2-13x-5$ for problems 12-18

12. Solve for x when f(x) = h(x).
13. Find the roots, y-intercepts, asymptotes, crossings, holes, domain, and range of the following and graph.
14. f(x + 1) – 4
15. f(h(x))
16. $\frac{g(x)}{h(x)}$
17. $f(x)*\frac{g(x)}{h(x)}$
18. Simplify f(g(x)) + f(h(x))
19. Let f(x) = x^2-8x+2, find $f^{-1}(x)$ and $[f(x)]^{-1}$. Graph all three.
20. Let g(x) = $\frac{2x-1}{x+4}$, find $g^{-1}(x)$ and $[g(x)]^{-1}$
21. What is the expression of the volume of a cube, V, as a function of its surface area, S?
22. The area of a rectangle is 900 square ft. Write the equation of the perimeter of a rectangle, P, in terms of its width, w.

23. Solve for t when $|10-3t| = t+4$

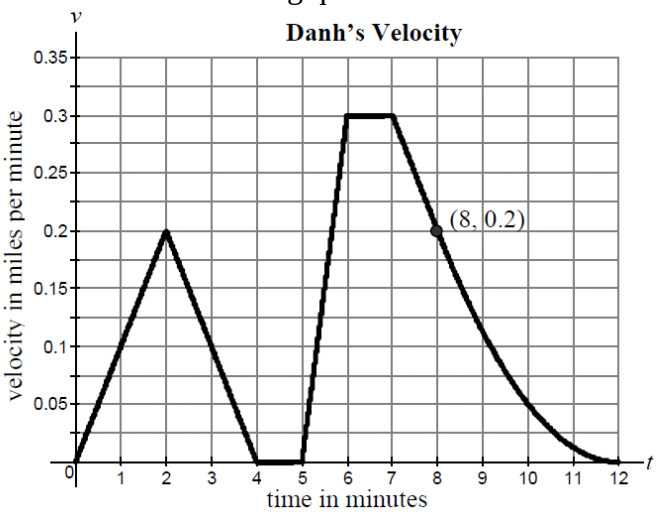
24. On a number line, graph the possible solutions of t when $|5t-9|+9<10$.

The following table shows the maximum amount of water vapor (g/m^3) that the air can hold at a particular air temperature (C).

Air Temperature (^oC)	–20	–10	0	5	10	20	30	40	45
Maximum Water Vapor in the Air (g/m^3)	0.75	2.0	3.5	5.0	7.0	14.0	26.5	47.0	67.0

25. Draw a scatterplot of the data and use a graphing calculator to find the line of best fit.

The following piece-wise graph and table models Danh's velocity on a bike ride. Use the graph and table to answer the following questions.



t	$v(t)$
0	0
2	0.2
4	0
5	0
6	0.3
7	0.3
8	0.2
12	0

26. For the first 4 minutes, the graph consists of an absolute value function. Find the equation for the absolute value function shown and write it in the appropriate position below.

$$v(t) = \begin{cases} \_\_\_\_\_ & 0<t<4 \\ \_\_\_\_\_ & 4<t<5 \\ \_\_\_\_\_ & 5<t<6 \\ \_\_\_\_\_ & 6<t<7 \\ \_\_\_\_\_ & 7<t<8 \\ \_\_\_\_\_ & 8<t<12 \end{cases}$$

27. From t = 4 to t = 8, there are four linear segments. Find those equation and write them in the appropriate positions.

28. From t = 8 to t = 12, the graph is parabolic with a vertex at (12, 0). Find the equation of the parabolic function and write it in the appropriate position.

29. At what times is Danh's velocity .1 miles per minute?

Let $A = \begin{bmatrix} 5 & 7 \\ -7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & -5 \\ 4 & 7 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -1 \\ 13 \\ -4 \end{bmatrix}$, and $E = \begin{bmatrix} -13 \\ 30 \end{bmatrix}$

30. Find the following based on the above:

AB	CD	|C|
AC	|A|	A^{-1}

Solve the following systems by elimination or substitution

31. $x + y + z = 5$
$2x + 2y + 3z = 14$
$2x - 3y + 2z = -5$

32. $4x + 2y - 5z = -21$
$2x - 2y + z = 7$
$4x + 3y - z = -1$

33. Solve the following system of equations (you may use matrices and a graphing calculator)
$w - 2x + 3y + z = 3$
$2w - x - y + z = 4$
$w + 2x - 3y - z = 1$
$3w - x + y - 2z = -4$

34. Carnegie Vanguard is selling tickets to a theatre performance. On the first day of ticket sales the school sold 3 student tickets and 1 adult ticket for a total of $38. The school took in $52 on the second day by selling 3 student tickets and 2 adult tickets. Find the price of an adult ticket and the price of a student ticket.

35. When selling tickets for Carnegie's theatre performance, ticket prices affect the number of people who buy tickets. If the prices are set at $12, then 300 people would attend over the course of the three performances. For every dollar cheaper the tickets are, 75 more people buy tickets to the show. What is the optimum price in order to maximize revenue? How much does Carnegie make at its maximum?

36. A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 3/4 lb. of clay and each plate uses one lb. of clay. She has 20 hours available for making the cups and plates and has 250 lbs. of clay on hand. She makes a profit of $2 on each cup and $1.50 on each plate. Use linear programming to determine how many cups and how many plates she should make in order to maximize her profit.

37. A company makes two types of sofas, regular and long, at two locations, one in Hickory and one in Lenoir. The plant in Hickory has a daily operating budget of $45,000 and can produce at most 300 sofas daily in any combination. It costs $150 to make a regular sofa and $200 to make a long sofa at the Hickory plant. The Lenoir plant has a daily operating budget of $36,000, can produce at most 250 sofas daily in any combination and makes a regular sofa for $135 and a long sofa for $180. The company wants to limit production to a maximum of 250 regular sofas and 350 long sofas each day. If the company makes a profit of $50 on each regular sofa and $70 on each long sofa, how many of each type should be made at each plant in order to maximize profit? What is the maximum profit?

Simplify so that there are no roots or *i* in the denominator.

38. $\dfrac{\sqrt{-18}+\sqrt{-98}}{\sqrt{-12}-\sqrt{49}}$

39. $\dfrac{(2+3i)(1+2i)}{(1-i)}$

Use long division or synthetic division to determine whether or not the given expression is a factor of the polynomial. Write out the resulting answer.

40. $x^3 + x^2 + 6x + 6$; $x + 1$
41. $2x^3 - 6x^2 + 2x - 14$; $2x^2 - 7$
42. $64x^3 - 8$; $4x - 3$
43. $-4x^3 + 8x^2 + 252x$; $x + 7$
44. $x^3 + 343$; $x^2 - 7x + 49$

45. List all of the possible rational roots of $6x^3 - 11x^2 - 24x + 9$ based on the rational roots theorem
46. Using Descartes Rule, show the possible combinations of positive, negative, and imaginary roots for
$12x^5 - 76x^4 + 17x^3 + 459x^2 - 575x + 100$

Factor, find the zeros, describe the end behaviour, and graph the even problems

47. $64x^3 - 8$
48. $2x^3 - 20x^2 + 18x$
49. $-8x^4 + 8x^3 + 27x - 27$
50. $x^4 + 12x^3 + 35x^2 - 12x - 36$
51. $x^4 + 13x^2 - 48$
52. $-10x^4 + 27x^3 - 25x^2 + 9x - 1$
53. $x^6 - x^4 - 81x^2 + 81$
54. $6x^5 - 17x^4 - 2x^3 + 24x^2 - 4x - 7$
55. $-4x^4 + 27x^2 + 21x - 2$

Find the inverse of the following and graph it

56. $y = 4x^2 - 8x - 60$
57. $y = 2x^2 + 12x - 4$
58. $y = (x + 3)^3 - 9$

Suppose the following are zeros for a polynomial. Find the equation of the polynomial that has a leading coefficient of 1.

59. 3 + i, 4, 1
60. $2 - \sqrt{3}$, -2, 2

Given the following data, find the polynomial equation that fits the data.

61. (1, -2) (2, 7) (3, 18) (4, 31) (5, 46) (6, 63)
62. (-1 -48) (0, -50) (1, -36) (2, 0) (3, 64) (4, 162)

Solve the following for x

63. $\sqrt{x-1} = x - 7$
64. $\sqrt{x-3} - \sqrt{x} = 3$
65. $\sqrt{2x+9} - \sqrt{x+1} = \sqrt{x+4}$
66. $8^{x-5} = 32^{x+1}$
67. $3^{5x+1} = 9^{x^2-4x-7}$

Rationalize or simplify

68. $\sqrt[3]{\frac{x}{y^8}}$
69. $\frac{6xy^{1/2}z^0}{3x^{1/2}y^{3/4}}$
70. $\sqrt[5]{\frac{486}{8}}$

71. Solve the following for x

a. $10^{x+5}=100$
b. $3^{2x-1} = 27^x$
c. $6(2)^{x+3} = 30$

72. Suppose you put $16,000 in a bank with an interest rate of 2% per year. How much money would you have after 50 years if the money is compounded yearly, quarterly, monthly, daily, continuously?

73. Condense or expand the following:

a. $5\log(x) + 7\log(y) - 3\log(z)$
b. $-3\log(x) + 6\log(y) + 2\log(z + 3)$
c. $\ln\frac{x^7}{y^3z^8}$
d. $\log_5\sqrt[3]{\frac{25r^2}{s^4t^5}}$

Solve for x:

74. $\ln(x - 3) + \ln(x - 2) = \ln(2x - 24)$
75. $\log(x+21) + \log(x) = 2$
76. $\log(2x - 1) + \log(x) = \log(4x - 3)$
77. $\log_2(x) + \log_2(x + 2) = 3$
78. $\log(2x-3) + \log(x-2) = 2\log(x)$
79. $\log_2(3x-5) - \log_2(2x+5) = 1$
80. $\log(x^2-5) - \log(x-2) = \log(x+3)$

81. Given the following data, find the exponential equation that fits: (-1, 1/8), (3, 2). Then find x when y = 10.

82. t varies directly with x squared and inversely with b and s. Find the expression relating the variables if when x = 15, b = 6, and s = 5, then t =15. Then find x when t = 24, b = 3 and s = 4.
83. m varies jointly with p and s and inversely with c squared. Find the expression relating the variables if when m = 1/3, p = 3, s = 2, and c = 3. Then find c when m = 3, p = 15, and s = 10.

Find the important information (including intercepts, asymptotes and crossings, holes, domain, and range) and graph the following.

84. $y = \frac{x-3}{x+2}$

85. $y = \frac{x+4}{2x+1}$

86. $y = \frac{2x-8}{x+4}$

87. $y = \frac{3x-3}{x-1}$

88. $y = \frac{x-8}{x^2+4}$

89. $y = \frac{2x^2-x-1}{x^2-4}$

90. $y = \frac{4x^2-5x-6}{3x^2-4x-4}$

91. $y = \frac{2x^2-3x-9}{x^2+4x-5}$

92. $y = \frac{3x^2+x-4}{3x^3-2x^2-3x+2}$

93. $y = \frac{15x^2+2x-8}{4x^3-5x^2-2x+3}$

94. $y = \frac{4x^3+3x^2-16x-12}{3x^2+10x+8}$

95. $y \frac{2x^3+x^2-16x-15}{4x^4+12x^3+5x^2-12x-9}$

Solve the following Rational Equations.

96. $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

97. $\frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2}$

98. $\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2-6x+8}$

Simplify the following.

99. $\frac{x+1}{x+3} + \frac{2x+5}{x-2}$

100. $\frac{2}{x^2-25} - \frac{8}{x^2+10x+25}$

101. $\frac{x}{x+2} + \frac{1}{2x+4} - \frac{3}{x^2+2x}$

102. $\frac{4+\frac{1}{x}}{3+\frac{2}{x^2}}$

103. $\frac{\frac{1}{x-1}+x+3}{x-3+\frac{1}{x+4}}$

104. $\frac{\frac{y}{x}-\frac{x}{y}}{\frac{x+y}{xy}}$

105. Graph the following conics:

a. $y = (x+1)^2 - 6$

b. $25(x+3)^2 + 9(y+1)^2 = 225$

c. $4x^2 - y^2 = 9$

d. $x^2 + y^2 - 10x = 39$

106. Find the intersections of 17a and b with a calculator and 17 c and d without a calculator.

Find the sums

107. $\sum_{i=0}^{5} i^2 - 2i$

108. $\sum_{i=1}^{50} -2i + 14$

109. $\sum_{i=1}^{6} 3\left(\frac{2}{3}\right)^i$

110. $\sum_{i=1}^{\infty} \left(-\frac{5}{2}\right)^{i-1}$

Determine if the following are arithmetic, geometric or neither, write their rules in explicit form.

111. 517, 501, 485, 469, …

112. 3, -9, 27, -81, …

113. 1, 4, 9, 16, …

Find the formula that describes the following

114. arithmetic sequence; $a_6 = 13$ and $a_{14} = 25$

115. geometric sequence; $a_6 = -\frac{25}{64}$ and $a_3 = 25$

116. Write the recursive rule for 0, 1, 1, 2, 3, 5, 8, …

117. Write .373737373737…. as a fraction

118. Solve for n when ${}_nP_8 = 12({}_nP_6)$

119. ${}_{15}C_5$

120. You need to choose 5 representatives from a class of 21. How many ways can they be chosen?

121. How many ways can 100 racers finish 1st - 4th?

122. How many ways can you arrange the letters in the word *hippopotamus*.

123. Write the binomial expansion for $(x^2 - 2y)^7$

124. What is the probability of drawing a king or a spade in a standard deck of cards? The odds?

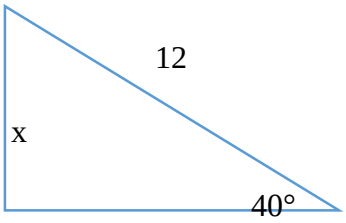
125. Define based on opposite, adjacent, and hypotenuse

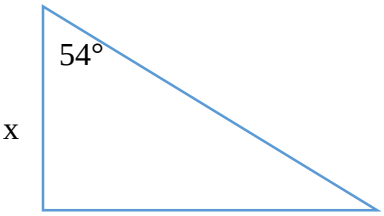
a. sin(x)

b. cos(x)

c. tan(x)

126. Use a calculator to find x in each of the following triangles:





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127. For x = 30°, 45°, and 60 °, use special triangles to find sin(x), cos(x), and tan(x)

x	30°	45°	60°
sin(x)			
cos(x)			
tan(x)			

128. Use Law of Sines and/or Law of Cosines to solve the following triangles (find all unknown sides and angles). You may use a graphing calculator.

(a) A = 43°, C = 83°, b = 12 (b) C = 55°, a = 17, c = 15 (c) B = 63°, a = 11, b = 8

(d) a = 24 b = 33 c = 24 (e) C = 101°, a = 23, b = 19