Linear Functions

Unit Overview
In this unit, you will build linear models and use them to study functions, domain, and range. Linear models are the foundation for studying slope as a rate of change, intercepts, and direct variation. You will learn to write linear equations given varied information and express these equations in different forms.

Academic Vocabulary
Add these words and others you encounter in this unit to your portfolio glossary.
- dependent variable
- direct variation
- domain
- function
- independent variable
- inverse variation
- linear equation
- range
- x-intercept
- y-intercept

Essential Questions
- How can you show mathematical relationships?
- Why are linear functions useful in real-world settings?

EMBEDDED ASSESSMENTS
This unit has three embedded assessments, following Activities 2.3, 2.5, and 2.8. They will give you an opportunity to demonstrate what you have learned.

Embedded Assessment 1
Representations of Functions p. 97

Embedded Assessment 2
Linear Functions and Equations p. 113

Embedded Assessment 3
Linear Equations and Slope as Rate of Change p. 131
1. Complete the table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>11</td>
<td>29</td>
</tr>
</tbody>
</table>

2. List the integers that make this statement true.

\[-3 \leq x < 4\]

3. Evaluate for \(a = 3\) and \(b = -2\).
   a. \(2a - 5\)  
   b. \(3b + 4a\)

4. Name the point for each ordered pair.
   a. \((-3, 0)\)  
   b. \((-1, 3)\)  
   c. \((2, -2)\)

5. Explain how you would plot \((3, -4)\) on a coordinate plane.

6. Which of the following equations represents the data in the table?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

A. \(y = 2x - 1\)  
B. \(y = 3x - 1\)  
C. \(y = x + 1\)  
D. \(y = 2x + 1\)

7. If \(2x + 6 = 2\), what is the value of \(x\)?
   A. 4  
   B. 2  
   C. 0  
   D. -2

8. Which of the following are the coordinates of a point on this line?

A. \((-1, 3)\)  
B. \((1, -3)\)  
C. \((-1, -3)\)  
D. \((1, 3)\)
Use this machine to answer the questions on the next page.

**DVD Vending Machine**

Insert money and push the buttons below.

Remove Purchased DVDs Here
1. Suppose you inserted your money and pressed A1. What item would you receive?

2. Suppose you inserted your money and pressed C2. What item would you receive?

3. Suppose you inserted your money and pressed B3. What item would you receive?

4. If the machine were filled properly, what would happen if you pressed any of those same buttons again?

Each time you press a button, an input, you may receive a DVD, an output.

5. In the DVD vending machine situation, does every input have an output? Explain your response.

6. Each combination of input and output can be expressed as a mapping written input → output. For example, B2 → Wizard of Gauze]

a. Write as mappings each of the possible combinations of buttons pushed and DVDs received in the vending machine.
b. Mappings relating values from one set of numbers to another set of numbers can be written as ordered pairs. Write the following numerical mappings as ordered pairs.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Ordered Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−2</td>
<td>(1, −2)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

A relation is a set of ordered pairs. The list of ordered pairs that you wrote in Item 6(b) is a relation.

Relations can have a variety of representations. Consider the relation \{(1, 4), (2, 3), (6, 5)\}, shown here as a set of ordered pairs. This relation can also be represented in these ways.

<table>
<thead>
<tr>
<th>Table</th>
<th>Mapping</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(y)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

7. You represented the vending machine situation using mappings in Item 6. Other representations can also be used to illustrate how the inputs and outputs of the vending machine are related.

a. Create a table to illustrate how the inputs and outputs of the vending machine are related.

b. In representing the vending machine inputs and outputs, what decisions would need to be made to create the graph?
A **function** is a relation in which each input is paired with exactly one output.

8. Compare and contrast the DVD Vending Machine with a function.

9. Suppose when pressing button C1 button on the vending machine both “Finding Dreamo” and “Raiders of the Mossed Bark” come out. How does this vending machine resemble or not resemble a function?

10. Imagine a machine where you input an age and the machine gives you the name of anyone who is that age. Compare and contrast this machine with a function. Explain by using examples and create a representation of the situation.

11. Create an example of a situation (math or real-life) that behaves like a function and another that does not behave like a function. Explain why you chose each example to fit the category.

   a. Behaves like a function:

   b. Does not behave like a function:
12. Identify whether each list of ordered pairs represents a function. Explain your answers.

a. \{(5, 4), (6, 3), (7, 2)\}

b. \{(4, 5), (4, 3), (5, 2)\}

c. \{(5, 4), (6, 4), (7, 4)\}

13. Using positive integers, write two relations as a list of ordered pairs below, one that is a function and one that is not a function.

Function:

Not a function:

The set of all inputs for a function is known as the **domain** of the function. The set of all outputs for a function is known as the **range** of the function.

14. Consider a vending machine where inserting 25 cents dispenses one pencil, inserting 50 cents dispenses 2 pencils, and so forth up to and including all 10 pencils in the vending machine.

a. What is the domain in this situation?

b. What is the range in this situation?

**ACADEMIC VOCABULARY**

- **domain**
- **range**

**WRITING MATH**

The **domain** and **range** of a function can be written using set notation.

For example for the function \{(1,2), (3,4),(5,6)\} the domain is \{1,3,5\} and the range is \{2,4,6\}. 
15. For each function below, identify the domain and range.

a. | input | output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Domain:
Range:

b. Domain:
Range:

c. Domain:
Range:

d. Domain:
Range:

16. Each of the functions that you have seen has a finite number of ordered pairs. There are functions that have an infinite number of ordered pairs. Describe any difficulties that may exist trying to represent a function with an infinite number of ordered pairs using the four representations of functions that have been described thus far.
17. Sometimes, machine diagrams are used to represent functions. In the function machine below, the inputs are labeled \( x \) and the outputs are labeled \( y \). The function is represented by the expression \( 2x + 5 \).

\[ x \rightarrow 2x + 5 \rightarrow y \]

\( a. \) If \( x = 7 \) is used as an input, what is the output?

\( b. \) If \( x = -2 \) is used as an input, what is the output?

\( c. \) If \( x = \frac{1}{2} \) is used as an input, what is the output?

\( d. \) Is there any limit to the number of input values that can be used with this expression? Explain.

Consider the function machine below.

\[ x \rightarrow x^2 + 2x + 3 \rightarrow y \]

18. Use the diagram to find the (input, output) ordered pairs for the following values.

\( a. \) \( x = -5 \)

\( b. \) \( x = \frac{3}{5} \)

\( c. \) \( x = -10 \)
ACTIVITY 2.1 Continued

**Introduction to Functions**

**Vending Machines**

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Activating Prior Knowledge, Group Discussion

19. Make a function machine for the expression $10 - 5x$. Use it to find ordered pairs for $x = 3$, $x = -6$, $x = 0.25$, and $x = \frac{3}{4}$.

Creating a function machine can be time consuming and awkward. The function represented by the diagram in Item 17 can also be written algebraically as the equation $y = 2x + 5$.

20. Evaluate each function for $x = -2$, $x = 5$, $x = \frac{2}{3}$, and $x = 0.75$. For each $x$-value, find the corresponding $y$-value. Place the results in a table.

   a. $y = 9 - 4x$
   
   b. $y = \frac{1}{x}$

When referring to the functions in Item 20, it can be confusing to distinguish among them since each begins with “$y =$.” Function notation can be used to help distinguish among different functions.

For instance, the function $y = 9 - 4x$ in Item 20(a) can be written:

$f(x) = 9 - 4x$

This is read as “$f$ of $x$” and $f(x)$ is equivalent to $y$.

- “$f$” is the name of the function.
- $x$ is the input variable.
21. To distinguish among different functions, it is possible to use different names. Use the name *h* to write the function from Item 20b using function notation.

Function notation is useful for evaluating functions for multiple input values. To evaluate \( f(x) = 9 - 4x \) for \( x = 2 \), you substitute 2 for the variable \( x \) and write \( f(2) = 9 - 4(2) \). Simplifying the expression yields \( f(2) = 1 \).

22. Use function notation to evaluate \( f(x) \) shown above at \( x = 5 \), \( x = -3 \), and \( x = 0.5 \).

23. Use the values for \( x \) and \( f(x) \) from Item 22. Display the values using each representation.

   a. list of ordered pairs
   b. table of values

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

c. mapping
d. graph
24. Evaluate each function for \( x = -5, x = \frac{4}{3} \).

   a. \( f(x) = 2x - 7 \)
   b. \( g(x) = 6x - x^2 \)
   c. \( h(x) = \frac{2}{x^2} \)

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. The set \{ (3,5), (-1,2), (2,2), (0,-1) \} represents a function. Identify the domain and range of the function. Then display the function using each representation.
   a. a table
   b. a mapping
   c. a graph

2. Explain why each of the following is not a function.
   a. The graph shown
   b. The table shown

3. Evaluate the functions for each domain value indicated.
   a. \( p(x) = 3x + 14 \)
      \( x = -5, 0, 4 \)
   b. \( h(t) = t^2 - 5t \)
      \( t = -2, 0, 5, 7 \)

4. Which representation of a function do you feel is most useful? Why? Which one do you feel is least useful? Why?
Roller coasters are scary and fun to ride. Wooden roller coasters shake and rattle as part of the thrill of the ride. Below is the graph of the heights reached by the cars of the wooden roller coaster, Thunderball, over its first 1250 feet of track. The graph displays a function because each input value has one and only one output value. You can see this visually using the vertical line test.

Study this graph to determine the domain and range.

The domain gives all values of the independent variable: distance along the track in feet. These values are graphed along the horizontal or x-axis.

The domain can be written in set notation as:

\[ \{ \text{all real values of } x : 0 \leq x \leq 1250 \} \]

Read this notation as: the set of all real values of \( x \), between 0 and 1250, inclusive.

The range gives the values of the dependent variable: height above the ground in feet. The values are graphed on the vertical or y-axis.

The range can be written in set notation as:

\[ \{ \text{all real values of } y : 10 \leq y \leq 110 \} \]

Read this notation as: the set of all real values of \( y \), between 10 and 110, inclusive.

The graph above shows data that are continuous. The points in the graph are connected, indicating that domain and range are sets of real numbers with no breaks in between. A graph of discrete data consists of individual points that are not connected by a line or curve.

**Academic Vocabulary**

An independent variable is the variable for which input values are substituted in a function. A dependent variable is the variable whose value is determined by the input or value of the independent variable.
1a. Use set notation to write the domain and range for the graph below. Does this graph appear to represent a function? Justify your answer. Are the data discrete or continuous? Why?

1b. The graph below shows the relationship between $t$, the length of time of the bath (from the time water starts running through the time the tub is drained) and $d$, the depth of the water in the bath tub. The graph represents function $d$ (bath water depth). What are the dependent and independent variables? Explain. Use set notation to write the domain and range of function $d$. Are the data discrete or continuous and why?
EXAMPLE

Give the domain and range of the function \( f(x) = \frac{1}{(x - 2)^2} \) graphed below.

Step 1: Study the graph.
The sketch of this graph is a portion of the function represented by the equation \( f(x) = \frac{1}{(x - 2)^2} \).

Step 2: Look for values for which the domain causes the function to be undefined. Look how the graph behaves near \( x = 2 \).

Solution: The domain and range for \( f(x) = \frac{1}{(x - 2)^2} \) can be written:

- Domain: \( \{ \text{all real values of } x : x \neq 2 \} \)
- Range: \( \{ \text{all real values of } y : y > 0 \} \)

TRY THESE

a. Give the domain and range of the function \( f(x) = 8 + 2x - x^2 \) graphed below.
TRY THESE (continued)

b. Give the domain and range for the equation \( y = 2x - 1 \). Explain whether this equation represents a function and how you determined this.

Technology Time

- Work with a partner to investigate the equations listed in the chart using graphing technology. Every equation given here is a function.
- Determine the domain and range for each function from the possibilities listed below the chart.
- Select the appropriate domain from choices 1–6 and record your answer in the Domain column. Then select the appropriate range from choices a–f and record the appropriate range in the Range column.
- When the chart is complete, compare your answers with those from another group.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( y = -3x + 4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. ( y = x^2 - 6x + 5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. ( y = 9x - x^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. ( y =</td>
<td>x + 1</td>
<td>)</td>
</tr>
<tr>
<td>5. ( y = 3 + \sqrt{x} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. ( y = \frac{4}{x} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Possible Domains:
1) all real numbers  
2) all real \( x \), such that \( x \neq -2 \)  
3) all real \( x \), such that \( x \neq 0 \)  
4) all real \( x \), such that \( x \neq 2 \)  
5) all real \( x \), such that \( x \geq 0 \)  
6) all real \( x \), such that \( x \leq 0 \)

Possible Ranges:
1) all real numbers  
2) all real \( y \), such that \( y \neq 0 \)  
3) all real \( y \), such that \( y \geq -4 \)  
4) all real \( y \), such that \( y \geq 0 \)  
5) all real \( y \), such that \( y \geq 1 \)  
6) all real \( y \), such that \( y \geq 3 \)

**Math Tip:**
The domain is restricted to avoid situations where division by zero or taking the square root of a negative number would occur.
ACTIVITY 2.2
continued

Domain and Range of Continuous Functions

Shake, Rattle, and Roll

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Give the domain and range for the function graphed below. Explain why this graph represents a function.

2. A student calculates how far away a lightning strike is, based on when the thunder is heard. The student makes the table below using $\frac{1}{3}$ km/sec as the average speed of sound under rainy conditions. If the thunder is only heard when the lightning strike is within 15 km of the listener, what are the domain and range for this model? Is this relation a function? How do you know?

<table>
<thead>
<tr>
<th>Time until thunder is heard (sec)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from lightning strike (km)</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>1</td>
<td>$1\frac{1}{3}$</td>
<td>$1\frac{2}{3}$</td>
<td>2</td>
</tr>
</tbody>
</table>

3. Give the domain and range of the function $f(x) = -4x - 5$.

4. The graph below shows five points that make up the function $h$. Give the domain and the range for the function $h$.

5. Jeff walks at an average rate of 125 yards per minute. Mark's house is located 2000 yards from Jeff's house. The graph below shows how far Jeff still needs to walk to reach Mark's house. Give the domain and range for this model. Is this model a function? Explain.
CHECK YOUR UNDERSTANDING (continued)

6. Capital letters sketched in the coordinate plane may or may not be functions. Pick one letter that represents a function and two that do not. Use the vertical line test as part of the explanation for your selections.

7. **MATHEMATICAL REFLECTION** Describe at least three different methods for determining if a relation is a function. Which method do you prefer and why?
Jonas and Margo’s dad explained that since their grandparents were moving in with them, he needed to make it easier for them to get in and out of the house. Their dad asked Jonas and Margot to research the specifications for building stairs and wheelchair ramps. They decided to look at the government website that gave the Americans with Disabilities Act (ADA) accessibility guidelines for wheelchair ramps and found the following diagram:

The chart below gives information from the ADA website about the slope of wheelchair ramps.

<table>
<thead>
<tr>
<th>Slope</th>
<th>Maximum rise</th>
<th>Maximum run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in.</td>
<td>mm</td>
</tr>
<tr>
<td>$\frac{1}{16} &lt; m \leq \frac{1}{12}$</td>
<td>30</td>
<td>760</td>
</tr>
<tr>
<td>$\frac{1}{20} \leq m &lt; \frac{1}{16}$</td>
<td>30</td>
<td>760</td>
</tr>
</tbody>
</table>

Then, they decided to look for the requirements for building stairs and found the following diagram:
1. What do you think is meant by the terms *rise* and *run* in this context?

Consider the line in the graph below:

2. What is the vertical change between:
   - a. points A and B?
   - b. points A and C?
   - c. points C and D?

3. What is the horizontal change between:
   - a. points A and B?
   - b. points A and C?
   - c. points C and D?

The ratio of the vertical change to the horizontal change determines the **slope** of the line.

\[
slope = \frac{\text{vertical change}}{\text{horizontal change}}
\]

4. Find the slope of the segment of the line connecting:
   - a. points A and B
   - b. points A and C
   - c. points C and D
5. What do you notice about the slope of the line in parts (a), (b), and (c) of item 4?

6. Slope is sometimes referred to as $\frac{\text{rise}}{\text{run}}$. Explain how the ratio $\frac{\text{rise}}{\text{run}}$ relates to the ratios for finding slope mentioned above.

7. Would the slope change if you counted the run (horizontal change) before you counted the rise (vertical change)? Explain your reasoning.

8. Vertical change can be represented as a change in $y$, and horizontal change can be represented by a change in $x$.
   These movements can also be written as the ratio $\frac{\text{change in } y}{\text{change in } x}$ or $\frac{\Delta y}{\Delta x}$. Using this new terminology, explain how to move along the grid to get from one point to another.
   
   From A to C: $\Delta y = \ldots$ and $\Delta x = \ldots$ Ratio $\frac{\Delta y}{\Delta x} = \ldots$
   
   From B to D: $\Delta y = \ldots$ and $\Delta x = \ldots$ Ratio $\frac{\Delta y}{\Delta x} = \ldots$
   
   From A to D: $\Delta y = \ldots$ and $\Delta x = \ldots$ Ratio $\frac{\Delta y}{\Delta x} = \ldots$

9. What do you notice about these ratios?

**WRITING MATH**

In mathematics the Greek letter $\Delta$ (delta) represents a change or difference between mathematical values.
My Notes

10. Describe the movement along the grid to get from B to A and then from D to B.
   From B to A: $\Delta y = \square$ and $\Delta x = \square$
   From D to B: $\Delta y = \square$ and $\Delta x = \square$

11. What kind of number represents the change in $y$, $\Delta y$, described in Item 10?

12. What kind of number represents the change in $x$, $\Delta x$, described in Item 10?

13. Write ratios for $\frac{\Delta y}{\Delta x}$ using these numbers for the movement from B to A and then from D to B.
   B to A: 
   D to B: 

14. How do these ratios compare to those you found in Item 8?

15. When the points in a scatter plot lie on a line, a ratio $\frac{\Delta y}{\Delta x}$ tells you the slope of a line through those points. What do you think is true about slope ratios between any two points on a line?
Jonas and Margo have drawn this design for the wheelchair ramp.

Recall the chart from the ADA website about the slope of wheelchair ramps.

<table>
<thead>
<tr>
<th>Slope</th>
<th>Maximum rise in.</th>
<th>Maximum run ft</th>
<th>Maximum run m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{16} &lt; m \leq \frac{1}{12} )</td>
<td>30</td>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>( \frac{1}{20} \leq m \leq \frac{1}{16} )</td>
<td>30</td>
<td>40</td>
<td>12</td>
</tr>
</tbody>
</table>

16. Jonas and Margo need to check that the ramp that they designed meets the ADA recommendations.

a. In feet, what is the rise of the ramp they designed?

b. What is the slope of the ramp that they designed?

c. Which row of the chart do Jonas and Margo need to check? Explain.

d. Does the ramp they designed meet the recommendations for rise? Explain.

e. Does the ramp they designed meet the recommendation for run? Explain.

f. Does the ramp they designed meet the ADA recommendations?
17. Determine the slope of the line graphed below.

\[
\begin{array}{c|c|c}
\text{change in } y & \text{change in } x \\
\hline
1 & 1 \end{array}
\]

Although the slope of a line can be calculated by looking at a graph and counting the vertical and horizontal change, it can also be calculated numerically.

18. Recall that the slope of a line is found by the ratio \(\frac{\text{change in } y}{\text{change } x}\).

a. Find two points on the graph above and record the coordinates of the two points that you selected.

\[
\begin{array}{c|c|c}
\text{point} & x\text{-coordinate} & y\text{-coordinate} \\
\hline
1^{\text{st}} & \_ & \_ \\
2^{\text{nd}} & \_ & \_ \\
\end{array}
\]

b. Which coordinates relate to the vertical change on a graph?

c. Which coordinates relate to the horizontal change on a graph?
ACTIVITY 2.3 continued

Slope as Rate of Change
Ramp It Up!

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation

d. Find the vertical change by subtracting the \( y \)-coordinate of the first point from the \( y \)-coordinate of the second point.

e. Find the horizontal change by subtracting the \( x \)-coordinate of the first point from the \( x \)-coordinate of the second point.

f. Calculate the slope of the line. How does this slope compare to the slope that you found in Item 17?

g. If other students in your class selected different points for this problem, should they have gotten different values for the slope of this line? Explain.

19. It is customary to label the coordinates of the first point \((x_1, y_1)\) and the coordinates of the second point \((x_2, y_2)\).

a. Write an expression to calculate the vertical change, \( \Delta y \), of the line through these two points.

b. Write an expression to calculate the horizontal change, \( \Delta x \), of the line through these two points.

c. Write an expression to calculate the slope of the line through these two points.
20. Margo and Jonas went to the lumber yard to buy supplies to build the wheelchair ramp. They know that they will need many pieces of wood. Each piece of wood costs $3.

a. Write a function, \( f(x) \), for the total cost of the wood pieces if Jonas and Margo buy \( x \) pieces of wood.

b. Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Pieces of Wood, ( x )</th>
<th>Total cost, ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph](image)

Total Cost of Wood

\[ f(x) \]

\[ x \]

\[ \text{Total Cost (in dollars)} \]

\[ \text{Pieces of Wood} \]

\[ -4 \quad -2 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \]

C. What is the slope of the line that you graphed? Use the formula that you developed in Item 19 and then use the graph to check your work.

D. How does the slope of this line relate to the situation with the pieces of wood?

E. Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.
21. Margo is going to work with a local carpenter during the summer. Each week she will earn $10.00 plus $2.00 per hour.

a. Write a function, \( f(x) \), for Margo's total earnings if she works \( x \) hours in one week.

b. Make an input/output table of ordered pairs and then graph the function. Label your axes.

<table>
<thead>
<tr>
<th>Hours, ( x )</th>
<th>Earnings, ( f(x) ) (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph](image)

f(x)

20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-4
-2
0
2
4
6
8
10
x

22. What is the slope of the line that you graphed? Use the formula you developed in Item 19 and then use the graph to check your work.

d. How does the slope of this line relate to Margot's job?

e. Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.

f. How much will Margot earn if she works for 8 hours in one week?
22. By the end of the summer Margot has saved $375. Recall that each of the small pieces of wood costs $3.

a. Write a function, \( f(x) \), for the amount of money that Margo still has if she buys \( x \) pieces of wood.

b. Make an input/output table of ordered pairs and then graph the function.

\[
\begin{array}{|c|c|}
\hline
\text{Pieces of wood, } x & \text{Money remaining, } f(x) \quad \text{(in dollars)} \\
\hline
\ldots & \ldots \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Money Remaining (in dollars)} & \text{Margot's savings} \\
\hline
500 & \ldots \\
450 & \ldots \\
400 & \ldots \\
350 & \ldots \\
300 & \ldots \\
250 & \ldots \\
200 & \ldots \\
150 & \ldots \\
100 & \ldots \\
50 & \ldots \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Pieces of Wood} & \text{Money Remaining (in dollars)} \\
\hline
50 & \ldots \\
100 & \ldots \\
150 & \ldots \\
200 & \ldots \\
\hline
\end{array}
\]

c. How does this line differ from the other lines you have seen in this activity?

d. What is the slope of the line that you graphed? Use the formula you developed in Item 19.

e. How does this slope differ from the other slopes that you have seen in this activity?

f. How does the slope of this line relate to Margot's savings?
23. Consider the graph of the line below:

```
  A       B
 /|
 / |
 /  |
 /___|
```

a. What is the vertical change from point A to point B?

b. What is the horizontal change from point A to point B?

c. What is the slope of the line?

24. Describe the slope of any line that rises as you view it from left to right. An example is shown below.

```
  /|
 / |
/  |
/___|
```
25. Describe the slope of any line that falls as you view it from left to right. An example is shown below.

![Graph showing a line with negative slope]

26. What is the slope of a horizontal line? Choose any two points on the line and calculate the rise and the run.

![Graph showing a horizontal line]

27. What is the slope of a vertical line? Choose any two points on the line and calculate the rise and the run.

![Graph showing a vertical line]
28. One point that is on the graph of \( y = \frac{2}{3}x + 3 \) is shown on the grid below. Use your knowledge of slope to place three more points on the graph and then give the coordinates of your points.

![Graph](image)

29. Margot and Jonas used the diagram from the first page of this activity as a model for the staircase that they are going to build.

![Staircase Diagram](image)

a. If the height the entire staircase needs to cover is 56.7 inches, what should the height of each step be?

b. If the length the entire staircase needs to cover is 67.9 inches, what should the length of each tread be?
**ACTIVITY 2.3 continued**

**Slope as Rate of Change**

**Ramp It Up!**

**SUGGESTED LEARNING STRATEGIES:** RAFT

**Writing Math**

When writing your response to Item 30, you can use a RAFT.

- **Role**
- **Audience**
- **Format**—a letter
- **Topic**—wheelchair ramps and stairs

**CHECK YOUR UNDERSTANDING**

1. Find $\Delta x$ and $\Delta y$ for the points $(7, -2)$ and $(9, -7)$.

2. Find the slope given a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

3. Find the slope of a line that passes through the points $(0, 4)$ and $(3, 9)$.

4. Find the slope of a line that passes through the points $(-2, 4)$ and $(3, -3)$.

5. Find two points that make a slope of $-\frac{5}{7}$.

6. Determine the slope of the given line.

7. Sketch a line that has a positive slope.

8. Sketch a line that has a negative slope.

9. Sketch a line that has a zero slope.

10. Given the linear equation $y = -\frac{2}{5}x + 1$, determine the slope.

11. One point on the graph of $y = \frac{3}{4}x - 2$ is given. Place two more points on the graph of the equation.

12. **Mathematical Reflection**

Explain three different ways to find the slope of a line and how these methods are the same and different.

**c.** What is the slope of each step? What is the slope of entire staircase?

30. Write a letter to Jonas and Margot explaining how rise and run relate to slope and why these topics are an important aspect of building wheelchair ramps and stairs.
Representations of Functions

BRYCE CANYON HIKING

While on vacation, Jorge and Jackie traveled to Bryce Canyon National Park in Utah. They were impressed by the differing elevations at the viewpoints along the road. The graph describes the elevations for several viewpoints in terms of the time since they entered the park.

1. The graph above represents a function $E(t)$. Describe why the graph represents a function. Identify the domain and range of the function.

2. Is this discrete or continuous data? Explain.

While at Bryce Canyon National Park, Jorge and Jackie went hiking on the Under the Rim trail to Yellow Creek. The table shows their progress down to Yellow Creek. The grid is provided for optional use to help you answer the questions below.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Elevation (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7900</td>
</tr>
<tr>
<td>10</td>
<td>7500</td>
</tr>
<tr>
<td>20</td>
<td>7100</td>
</tr>
<tr>
<td>30</td>
<td>6700</td>
</tr>
</tbody>
</table>

3. Find the slope for the data in the table. Interpret the slope as a rate of change, including units.

4. On the descent, what was the elevation 18 min after Jorge and Jackie began? Justify your answer.

5. On the descent, when were they at 7000 ft? Justify your answer.
## Properties and Solving Equations

### Bryce Canyon Hiking

<table>
<thead>
<tr>
<th>Math Knowledge #1, 2, 3</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student correctly:</td>
<td>The student attempts the three tasks but answers only two correctly.</td>
<td>The student attempts only two tasks and answers only one correctly.</td>
<td></td>
</tr>
<tr>
<td>• Identifies domain and range. (1)</td>
<td>• Determines whether the function is discrete or continuous. (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Determines whether the function is discrete or continuous. (2)</td>
<td>• Finds the slope. (3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #3, 4, 5</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student correctly:</td>
<td>The student attempts the three tasks but answers only two correctly.</td>
<td>The student attempts only two tasks and answers only one correctly.</td>
<td></td>
</tr>
<tr>
<td>• Interprets the slope as a rate of change; interpretation is correct based on the slope given. (3)</td>
<td>• Identifies the elevation. (4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Identifies the elevation. (4)</td>
<td>• Determines when they were at 7000 ft. (5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communication #1, 2, 4, 5</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student gives a complete and accurate:</td>
<td>The student attempts the description, explanation, and justification, but only two are complete and correct. There are no mathematical errors.</td>
<td>The student attempts the description, explanation, or justification. Only one is complete and correct.</td>
<td></td>
</tr>
<tr>
<td>• Description. (1)</td>
<td>• Explanation. (2)</td>
<td>• Justification for both answers. (4, 5)</td>
<td></td>
</tr>
</tbody>
</table>
You are the packaging director for a paper products company. Your company is introducing a new type of paper cups. Your design team must design a cardboard container to use when packaging the cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, your team will collect data on two types of cups found around the office.

1. Use two different types of cups to complete the tables below.

<table>
<thead>
<tr>
<th>CUP 1</th>
<th>CUP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cups</td>
<td>Height of Stack</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?
Use your data for Cup 1 to complete Items 3–6.

3. Make a graph of the data you collected.

4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

6. Write an equation that gives the height of a stack of cups, \( h \), in terms of \( n \), the number of cups in the stack.
7. Use your equation from Item 6 to find $h$ when $n = 16$ and when $n = 50$. Do your answers to this question agree with your predictions in Items 4 and 5?

8. Sketch the graph of your equation from Item 6.

9. How are the graphs you made in Items 3 and 8 the same? How are they different?
10. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

11. Find the dimensions of a container that will hold a stack of 25 cups.

12. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
   - the equation your team discovered to find the height of the stack of Cup 1 style cups
   - a description of how your team discovered the equation and the minimum number of cups needed to find it
   - an explanation of how the numbers in the equation relate to the physical features of the cup
   - an equation that could be used to find the height of the stack of Cup 2 style cups
After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The cup will be the same basic shape as described in your report. The company will use the function $S(n) = 0.5n + 12.5$.

13. What do $S$, $S(n)$, and $n$ represent?

14. What do the numbers 0.5 and the 12.5 in the function $S$ tell you about the physical features of the cup?

15. Evaluate $S(1)$ to find the height of a single cup.

16. How tall is a stack of 35 cups? Show your work using function notation.
SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Quickwrite, Look for a Pattern, Marking the Text

17. If you add 2 cups to a stack, how much does the height of the stack increase?

18. If you add 20 cups to a stack, how much does the height of the stack increase?

19. A member of one of the teams stated: “If you double the number of cups in a stack, then the height of the stack is also doubled.” Is this statement correct? Explain.

20. If you were to graph the linear function $S(n) = 0.5n + 12.5$ and connect the points, you would see that they lie in a straight line. The slope of a line is a measure of the steepness of a line and indicates a rate of change.

   a. What is the slope of this line?

   b. Interpret the slope of the line as a rate of change that compares a change in height to a change in the number of cups.
21. **a.** The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?

**b.** If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?

**c.** How many cups fit in a container that is 36 cm tall?

**d.** How many cups fit in a container that is 50 cm tall?

22. The function \( S(n) = 0.5n + 12.5 \) describes height \( S \) in terms of the number of cups \( n \).

**a.** Solve this equation for \( n \) to describe the number of cups \( n \) in terms of the height \( S \).
b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Item 21 parts (c) and (d).

c. What is the slope of the line represented by your equation in part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 20(b).

5. The equation for the cost $C$ of a cab ride of $m$ miles is $C = 2.5m + 3.5$.
   a. What is the cost of a 6-mile ride?
   b. What is the cost of a 7-mile ride?
   c. How is the price difference between a 6-mile ride and a 7-mile ride related to the numbers in your equation?

6. In the equation $S = 0.25n + 8.5$, $S$ is the height in inches of a stack of jumbo cups and $n$ is the number of cups.
   a. How many cups would it take to make a stack 1 inch higher?
   b. How many cups would fit in a carton that is 18 inches high?
   c. Interpret the slope as a rate of change.

7. **MATHEMATICAL REFLECTION** What did you learn about creating a linear model? How can you recognize and interpret a constant rate of change?
You work for a packaging and shipping company. As part of your job there, you are part of a package design team deciding how to stack boxes for packaging and shipping. Each box is 10 cm high.

1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height of the Stack (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

2. Write a function to represent the data in the table and graph above.

3. What do the $f(x)$, or $y$, and the $x$ represent in your equation from Item 2?

4. What patterns do you notice in the table and graph representing your function?
5. The number of boxes is directly proportional to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.

6. When two values are directly proportional, there is a direct variation. In terms of stacking boxes, the \( \text{number of boxes} \) varies directly as the \( \text{height of the stack} \).

Therefore, this function is called a direct variation.

7. Using variables \( x \) and \( y \) to represent the two values, you can say that \( y \) varies directly as \( x \). Use your answer to Item 6 to explain this statement.

8. Direct variation is defined as \( y = kx \), where \( k \neq 0 \) and the coefficient \( k \) is the constant of variation.

   a. Consider your answer to Item 2. What is the constant of variation in your function and why do you think it is called that?

   b. Why can’t \( k \) equal zero?

   c. Write an equation for finding the constant of variation by solving the equation \( y = kx \) for \( k \).

9. a. What does the point \((0, 0)\) mean in your table and graph?

   b. True or False? Explain your answer.
   “The graphs of all direct variations are lines that pass through the point \((0, 0)\).”
Now use what you have learned about direct variation to answer questions about stacking and shipping your boxes.

10. The height $y$ of a different stack of boxes varies directly as the number of boxes $x$. For this type of box, 25 boxes are 500 cm high.

   a. Find the value of $k$.

   b. Write a direct variation equation that relates $y$, the height of the stack, to $x$, the number of boxes in the stack.

   c. How high is a stack of 20 boxes? Use your equation to answer this question.

11. At the packaging and shipping company, you get paid each week. One week you earned $48 for 8 hours of work. Another week you earned $30 for 5 hours of work.

   a. Write a direct variation equation that relates your wages to the number of hours you worked each week.

   b. How much do you earn per hour?

   c. How much would you earn if you worked 3.5 hours in one week?

When packaging a different product, the team determines that all boxes will have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.

Math Tip
The volume of a rectangular prism is found by multiplying length, width, and height:
$V = lwh$. 
12. To explore the relationship between length and width in the situation on the previous page, complete the table and make a graph of the points.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

13. How did you figure out the lengths and widths in Item 12?

14. Write a function to represent the data in the table and graph above.

15. What do the $f(x)$, or $y$, and the $x$ represent in your equation from Item 14?

16. What patterns do you notice in the table and graph representing your function?

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an indirect variation, also known as inverse variation.

17. Compare and contrast direct and inverse variation.
18. Recall that direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient $k$ is the constant of variation.

   a. How would you define inverse variation in terms of $y$, $k$, and $x$?

   b. Are there any limitations on these variables as there are on the $k$ in direct variation? Explain.

   c. Write an equation for finding the constant of variation by solving for $k$ in your answer to part (a).

19. Use your equation in 14 to determine the following measurements for your company.

   a. Find the length of a box whose width is 80 inches.

   b. Find the length of a box whose width is 0.5 inches.

20. The time, $y$, to finish loading the boxes varies inversely as the number of people, $x$, working. If 10 people work, the job is completed in 20 h.

   a. Find the value of $k$.

   b. Write an inverse variation equation that relates the time to finish loading the boxes to the number of people working.

   c. How long does it take 8 people to finish loading the boxes? Use your equation to answer this question.
21. The cost for the company to ship the boxes varies inversely with the number of boxes being shipped. If 25 boxes are shipped at once, it will cost $10 per box. If 50 boxes are shipped at once, the cost will be $5 per box.

a. Write an inverse variation equation that relates the cost per box to the number of boxes being shipped.

b. How much would it cost to ship only 10 boxes?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. In the equation $y = 15x$, what is the constant of variation?

2. Identify the examples of direct variation from tables, graphs and equations below. Explain how you made your decision.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
</tbody>
</table>

3. $y$ varies directly as $x$. If $y = 300$ when $x = 20$, find $y$ when $x = 7$.

4. The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

5. In the equation $y = \frac{80}{x}$ what is the constant of variation?

6. Which equations are examples of inverse variation? Explain your answers.

   a. $y = 2x$
   b. $y = \frac{x}{2}$
   c. $y = \frac{2}{x}$
   d. $xy = 2$

7. $y$ varies inversely as $x$. If $y = 8$ when $x = 20$, find $y$ when $x = 10$.

8. MATHEMATICAL REFLECTION Create a graphic organizer that helps you to compare and contrast direct and inverse variation equations.
Linear Functions and Equations

TEXT MESSAGE PLANS

Pedro is planning to add the text messaging feature to his cell phone. He has gathered information about the two different plans offered by the phone company.

Plan A: $4.00 per month plus 4 cents for each message
Plan B: 5 cents per message

1. Use the mathematics you have been studying in this unit to provide Pedro with the following information for each plan.

   a. Plan A
      • a table of data
      • a graph of the data
      • the linear function that fits this plan
      • the domain and range of the function

   b. Plan B
      • a table of data
      • a graph of the data
      • the linear function that fits this plan
      • the domain and range of the function

2. If Pedro sends 360 messages on average each month, which plan would you recommend that he choose? What information or mathematics can you use to support your recommendation?

3. If Pedro knows that his average usage is going to increase to 500 text messages per month on average, should he change to a different plan? Explain your reasoning.

4. Do either of the plans represent a direct variation? Explain.

5. Pedro’s friend Chenetta is considering another text messaging plan that advertises: “Monthly text plan varies inversely as the number of text messages sent.” The sales person gives Chenetta the example that she can send 450 messages for 80 cents per message.

   a. Write an inverse variation equation that relates the per-message rate and the number of text messages.

   b. Chenetta knows that she has about 1800 text messages per month. In a short note, use an example to let Chenetta know if you think this plan would be a good deal for her.
## Properties and Solving Equations

### TEXT MESSAGE PLANS

<table>
<thead>
<tr>
<th>Math Knowledge #1a, 1b, 4, 5a, 5b</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>• The domain and range are correct, based on the equations given.</td>
<td>• Either the decision or the equation is correct.</td>
<td>• The domain and/or range are incorrect or missing.</td>
</tr>
<tr>
<td>The student:</td>
<td>• Correctly decides whether either plan represents a direct variation.</td>
<td>• Either the decision or the equation is correct.</td>
<td>• Either the decision or the equation is correct.</td>
</tr>
<tr>
<td>The student:</td>
<td>• Writes a correct equation; payment is correct based on the equation given.</td>
<td>• The payment amount is correct, based on the equation given.</td>
<td>• The payment amount is not correct based on the equation given.</td>
</tr>
<tr>
<td></td>
<td>(1a, b)</td>
<td>(5a, b)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #2, 3</th>
<th>The student:</th>
<th>The student:</th>
<th>The student:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>• Chooses the most efficient plan.</td>
<td>• Chooses the most efficient plan</td>
<td>• The choice of plan is missing OR The student makes no decision about changing plans.</td>
</tr>
<tr>
<td>The student:</td>
<td>• Decision of whether to change is correct based on the information given.</td>
<td>• Correctly decides whether to change plans.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representations #1a, 1b</th>
<th>The student provides a table, a graph, and an equation that correctly represent both Plan A and Plan B.</th>
<th>Only two of the table, graph, and equation are correct for both plans OR All three answers are correct for one of the plans.</th>
<th>Only one of the three representations is correct for both plans OR Two of the three representations are correct for one of the plans.</th>
</tr>
</thead>
</table>

| Communication #2, 3, 4 | • The student provides support for the plan chosen with no mathematical errors | Support for the plan chosen is incomplete; the explanation is complete for only one of the answers given. | Support for the plan chosen is missing or contains mathematical errors; the explanations are incomplete or contain mathematical errors. |
|                       | (2) | (3, 4) |              |
When a diver descends in a lake or ocean, pressure is produced by the weight of the water on the diver. As a diver swims deeper into the water, the pressure on the diver’s body increases at a rate of about 1 atmosphere of pressure per 10 meters of depth. The table and graph below represent the total pressure, $y$, on a diver given the depth, $x$, under water in meters.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

1. Write an equation describing the pressure exerted on a diver when under water.

2. What is the slope of the equation of the line that you found? What are the units of the slope?

3. What is the $y$-intercept of the line?

**Slope-Intercept Form of a Linear Equation**

$y = mx + b$

where $m$ is the slope of the line and $b$ is the $y$-intercept.

4. Identify the slope and $y$-intercept of the line described by the equation $y = -2x + 9$. 

**CONNECT TO SCIENCE**

*Pressure* is the measure of a force against a surface, and is usually expressed as a force per unit area. *Atmospheric pressure* is defined using the unit atmosphere. 1 atm is 14.6956 pounds per square inch.

**ACADEMIC VOCABULARY**

A linear equation is an equation of the form $Ax + By = C$ where $A$, $B$, and $C$ are constants and $A$ and $B$ cannot both be zero.

**Math Tip**

Linear equations can be written in several forms.

**ACADEMIC VOCABULARY**

The $y$-intercept of a line is the $y$-coordinate of the point where the line crosses the $y$-axis, which is when the $x$-coordinate is 0.
SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Group Presentation, Note Taking

5. Create a table of values for the equation $y = -2x + 9$. Then plot the points and graph the line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

6. Explain how to find the slope from the table.

7. Explain how to find the $y$-intercept from the table.

8. Explain how to find the slope from the graph.

9. Explain how to find the $y$-intercept from the graph.

EXAMPLE 1
Write the equation, in slope-intercept form, of the line that passes through the point $(1, 4)$ and has a slope of $-3$.

Step 1: Find the $y$-intercept by substituting the coordinates of the point and the slope in the equation.

$y = mx + b$

$4 = -3(1) + b$ Substitute $-3$ for $m$, $1$ for $x$, and $4$ for $y$.
$4 = -3 + b$ $4 + 3 = -3 + b + 3$
$7 = b$ The $y$-intercept is $7$.

Step 2: Substitute the slope and $y$-intercept into the slope-intercept form.

$y = mx + b$
$y = -3x + 7$

Solution: $y = -3x + 7$
TRY THESE A

a. Write the equation, in slope-intercept form, of the line with a slope of 4 and a y-intercept of 5.

b. Find the equation, in slope-intercept form, of the line that passes through the point (−3, 7) and has a slope of \(-\frac{2}{3}\).

c. Write an equation of the line shown in the graph at the right.

Point-Slope Form of a Linear Equation

\[ y - y_1 = m(x - x_1) \]

where \(m\) is the slope of the line and \((x_1, y_1)\) is a point on the line.

You get this form of the equation by solving the slope formula

\[ m = \frac{y - y_1}{x - x_1} \]

for \(y - y_1\), by multiplying both sides by \(x - x_1\).

The variable \(y\) is the dependent variable, and \(x\) is the independent variable. You may use this form when you know a point on the line and the slope.

EXAMPLE 2

Write an equation of a line with a slope of \(\frac{1}{2}\) that passes through the point (2, 5).

**Step 1:** Substitute the given values into point-slope form.

\[ y - y_1 = m(x - x_1) \]

\[ y - 5 = \frac{1}{2}(x - 2) \]

**Step 2:** Solve for \(y\).

\[ y = \frac{1}{2}(x - 2) + 5 \]

**Solution:** \(y = \frac{1}{2}(x - 2) + 5\)

CONNECT TO AP

In calculus, the point-slope form of a line is used to write the equation of the line tangent to a curve at a given point.

Math Tip

If you needed to express the solution to Example 2 in slope-intercept form, you could apply the distributive property and combine like terms.
TRY THESE B
Find an equation of the line given a point and the slope.

a. \((-2, 7), m = \frac{2}{3}\)  

b. \((6, -1), m = -\frac{5}{4}\)  

c. \((86, 125), m = -18\)

The town of San Simon charges its residents for trash pickup and water usage on the same bill. Each month the city charges a flat fee for trash pick-up and a $0.25 per gallon usage fee for water. In January, one resident used 44 gallons of water, and received a bill for $16.

10. If \(x\) is the number of gallons of water used during a month, and \(y\) represents the bill amount in dollars, write a point \((x_1, y_1)\).

11. What does the $0.25 per gallon represent?

12. Use point-slope form to write an equation that represents the bill cost \(y\) in terms of the number of gallons of water \(x\) used in a month.

13. Write the equation in Item 12 in slope-intercept form. What does the \(y\)-intercept represent?

EXAMPLE 3
Write an equation of the line that passes through the points \((6, 4)\) and \((3, 5)\).

Step 1: Find the slope by substituting the two points into the slope formula.
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 4}{3 - 6} \quad \text{Substitute (6, 4) for } (x_1, y_1) \text{ and (3, 5) for } (x_2, y_2).
\]
\[
m = \frac{5 - 4}{3 - 6} = \frac{1}{-3} = -\frac{1}{3}
\]

Step 2: Substitute the slope and one of the points into point-slope form.
\[
y - y_1 = m(x - x_1)
\]
\[
y - 5 = -\frac{1}{3}(x - 3) \quad \text{Substitute (3, 5) for } (x_1, y_1) \text{ and } -\frac{1}{3} \text{ for } m.
\]
\[
y = -\frac{1}{3}(x - 3) + 5 \quad \text{Solve for } y.
\]

Solution: \(y = -\frac{1}{3}(x - 3) + 5\)
TRY THESE C

a. Find the equation in point-slope form of the line shown in the graph.

b. Find the equation in point-slope form of the line with a slope of 5 that passes through the point (1, -3).

c. Find the slope and a point on the line whose equation is \( y = 3 - \frac{2}{3}(x + 3) \).

d. Find the equation of the line that passes through the points \((-2, 1)\) and \((2, 3)\).

e. Write the equation of the line from Item (d) above in slope-intercept form.

**Standard Form of a Linear Equation**

\[ Ax + By = C \]

where \( A \geq 0 \), \( A \) and \( B \) cannot both be zero, and \( A, B, \) and \( C \) are integers whose greatest common factor is one.

14. You can use the coefficients of this form of an equation to find the slope, \( x \)-intercept and \( y \)-intercept of a line.

a. Write \( Ax + By = C \) in slope-intercept form to find the slope.

b. Find the \( x \)-intercept.

c. Find the \( y \)-intercept.

**MATH TERMS**

The **greatest common factor** of two or more integers is the greatest integer that is a divisor of all the integers.

**ACADEMIC VOCABULARY**

The **\( x \)-intercept** is the \( x \)-coordinate of the point where a line crosses the \( x \)-axis.
15. Find the slope and \(y\)-intercept of the line described by the equation \(3x + 2y = 8\).

16. Write \(3x + 2y = 8\) in the slope-intercept form of an equation.

17. Given the equation \(y = \frac{1}{2}x + \frac{2}{5}\) of a line, write it in the standard form of a linear equation.

---

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper. Show your work.

1. Write the standard form of the equation of the line with a slope of 7 that passes through the point \((1, 2)\).

2. Write the equation \(3x - 2y = 16\) in slope-intercept form.

3. Write the equation \(y = -4 + 6(x + 1)\) in slope-intercept form.

4. Write the equation in standard form of the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

5. As a diver descends into fresh water, we have determined that for every 10 meters of depth, the pressure on the diver increases by one atmosphere. Find the equation of the line representing this relationship if we know that when the diver is at a depth of 25 meters, there is a pressure of 3.5 atmospheres.

6. Rate yourself on a scale of 1 to 5 on your perceived understanding of equations of lines for Slope-intercept, Point-slope, and Standard forms of a line. The lower the rating the lower the level of understanding. What can you do to bring your level of understanding to a higher level?
Frank is reading Harper Lee’s *To Kill a Mockingbird* for his English class. The book is 280 pages long. He estimates he will need seven hours of reading time to finish the book. Displaying \( y \) on the vertical axis as pages left to read and \( x \) on the horizontal axis as time spent reading in hours, the **intercepts** have been connected to form a line.

1. Identify the \( x \)-intercept, \( y \)-intercept, and slope from this model. What does each represent in the problem situation?

2. Write the equation of this line using the intercepts. Identify the form of the equation that you have created.

3. Write the equation of the line in the standard form.
4. Divide each side of the equation in Item 3 by the constant term and simplify so that there will be a 1 on one side of the equal sign.

5. What do you recognize about the denominators of the equation you wrote in Item 4?

6. The equation below is a variable representation of the equation that you created in Item 4. As a historical mathematician, you have the honor of naming this form of the line. Write your choice of name for this form and explain your decision using mathematical vocabulary.

\[ \frac{x}{a} + \frac{y}{b} = 1 \]

**EXAMPLE**

The graph of \( y = 2x - 6 \) below shows the line crossing the \( x \)-axis at 3 and the \( y \)-axis at \(-6\).

a. Verify the intercepts based on the definition.

b. Then write the equation in the form shown in Item 6.
Step 1: Use the definition of intercept.

From the definition, the \( y \)-intercept is the \( y \)-coordinate where \( x = 0 \).

\[
y = 2x - 6
\]

\[
y = 2(0) - 6 \quad \text{Substitute 0 for } x.
\]

\[
y = -6
\]

Write as an ordered pair \((0, -6)\). The \( y \)-intercept is \(-6\).

From the definition, the \( x \)-intercept is the \( x \)-coordinate where \( y = 0 \).

\[
y = 2x - 6
\]

\[
0 = 2x - 6 \quad \text{Substitute 0 for } y.
\]

\[
0 - 2x = 2x - 6 - 2x \quad \text{Solve for } x \text{ by subtracting } 2x \text{ from both sides.}
\]

\[
-2x = -6 \quad \text{Simplify by dividing both sides by } -2.
\]

\[
x = 3
\]

Write as an ordered pair \((3, 0)\). The \( x \)-intercept is 3.

Step 2: Use algebra to write the equation in the form used in Item 6.

To write \( y = 2x - 6 \) in the form \( \frac{x}{a} + \frac{y}{b} = 1 \), isolate the constant to the right of the equal sign. Then divide through by an appropriate number to create the 1 to the right of the equals sign.

\[
y = 2x - 6
\]

\[
-2x - 2x \quad \text{Subtract } 2x \text{ from each side of the equation.}
\]

\[
-2x + y = -6 \quad \text{Simplify the zero pair } (2x - 2x = 0).
\]

\[
\frac{-2x + y}{-6} = \frac{-6}{-6} \quad \text{Divide each term by } -6.
\]

\[
\frac{x}{3} + \frac{y}{-6} = 1 \quad \text{Simplify.}
\]

Solution: The \( y \)-intercept is \(-6\), and the \( x \)-intercept is 3. The equation of the line is \( \frac{x}{3} + \frac{y}{-6} = 1 \)

TRY THESE A

a. Write the equation \( 3x - 4y = 24 \) in intercept form. Verify the intercepts for this equation algebraically.

MATH TERMS

The intercept form of an equation of a line is

\[
\frac{x}{a} + \frac{y}{b} = 1
\]

where \( a \) is the \( x \)-intercept and \( b \) is the \( y \)-intercept.
TRY THESE A (continued)

b. Write the equation \( y = \frac{-2}{3}x - 4 \) in intercept form.

c. Write the intercept form of the equation that has a \( y \)-intercept of \(-5\) and an \( x \)-intercept of \(-4\).

The Booster Club is planning to sell refreshments as a project at the upcoming Fall Bazaar. Tim makes the following proposals to the Club.

Tim’s proposal:
Buy 120 bottles of water for $21 to sell for $1.50 each.

To support his proposal, Tim has developed a function to calculate the profit \( P \) based on \( n \) bottles sold.

Tim’s function: \( P = 1.50n - 21.00 \)

7. What are the intercepts for Tim’s function?

8. Graph Tim’s function.

9. What is the zero of Tim’s function? What does it represent in terms of the problem situation?
10. What is the domain of Tim’s function from the previous page?

11. Write Tim’s function in its intercept form.

**TRY THESE B**

a. Find the zero of the function \( g(x) = -\frac{3}{4}x + \frac{1}{2} \).

b. Estimate the zero of the function graphed below.

![Graph of a linear function with x and y axes ranging from -7 to 5 and -7 to 10, respectively.]

C. If the equation of the function in part b above is \( f(x) = 3x + 7 \), would you consider your estimate of the zero to have been a good one? Explain.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. Matt sells used books on the Internet. The cost of the weekly website fee is $7.50, and he earns $1.50 on each book that he sells. Matt uses the linear equation $y = 1.5x - 7.5$ to figure his weekly earnings where $y =$ earnings in dollars and $x =$ the number of books that he sells.

   a. Graph this function. Set up your axes as shown below.

   b. Write the equation in intercept form.

   c. What meaning do the $x$- and $y$-intercepts have in the context of this problem?

2. Assume that each line graphed in the next column crosses the $x$- and $y$-axes at integer values. Match lines $c$ and $d$ on the graphs with the intercept form of the equations.

   a. $\frac{x}{8} + \frac{y}{4} = 1$

   b. $\frac{x}{7} + \frac{y}{3} = 1$

   c. $\frac{x}{4} + \frac{y}{-8} = 1$

   d. $\frac{x}{3} + \frac{y}{-7} = 1$

3. Given the intercepts $(0, 5)$ and $(-4, 0)$.

   a. Graph the line that contains the points.

   b. Write the intercept form of this line.

4. The slope-intercept form of a line is $y = -\frac{2}{3}x - 6$. Write this equation in the intercept form. Give the zero of this function.

5. Does the point $(2, 210)$ lie on the line $\frac{x}{10} + \frac{y}{300} = 1$? Explain your reasoning.

6. Give two instances when knowing the intercept form of the line would be most beneficial. Justify your answer.
Equations from Data
Pass the Book

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Visualization, Use Manipulatives, Create Representations, Quickwrite

How fast can you and your classmates pass a textbook from one person to the next until the book has been relayed through each person in class?

1. Suppose your entire class lined up in a row. Estimate the length of time you think it would take to pass a book from the first student in the row to the last. Assume that the book starts on a table and the last person must place the book on another table at the end of the row.

Estimated time to pass the book: ___________________

2. As a class, experiment with the actual time it takes to pass the book using small groups of students in your class. Use the table below to record the times.

<table>
<thead>
<tr>
<th>Number of Students Passing the Book</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to Pass the Book (nearest tenth of a second)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Based on the data you recorded in the table above, would you revise your estimated time from Item 1? Explain the reasoning behind your answer.
4. Graph the data in your table from Item 2 as a **scatter plot** on the coordinate grid.

![Scatter plot](image)

5. Are the data that you collected linear data?
   a. Explain your answer using the scatter plot.
   b. Explain your answer using the table of data.

6. Describe how the time to pass the book changes as the number of students increases.

7. Work as a group to predict the number of seconds it will take to pass the book through the whole class.
   a. Place a **trend line** on the scatter plot in Item 4 in a position that your group feels best models the data. Then, mark two points on the line.
   b. In the spaces provided below, enter the coordinates of the two points identified in Part (a).

   Point 1: (______, ______)  Point 2: (______, ______)
c. Why does your group think that this line gives the best position for modeling the scatter plot data?

8. Use the coordinate pairs you recorded in Item 7(b) to write the equation for your trend line (or linear model) of the scatter plot.

9. Explain what the variables in the equation of your linear model represent.

10. What is the meaning of the slope in your linear model?

11. Use your equation to predict how long it would take to pass the book through all the students in your class.

   Predicted time to pass the book: ______________________

12. Using all of the students in your class, find the actual time it takes to pass the book.

   Actual time to pass the book: ______________________
**ACTIVITY 2.8 continued**

**Equations from Data**

**Pass the Book**

**SUGGESTED LEARNING STRATEGIES:** Think/Pair/Share

**My Notes**

13. How do your estimate from Item 1 and your predicted time from Item 11 compare to the actual time that it took to pass the book through the entire class?

14. Suppose that another class took 1 minute and 47 seconds to pass the book through all of the students in the class. Use your linear model to estimate the number of students in the class.

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper or grid paper. Show your work.

The table shows the number of days absent and the grades for several students in Ms. Reynoso’s Algebra 1 class. Use the table for Items 1–8.

<table>
<thead>
<tr>
<th>Days Absent</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade (percent)</td>
<td>98</td>
<td>88</td>
<td>69</td>
<td>89</td>
<td>90</td>
<td>86</td>
<td>77</td>
</tr>
</tbody>
</table>

1. Create a scatter plot of the data using days absent as the independent variable.
2. Are the data linear? Explain using the scatter plot and the table of data.
3. Based on the data, how do grades change as the number of days absent increases?
4. Draw a trend line on your scatter plot. Identify two points on the trend line and write an equation for the line containing those two points.
5. What is the meaning of the \(x\) and \(y\) variables in the equation you wrote?
6. What is the meaning of the slope and the \(y\)-intercept of the trend line you drew?
7. Use your equation to predict the grade of a student absent for 5 days.
8. Sixty percent is a passing grade in Ms. Reynoso’s class. Use your equation to find how many days a student could be absent and still earn a passing grade.
9. Write the equation of a line passing through the given pairs of points.
   a. \((-2, 5)\) and \((5, 6)\)
   b. \((0, 6)\) and \((-4, -8)\)
10. **MATHEMATICAL REFLECTION** What would be the sign of the slope of a trend line on a scatter plot that compares the sale prices of cars to ages of cars? Why do you think so?
Linear Equations and Slope as Rate of Change

A 10K RUN

Jim was serving as a finish-line judge for the Striders 10K Run. He was interested in finding out how three of his friends were doing out on the course. He was able to learn the following information from racing officials at different locations along the course.

“Matuba is running at a strong, steady rate of 320 m every minute after running the first 1400 m in a time of 5 minutes.”

“Rodriguez ran the first 2000 m in 6 minutes, before he settled into his steady pace passing the 4400 m mark at 14 minutes.”

“According to his calculations, Donovan feels he can equal his best running time of 32 minutes for 10,000 m over this course.”

Answer Items 1–3 below based on the information Jim received about his three running friends. Use \( x \) as the number of minutes since the race began and \( y \) as the number of meters completed.

1. Create three linear models for each runner’s progress toward the finish line.

2. Explain the order in which the three runners will finish the race based on the models you formed using this information.

3. Using the models you formed, in what order would the runners have passed the 5K mark in the race?

For the linear models you created, find the following numeric answers. Explain the significance, if any, for each answer in the context of the problem situation.

4. the domain of the linear model for Donovan

5. the \( y \)-intercept for Matuba’s linear model

6. the slope of the linear model for Rodriguez

CONNECT TO METRIC MEASUREMENT

A “10K Run” means that the length of the course for the foot race is 10 kilometers, or 10,000 meters.

\[
10K = 10 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 10,000 \text{ m}
\]
### Linear Equations and Slope as Rate of Change

#### A 10K RUN

<table>
<thead>
<tr>
<th></th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Knowledge #4, 5, 6</strong></td>
<td>The student correctly finds the domain, y-intercept, and slope for all three models. (4), (5), (6)</td>
<td>Only two of the domain, y-intercept, and slope are correct.</td>
<td>Only one of the domain, y-intercept, and slope is correct.</td>
</tr>
<tr>
<td><strong>Problem Solving #2, 3</strong></td>
<td>The student correctly determines the correct order of finish at 10K and 5K. (2, 3)</td>
<td>The student determines the correct order of finish at 10K and 5K based on the linear models developed.</td>
<td>The student determines the correct order of finish for either 10K or 5K based on the linear models developed.</td>
</tr>
<tr>
<td><strong>Representations #1</strong></td>
<td>The student correctly creates the three linear models. (1)</td>
<td>The student creates only two correct linear models.</td>
<td>The student creates only one correct linear model.</td>
</tr>
<tr>
<td><strong>Communication #4, 5, 6</strong></td>
<td>Explanations are correct, based on the models found. (4, 5, 6)</td>
<td>Explanations are complete and correct for only two of the models.</td>
<td>The explanation is complete and correct for only one of the models.</td>
</tr>
</tbody>
</table>
ACTIVITY 2.1

For each problem, one is a function and one is not. Identify each and explain.

1. \{ (5, -2), (\neg 2, 5), (2, -5), (-5, 2) \}  
   \{ (5, -2), (-2, 5), (2, -5), (\neg 5, 2) \}

2. \( y = \lvert x \rvert + 5 \)

3. \( y = x^2 + 4x - 3 \)

4. \( y = \lvert x \rvert + 5 \)
   \( x = \lvert y \rvert \)

Given \( f(x) = 10 - 3x \); \( g(x) = x^2 - 3x + 2 \), and \( h(x) = 5x - 4 \), evaluate the following:

5. \( h(3) \)
6. \( f(-7) \)
7. \( g(5) \)
8. \( g\left(\frac{2}{3}\right) \)
9. \( h(0.8) \)
10. \( f(-2.5) \)

ACTIVITY 2.2

11. Give the domain and range for the equation \( y = \lvert x - 2 \rvert \) graphed below. Does this graph pass the vertical line test? Does this equation and its graph represent a function?

12. Write the domain and range for the equation \( y = x^2 + 4x - 3 \) graphed below. Is this relation a function? How do you know?

13. Give the domain and range for the function \( f(x) = 2 + 0.5x \). State whether this function is linear or not and explain how you determined this.
14. In the set of figures below, shaded tiles are added to create each new phase. Fill in the table below. If the domain is the set of natural numbers, give the range for this relation. Explain whether this relation represents a function.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Tiles Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

15. Give the domain and range for the function graphed below. Explain whether or not this graph represents a function.

16. Find \( \Delta x \) and \( \Delta y \) for the points \((-4, 6)\) and \((3, -7)\).

17. Someone claims that the slope of the line through \((-2, 7)\) and \((3, 0)\) is the same as the slope of the line through \((2, 1)\) and \((12, -13)\). Prove or disprove the claim, and explain your reasoning.

18. If an airplane descended to land at a constant rate of 300 feet per minute, and it took 20 minutes from the time it began its descent to land, at what altitude was the plane flying when it began its descent? Explain your reasoning.

19. Does the table represent data with a constant slope? Justify your answer.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
</tr>
</tbody>
</table>

20. How does the slope of a line through the points \((12, 11)\) and \((2, 7)\) compare with the slope of the line through \((5, 9)\) and \((1, 5)\)?

21. Find the slope of the line of the graph.
ACTIVITY 2.4

The height \( h \) in centimeters of a stack of \( n \) cups is given by the linear function \( h(n) = 0.4n + 10 \).

22. What is the slope of this function?
   a. 0.4  
   b. 9.6  
   c. 10  
   d. 10.4

23. How tall is a stack of 50 cups?
   a. 20  
   b. 30  
   c. 40  
   d. 60

Jeremy collected the following data on stacking chairs.

<table>
<thead>
<tr>
<th>Chairs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>45</td>
</tr>
</tbody>
</table>

24. Write a linear function that best models the data.

25. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack?

The formula to convert degrees Celsius \( C \) to degrees Fahrenheit \( F \) is \( F = \frac{9}{5} C + 32 \).

26. Use the formula to convert 100° C to degrees Fahrenheit.

27. Interpret the slope as a rate of change.

28. If the temperature is 50° F, what is the temperature in degrees Celsius?

29. Solve for \( C \) to derive the formula that converts degrees Fahrenheit to degrees Celsius.

ACTIVITY 2.5

30. Given \( y \) varies directly as \( x \) and \( y = 125 \) when \( x = 25 \). What is the value of \( y \) when \( x = 2 \)?
   a. \( y = \frac{1}{10} \)  
   b. \( y = \frac{1}{5} \)  
   c. \( y = 5 \)  
   d. \( y = 10 \)

31. Which equation does not represent a direct variation?
   a. \( y = \frac{x}{3} \)  
   b. \( y = \frac{2}{5}x \)  
   c. \( y = \frac{3}{x} \)  
   d. \( y = \frac{5x}{2} \)

32. Given \( y \) varies inversely as \( x \) and \( y = 20 \) when \( x = 10 \). What is the value of \( y \) when \( x = 40 \)?
   a. \( y = 5 \)  
   b. \( y = 8 \)  
   c. \( y = 50 \)  
   d. \( y = 80 \)

The Pete’s Pets chain of pet stores is growing. The table below shows the number of stores in business each month, starting with January (J).

<table>
<thead>
<tr>
<th>Month</th>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>M</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stores</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

33. According to the table, how many new stores open per month?
   a. 0  
   b. 3  
   c. 9  
   d. 18

34. How many stores will be in business by December?
   a. 18  
   b. 30  
   c. 33  
   d. 36

35. Are the Pete’s Pets data an example of inverse variation, direct variation, or neither? Explain your reasoning.

ACTIVITY 2.6

36. Write an equation of the line that has a slope of \( \frac{2}{3} \) and \( y \)-intercept of \(-5\).

37. Write an equation of the line that passes through the point \((6, -3)\), with a slope of 5.

38. Find the slope and the \( y \)-intercept of the line \( 4x - y + 6 = 0 \).

39. Write an equation of the line that passes through the points \((-3, 2), (5, -2)\).
40. Write the equation of the line given the graph in slope-intercept form, and in standard form.

![Graph of a line with coordinates](image)

ACTIVITY 2.7

41. Rita and Mitch are reading novels for their Language Arts class. The graphs below show information about their reading assignment.

![Graph showing the catcher in the rye](image)

The Catcher in the Rye

![Graph showing the diary of Anne Frank](image)

Diary of Anne Frank

a. Which student will need less time to read their novel and what graphical evidence supports your conclusion?

b. Write the intercept form of the line for the Diary of Anne Frank graph where \( y \) represents pages left to be read and \( x \) represents number of hours Rita has been reading.

42. Assume that each line crosses the \( x \)- and \( y \)-axes at integer values. Match lines \( e \) and \( f \) on the graphs with the intercept form of the equations.

![Graph with lines e and f](image)

\[ a. \quad \frac{x}{6} + \frac{y}{2} = 1 \quad b. \quad \frac{x}{7} + \frac{y}{4} = 1 \]
\[ c. \quad \frac{x}{4} + \frac{y}{7} = 1 \quad d. \quad \frac{x}{2} + \frac{y}{6} = 1 \]

43. Given the standard form of the line \( 3x - 4y = 18 \), write the intercept form and give the coordinates of the \( x \)- and \( y \)-intercepts.

44. The intercepts of a line have coordinates \( (5, 0) \) and \( (0, -2) \). Write the intercept form of the line and graph the line.

45. Verify that the point \( (10, 9) \) lies on the line \( \frac{x}{4} + \frac{y}{-6} = 1 \). Show your work.

46. If \( f(x) = 5x - 2 \), what is the zero of this function? How did you find it?
ACTIVITY 2.8
The scatter plot shows the relationship between the day of the month and total rainfall for January.

47. Use the data points (3, 1.2) and (13, 3.6) to write a linear equation to model the data.
48. If this trend continues, what would be the total rainfall as of January 31?

Given the points (–2, 3) and (1, 5).
49. What is the slope of the line containing these points?
   a. $-\frac{3}{2}$  b. $-\frac{2}{3}$  c. $\frac{2}{3}$  d. $\frac{3}{2}$
50. Write the equation of the line containing these two points.
An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

**Essential Questions**

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How can you show mathematical relationships?
   - Why are linear functions useful in real-world settings?

**Academic Vocabulary**

2. Look at the following academic vocabulary words:
   - dependent variable
   - independent variable
   - direct variation
   - inverse variation
   - domain
   - function
   - linear equation
   - range
   - x-intercept
   - y-intercept

Choose three words and explain your understanding of each word and why each is important in your study of math.

**Self-Evaluation**

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?
b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. In the $xy$-plane, the line $y = ax + 5$ has the same slope as the line $3x + 8y = 10$. What is the value of $a$?

A. $-6$
B. $\frac{-8}{3}$
C. $\frac{-3}{8}$
D. $\frac{3}{8}$

2. The cheetah is the fastest land animal, reaching speeds of 70 to 75 miles per hour and has the ability to accelerate from 0 to 68 miles per hour in 3 seconds. A cheetah ran at a rate of 72 mi/h to reach a stranded donkey. The cheetah caught the donkey in 25 seconds. If the cheetah ran at a constant speed, how many miles away was the donkey from the cheetah?

3. Given the function $f(x) = -2x - 5$, what is the value of $f(-3)$?
4. Josh is driving from St. Augustine, Florida, to Richmond, Virginia. He is traveling at an average rate of 50 miles per hour. Two hours after Josh began his drive, his cousin begins the drive from St. Augustine to Richmond using the same route. Josh’s cousin drives at an average rate of 62.5 miles per hour.

**Part A:** Assuming that $t$ represents the number of hours that Josh travels, write an equation to represent $d_1$, the distance that Josh travels, and write an equation to represent $d_2$, the distance that Josh’s cousin travels. Explain how you determined your equations.

**Part B:** After how many hours will Josh’s cousin catch up with Josh? Find how far Josh will be from St. Augustine when his cousin catches up with him. Show how you found your answers.