AP CALCULUS BC
SUMMER ASSIGNMENT

Work these problems on <u>notebook paper</u>. <u>**All work must be shown**</u>. Use your graphing calculator only on <u>problems 44-55, 80-83, and 127</u>.

Find the *x*- and *y*-intercepts and the domain and range, and sketch the graph. <u>No calculator</u>.

1. $y=\sqrt{x-1}$
2. $y=\sqrt{9-x^2}$
3. $y=\dfrac{|x|}{x}$
4. $y=\sin x,\ -2\pi\le x\le 2\pi$
5. $y=\cos x,\ -2\pi\le x\le 2\pi$
6. $y=\tan x,\ -2\pi\le x\le 2\pi$
7. $y=\cot x,\ -2\pi\le x\le 2\pi$
8. $y=\sec x,\ -2\pi\le x\le 2\pi$
9. $y=\csc x,\ -2\pi\le x\le 2\pi$
10. $y=e^x$
11. $y=\ln x$
12. $y=\begin{cases}-1, & \text{if } x\le -1\\ 3x+2, & \text{if } |x|<1\\ 7-2x, & \text{if } x\ge 1\end{cases}$
13. $y=\begin{cases}x^2+1, & \text{if } x>0\\ -2x+2, & \text{if } x\le 0\end{cases}$

Find the asymptotes (horizontal, vertical, and slant), symmetry, and intercepts, and sketch the graph. <u>No calculator</u>.

14. $y=\dfrac{1}{x-1}$
15. $y=\dfrac{1}{(x+2)^2}$
16. $y=\dfrac{2(x^2-9)}{x^2-4}$
17. $y=\dfrac{x^2-2x+4}{x-1}$

Solve. <u>No calculator</u>.

18. $x^2-x-12>0$
19. $(x-2)^2(x+1)^3(x-5)\le 0$
20. $\dfrac{3x-2}{x+4}\le 0$
21. $\dfrac{(2x+5)(x-1)^2}{(x+2)^3}\ge 0$

Evaluate. <u>No calculator</u>.

22. $\cos\dfrac{5\pi}{6}$
23. $\sin\dfrac{3\pi}{2}$
24. $\tan\dfrac{5\pi}{4}$
25. $\sin\dfrac{7\pi}{4}$
26. $\cos\pi$
27. $\tan\dfrac{2\pi}{3}$
28. $\sec\dfrac{4\pi}{3}$
29. $\csc\dfrac{\pi}{4}$
30. $\cot\dfrac{2\pi}{3}$

Evaluate. <u>No calculator</u>.

31. $\tan\left(\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right)$
32. $\sec\left(Arc\sin\left(-\dfrac{\sqrt{2}}{2}\right)\right)$
33. $\cos\left(\sin^{-1}(2x)\right)$
34. $\sec\left(Arc\tan(4x)\right)$

Solve. Give exact answers in radians, $0 \le x \le 2\pi$. <u>No calculator</u>.

35. $2\cos^2 x + 3\cos x - 2 = 0$
36. $2\sin^2 x - \cos x = 1$
37. $\sin(2x) = \cos x$
38. $2\cos(2x) + 1 = 0$
39. $2\csc^2 x + 3\csc x - 2 = 0$
40. $\tan^2 x - \sec x = 1$
41. $2\cos\left(\frac{x}{3}\right) - \sqrt{3} = 0$
42. $\tan(2x) = -\sqrt{3}$
43. $2\sin(3x) - \sqrt{3} = 0$

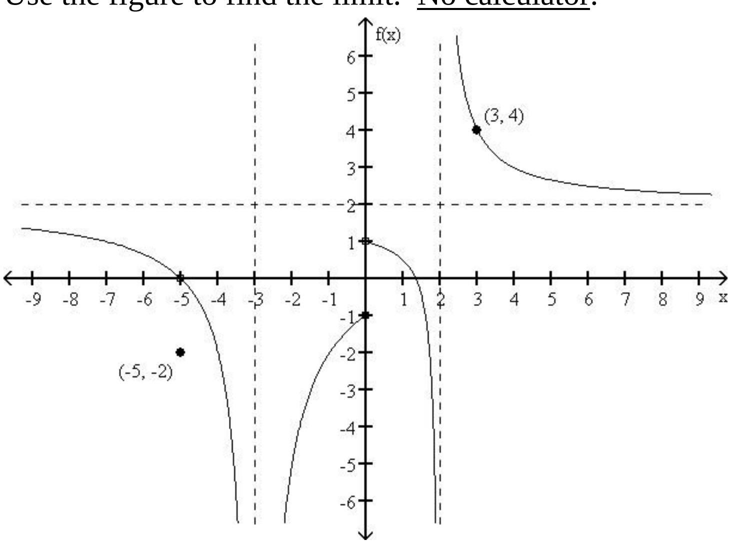
Solve. Show all steps. Give the exact answer and then use your calculator, and give decimal answers correct to <u>**three**</u> decimal places.

44. $e^{2x+3} = 37$
45. $e^{2x} - 5e^x + 6 = 0$
46. $e^x - 12e^{-x} - 1 = 0$
47. $\frac{50}{4 + e^{2x}} = 11$
48. $\log_4\left(x^2 - 3x\right) = 1$
49. $\ln(5x - 1) = 3$
50. $\log_2(x+3) + \log_2(x-1) = \log_2 12$
51. $\log_8(x+5) - \log_8(x-2) = 1$
52. $\log_6(\log_4(\log_2 x)) = 0$
53. $\log_3(\log_2(\log_5 25)) = x$

54. The number of students in a school infected with the flu t days after exposure is modeled by the function $P(t) = \frac{300}{1 + e^{-4t}}$.
   (a) How many students were infected after three days?
   (b) When will 100 students be infected?
   (c) What is/are the horizontal asymptote(s) of P(t)?

55. Exponential growth is modeled by the function $n = n_0 e^{kt}$. A culture contains 500 bacteria when $t = 0$. After an hour, the number of bacteria is 1200.
   (a) How many bacteria are there after four hours?
   (b) After how many hours will there be 8000 bacteria?

Use the figure to find the limit. <u>No calculator</u>.



56. $\lim_{x \to 3} f(x)$
57. $\lim_{x \to \infty} f(x)$
58. $\lim_{x \to 2^+} f(x)$
59. $\lim_{x \to 0} f(x)$
60. $\lim_{x \to -\infty} f(x)$
61. $\lim_{x \to -5} f(x)$

Evaluate. Show supporting work for each problem (algebraic steps or sketch). No calculator.

62. $\lim\limits_{x\to -3} \dfrac{x^2+x-6}{x+3}$

63. $\lim\limits_{x\to 0} \dfrac{(x-5)^2-25}{x}$

64. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

65. $\lim\limits_{x\to -6} \dfrac{x+6}{x^2+3x-18}$

66. $\lim\limits_{x\to 2} \dfrac{x^3+8}{x+2}$

67. $\lim\limits_{x\to \infty} \dfrac{3x-5x^2}{4x^2+1}$

68. $\lim\limits_{x\to 3^+} \dfrac{1}{x-3}$

69. $\lim\limits_{x\to 3^-} \dfrac{1}{x-3}$

70. $\lim\limits_{x\to 3} \dfrac{1}{x-3}$

71. $\lim\limits_{x\to 3} \dfrac{1}{(x-3)^2}$

72. $\lim\limits_{x\to 3^+} \lfloor x-1 \rfloor$

73. $\lim\limits_{x\to 3^-} \lfloor x-1 \rfloor$

74. $f(x)=\begin{cases} 1-x, & \text{if } x\le 1 \\ x^2, & \text{if } x>1 \end{cases}$ (a) $\lim\limits_{x\to 1^-} f(x)$ (b) $\lim\limits_{x\to 1^+} f(x)$ (c) $\lim\limits_{x\to 1} f(x)$

75. $f(x)=\begin{cases} \dfrac{x^2-x-6}{x-3}, & \text{if } x\ne 3 \\ 4, & \text{if } x=3 \end{cases}$ (a) $\lim\limits_{x\to 3} f(x)$ (b) $f(3)$

Use the definition of the derivative to find the derivative. No calculator.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ (You must **know** this formula.)

76. $f(x)=x^2-8x$

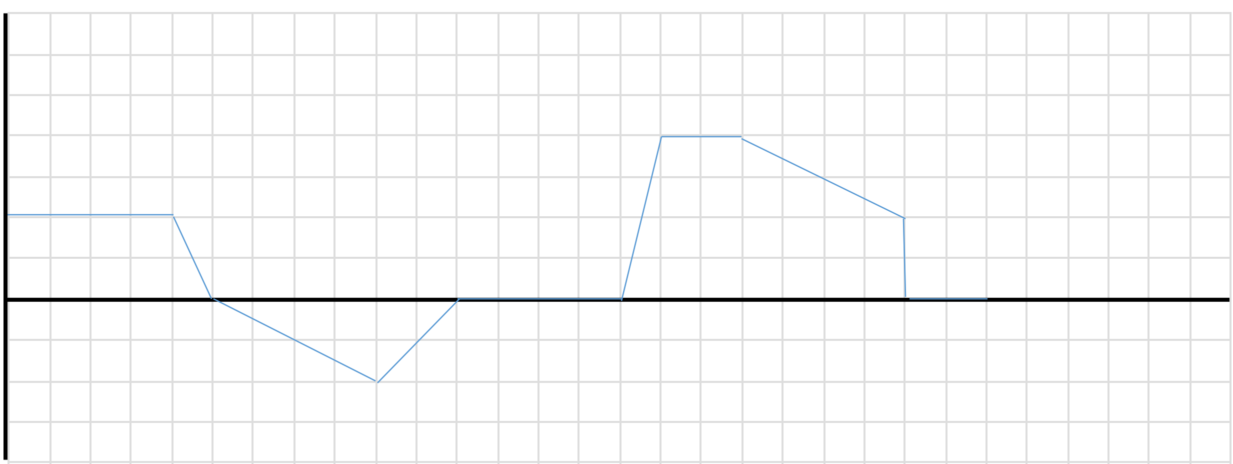
77. $f(x)=\sqrt{x+9}$

78. $f(x)=\dfrac{3}{x-4}$

79. $f(x)=x^3+2x^2-x+4$

Rates and Areas

The following graph gives the rate of rainfall (velocity) over a period of time that is collected in a basin.



Each x square is 1 hr

Each y square if .1 inch per pour

80. What does a negative rate mean?

81. How much water (position) is in the basin after 5 hours? 11 hours? 22 hours?

82. At what rate is the rate of rainfall changing (acceleration) at 21 hours?

The motion of a projectile follows the position equation $s(t) = -16t^2 + 96t + 256$

83. What it the average velocity of the projectile on the interval [3, 7]
84. Use the definition of the derivative to find the derivative at time $t = t_0$.
85. At what time does the projectile strike the ground?
86. What is its instantaneous velocity when it strikes the ground?
87. What is the maximum height the projectile reaches?
88. What is the instantaneous velocity at t = 4 seconds?

Use the differentiation rules (power rule, product rule, quotient rule, chain rule, logarithmic differentiation) to find the derivative. Do not leave negative exponents or complex fractions in your answers. No calculator.

89. $f(x) = 3x^4 - 5x^3 + \frac{2}{x} + 6x^{2/3} - 12$
90. $f(x) = \frac{2x^2 - 3x + 1}{x}$
91. $f(x) = \sqrt{x} + \sqrt[3]{x}$
92. $f(x) = (6x+5)(x^3 - 2)$
93. $f(x) = \frac{x^3 + 5x - 3}{x^2 - 1}$
94. $f(x) = \sin x$
95. $f(x) = \cos(2x)$
96. $f(x) = \tan(x^2 - 5)$
97. $f(x) = \cot(x^2 + x)$
98. $f(x) = \sec x$
99. $f(x) = \csc(-3x)$
100. $f(x) = \sin^{-1}(2x)$
101. $f(x) = \cos^{-1}(x^2)$
102. $f(x) = \tan^{-1}(-x^2 + 3x)$
103. $f(x) = \csc^{-1}(10x)$
104. $f(x) = \sec^{-1}\left(\frac{x-1}{2x}\right)$
105. $f(x) = \cot^{-1} x$
106. $f(x) = e^x$
107. $f(x) = e^{x^2 + 2x - 1}$
108. $f(x) = \ln x$
109. $f(x) = \ln(3x - 5)$
110. $f(x) = \ln(\sin x)$
111. $f(x) = x^{\tan x}$
112. $f(x) = xe^x \cos x$

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113. Given the function $f(x) = x^4 - 3x^2 + 7$ No calculator.
    (a) Use differentiation rules to find $f'(x)$
    (b) Write the equation of the tangent line to f at (1,5). Leave your equation in point-slope form.

Use implicit differentiation to find $\frac{dy}{dx}$. No calculator.

114. $9x^2 + 4y^2 + 24y = 0$
115. $-y^2 + 7\sqrt{xy} = -9$
116. $-4\sin(xy) + 2x = -5$

Identify all intervals on which the function is increasing, decreasing, concave up, and concave down, and identify all local extrema and inflection points. Sketch the graph. Justify your answers. No calculator.

117. $f(x) = x^3 + 2x^2 + x + 2$
118. $f(x) = x^3 - 9x^2 + 15x + 9$
119. $f(x) = \sin x + \cos x$
120. $f(x) = \frac{x^2}{x^2 - 9}$
121. $f(x) = xe^{-x}$
122. $f(x) = e^x \sin x$

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123. Find all global extrema of $f(x) = x^3 - 9x^2 + 15x + 9$ on $[0,10]$. Justify your answer. No calculator.

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124. A particle moving along the y-axis has position equation $y(t) = \frac{8}{\pi}\cos\left(\frac{\pi}{4}t\right) + 3$.

On $0 \le t \le 8$, identify all points at which the particle is stopped, and identify all intervals on which the particle is moving up, moving down, speeding up, and slowing down. Justify your answers. No calculator.

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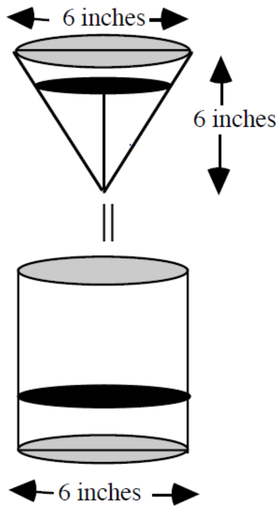
125. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 cubic inches per minute. The coffee filter has diameter 6 inches and height 6 inches, and the coffee pot has diameter 6 inches.



(a) What is the rate of change of the height of the coffee in the coffeepot at the instant when the coffee is 5 inches deep in the filter? No calculator.

(b) What is the rate of change of the height of the coffee in the coffeepot at the instant when the coffee is 2 inches deep in the filter? No calculator.

(c) What is the rate of change of the height of the coffee in the filter at the instant when the coffee is 5 inches deep in the filter? No calculator.

(d) What is the rate of change of the height of the coffee in the filter at the instant when the coffee is 2 inches deep in the filter? No calculator.

126. A 25-foot ladder leans against the side of a building. The bottom of the ladder is being pushed towards the wall at a rate of 3 feet per second and the ladder remains in contact with the wall.

(a) What is the speed that the top of the ladder is sliding up the wall at the instant when the bottom of the ladder is 7 feet from the wall? No calculator.

(b) What is the rate of change of the angle formed by the bottom of the ladder and the ground at the instant when the bottom of the ladder is 7 feet from the wall? G decimal answers correct to **three** decimal places.

(c) What is the speed that the top of the ladder is sliding up the wall at the instant when the bottom of the ladder is 15 feet from the wall? No calculator.

(d) What is the rate of change of the angle formed by the bottom of the ladder and the ground at the instant when the bottom of the ladder is 15 feet from the wall? Give decimal answers correct to **three** decimal places.

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127. Show all steps using Calculus to find an exact answer. Use your calculator, and give decimal answers correct to **three** decimal places.

The state wants to build a new stretch of highway to link an existing bridge with a turnpike interchange, located 8 miles to the east and 8 miles to the south of the bridge. There is a 5-mile-wide stretch of marshland adjacent to the bridge that must be crossed (see Figure 3.91). Given that the highway costs $10 million per mile to build over the marsh and only $7 million to build over dry land, how far to the east of the bridge should the highway be when it crosses out of the marsh?

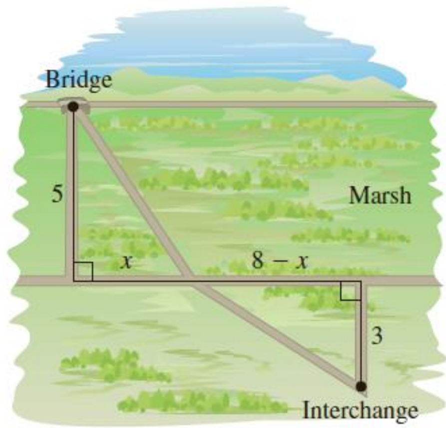

FIGURE 3.91
A new highway

Use L'Hopital's Rule to determine each limit.

128. $\lim_{x \to 0^+} \tan x \ln x$

129. $\lim_{x \to 0} \dfrac{\tan^{-1}(5x)}{\sin^{-1}(2x)}$

130. $\lim_{x \to \infty} x \ln\left(1 + \dfrac{3}{x}\right)$

131. $\lim_{x \to \infty} \dfrac{x^3}{e^x}$

132. $\lim_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^x$

133. $\lim_{x \to 0} \left(\dfrac{1}{\ln(x+1)} - \dfrac{1}{x}\right)$

134. $\lim_{x \to 4^+} \left|\dfrac{x-1}{x-4}\right|^{\sqrt{x^2-16}}$

Parametrics, Vectors, and Polars Review

135. Two particles move in the xy-plane. For any time $t \geq 0$, the position of particle A is given by $x = t - 2$ and $y = (t - 2)^2$, and the position of particle B is given by $x = 3t/2 - 4$ and $y = 3t/2 - 2$.

a. Find the position, velocity, and acceleration vectors for each particle at time t = 3.

b. Find the speed of each particle at time t = 3.

c. Find the line tangent to each graph at time t = 3

d. If the particles collide, find the time and location of collision.

136. Let the two polar graphs, $r = 2$ and $r = 2(1 - \sin\theta)$ lie on a plane.

a. Sketch each curve.

b. Find the line tangent to each curve at $\theta = \pi/12$

c. Find the intersection(s) between the two curves.