Unit Overview
In this unit you will extend your study of linear concepts to the study of piecewise defined functions and systems of linear equations and inequalities. You will learn to solve systems of equations and inequalities in a variety of ways.

Academic Vocabulary
Add these words to the academic vocabulary portion of your math notebook.

- Substitution Method
- Elimination Method
- Linear inequality
- Piecewise defined function
- System of linear equations
- System of linear inequalities

Essential Questions
Why would you use multiple representations of linear equations and inequalities?
How are systems of linear equations and inequalities useful in interpreting real world situations?

EMBEDDED ASSESSMENTS
This unit has two embedded assessments, following Activities 3-3 and 3-7. They will give you an opportunity to demonstrate what you have learned.

Embedded Assessment 1
Graphing Inequalities and Piecewise Functions p. 163

Embedded Assessment 2
Systems of Equations and Inequalities p. 193
1. Which of the following tables of values represents linear data?
   a. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   2 & 3 \\
   4 & 6 \\
   6 & 6 \\
   8 & 3 \\
   \end{array}
   \]
   b. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   1 & 1 \\
   2 & 4 \\
   3 & 7 \\
   4 & 10 \\
   \end{array}
   \]
   c. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   2 & 3 \\
   3 & 4 \\
   4 & 3 \\
   6 & 4 \\
   \end{array}
   \]
   d. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   1 & 1 \\
   2 & 6 \\
   3 & 12 \\
   4 & 24 \\
   \end{array}
   \]

2. Give an algebraic representation of these data.
   \[
   \begin{array}{c|c|c|c}
   x & 1 & 4 & 7 \\
   \hline
   y & 1 & 7 & 13 \\
   \end{array}
   \]

3. You open a savings account with a deposit of $100. Each month after your initial deposit, you add $25 to your account. Provide a table to display your first 4 deposits. Give a graphical and algebraic representation which would allow you to determine the amount of money \( A \) in dollars you have deposited after \( m \) months.

4. Graph \( 2x + 3y = 4 \).

5. Describe the graph of \( y = 3 \).

6. Which ordered pair is a solution of \( y > x + 5 \)?
   a. \( (2, 8) \)
   b. \( (-5, 0) \)
   c. \( (1, 6) \)
   d. \( (0, -5) \)

7. Compare and contrast the graphs of the two compound statements:
   \[-1 < x \leq 3 \]
   \[x < -1 \text{ or } x \geq 3.\]

8. Which of the following represents a constant rate of change?
   a. \[
   \begin{array}{c|c|c|c} 
   x & 1 & 4 & 16 \\
   \hline 
   y & 5 & 10 & 20 \\
   \end{array}
   \]
   b. \[
   \begin{array}{c|c|c|c} 
   x & 1 & 4 & 16 \\
   \hline 
   y & 5 & 10 & 15 \\
   \end{array}
   \]
   c. \( y = \frac{4}{x} \)
   d. \[
   \begin{array}{c|c|c|c} 
   x & 1 & 4 & 16 \\
   \hline 
   y & 5 & 10 & 15 \\
   \end{array}
   \]
Miriam has accepted a job at a veterinarian’s office. Her first assignment is to feed the dogs that are housed there. The bag of dog food was already torn open and she found only part of the label that described the amount of dog food to feed each dog.

<table>
<thead>
<tr>
<th>Barko Dog Food Feeding Chart</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight of Dog (pounds)</strong></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>20</td>
</tr>
</tbody>
</table>

Each dog has an information card that gives the dog’s name and weight. Using this information, Miriam fed each dog the amount of food she thought was appropriate.

1. How many ounces of dog food per day do you think Miriam gave each dog? Complete the chart below.

<table>
<thead>
<tr>
<th>Dog</th>
<th>Dog’s Weight (pounds)</th>
<th>Amount of Dog Food (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffy</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Trixie</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Rags</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Hercules</td>
<td>68</td>
<td></td>
</tr>
</tbody>
</table>

2. Based on the dog’s weight and the partial label at the top of the page, write a verbal rule that Miriam could use to determine the ounces of dog food per day to feed each dog.

3. Let $w$ be the weight of a dog in pounds and $A$ be the amount of food in ounces that a dog should be fed each day. Use the chart in Item 1 to write an equation that expresses $A$ in terms of $w$. 
After several hours, all of the dogs had finished their food except for Hercules. The vet asked, “How much food did you give Hercules?” Miriam answered that Hercules had been given 68 ounces based on the pattern she had observed on the partial label.

The vet told Miriam that she had overfed Hercules and suggested that Miriam check the complete feeding chart posted on the office wall.

Barko Dog Food Feeding Chart

<table>
<thead>
<tr>
<th>Weight of the Dog (pounds)</th>
<th>Daily Amount of Barko (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>over 100 pounds</td>
<td>60 ounces plus 1 ounce for each additional 10 pounds of weight</td>
</tr>
</tbody>
</table>

4. Miriam had assumed that each dog should be fed as many ounces of dog food as the dog weighed in pounds. Explain why the data in the new chart indicate that her original assumption was not correct.

Adult dogs that weigh less than 20 pounds are classified as *small* dogs. Dogs that weigh 20 to 100 pounds are classified as *mid-size* dogs. Finally, dogs that weigh more than 100 pounds are classified as *large* dogs. Miriam had not known that the formulas for feeding small, mid-size, and large dogs are all different.

5. The data for feeding mid-size dogs appear to be linear. Use the feeding chart above to explain why this may be true.
6. Miriam checked with the vet and found that the data were linear. Determine a rate of change that describes the number of additional ounces of food a mid-size dog should be fed for each pound of weight greater than 20. Explain how you found your answer.

7. Using the rate of change you found in Item 6 and the feeding chart, write an equation that expresses $A$, the ounces of dog food, in terms of $w$, the dog’s weight in pounds, for mid-size dogs.

8. Given that the equation from Item 7 only is true only for mid-size dogs, what inputs for $w$ would be appropriate?

9. Based on your answer to Item 7, how many ounces was Hercules overfed? Show work to support your answer.
Miriam knows that she also has several large dogs to feed. When she looks at the chart, she reads the instruction “60 ounces plus 1 ounce for each additional 10 pounds of weight.”

10. How much additional dog food should large dogs be fed for each pound of weight greater than 100?

11. A 140-pound Great Dane has arrived for a short stay at the kennel. Miriam determines that she should feed the dog 64 ounces of food daily. Do you agree? Show your work to justify your answer.

12. Write an equation that expresses the amount of food, $A$, in ounces that a large dog should be fed, as a function of $w$, the weight of the dog in pounds.
Miriam realizes that there are three different algebraic feeding rules to follow because dogs are different sizes. She organizes her feeding rules in a list so that she can quickly refer to them whenever she has to decide how much food to feed a dog.

**13.** Complete Miriam’s list by writing the appropriate equation. Indicate the domain by writing the appropriate inequality symbols.

\[
A = \underline{\text{___________}}, \text{ when } 0 \underline{\text{_______}} w \underline{\text{_______}} 20
\]

\[
A = \underline{\text{___________}}, \text{ when } 20 \underline{\text{_______}} w \underline{\text{_______}} 100
\]

\[
A = \underline{\text{___________}}, \text{ when } w \underline{\text{_______}} 100
\]

Miriam decides to make a table that lists the weight of the dogs she will be feeding, in the order that she will feed them. A portion of Miriam’s table is shown below.

**14.** Complete the table using the rules you wrote in Item 13.

<table>
<thead>
<tr>
<th>Dog’s Weight (pounds)</th>
<th>Amount of Dog Food (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
</tr>
<tr>
<td>107</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>
The vet compared Miriam’s list of rules in Item 13, and table in Item 14, to the summary she uses as a veterinarian:

The data can be written as a piecewise defined function. Every possible weight \( w \) has exactly one feeding amount \( A \) assigned to it, but the rule for determining that feeding amount changes for the different sized dogs. The different feeding rules, along with their domains, are considered to be the pieces of a single function called a piecewise defined function.

15. Does the vet’s summary agree with Miriam’s rules and table? Explain your answer.

16. When graphing a piecewise defined function, it is necessary to graph each piece of the function only for its appropriate interval of the domain. Graph the feeding function on the axes below. When finished, your graph should consist of three line segments.

![Graph of piecewise defined function](image_url)
When she looks at the graph, Miriam notices that for small dogs, each increase of 1 pound in weight causes an increase of 1 ounce of food. She concludes that the feeding rate of change for small dogs is 1 ounce per pound.

17. What is the feeding rate of change for large dogs?

18. Which size category of dog—small, mid-size, or large—has the greatest feeding rate of change?

19. How can the graph in Item 16 support your answer to Item 18?

20. What parts of the algebraic feeding rules in Item 13 support your answer to Item 18?

21. Dogs in one size category have a greater rate of change in feeding than other size dogs. Explain why you think this might be true.
ACTIVITY 3.1  
**Piecewise Linear Functions**

**Breakfast for Bowser**

**My Notes**

22. The vet has a German Shepard named Max, and Miriam knows that the vet feeds Max 63 ounces of food each day. Miriam also knows that the vet feeds her cocker spaniel named Min 32 ounces of food each day. If the two dogs are being correctly fed, what is each dog’s weight? Show your work and explain your reasoning.

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper or grid paper. Show your work.

These tables represent the parts of a piecewise defined function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>11</td>
<td>14</td>
</tr>
</tbody>
</table>

1. What is the function that represents the data in the first interval? What is the domain?

2. What is the function that represents the data in the second interval? What is the domain?

Use this piecewise defined function for Items 3–5.

\[ f(x) = \begin{cases} 
  x^2 - 1, & \text{if } x < 1 \\
  x + 2, & \text{if } x \geq 1 
\end{cases} \]

3. What is the domain for the first part of the function?

4. What is the domain for the second part of the function?

5. Sketch a graph of the function. Do the two pieces connect at \( x = 1 \)? Why or why not?

6. **Mathematical Reflection** How do piecewise defined functions differ from functions you have previously learned? What questions do you still have about piecewise defined functions?
Travis Smith and his brother, Roy, are co-owners of a trucking company. One of their regular weekly jobs is to transport fruit grown in Pecos, Texas, to a Dallas, Texas, distributing plant. Travis knows that his customers are concerned about the speed with which the brothers can deliver the produce, since fruit will spoil after a certain length of time.

1. Travis wants to address his customers’ concerns with facts and figures. He knows that a typical trip between Pecos and Dallas takes 7.5 hours. He makes the following graph.

   ![Graph](image)

   a. What information does the graph provide?

   b. Locate the point with coordinates of (3, 270) on the graph. Label it point A. Describe the information these coordinates provide.

CONNECT TO BUSINESS

Companies that ship fruit from distribution plants to stores around the country use refrigerated trucks to keep the fruit fresher longer. For example, the optimal temperature range for shipping cantaloupes is 36–41°F. What would the temperature range look like on a number line graph?
c. According to the graph, how many hours will it take Travis to reach Dallas from Pecos? How did you determine your answer?

d. Interstate 20 is the direct route between Pecos and Dallas. Based upon his graph, at what average speed does Travis expect to travel? How did you determine your answer?

2. Complete the table to show how far Travis is from Dallas at each hour.

<table>
<thead>
<tr>
<th>Hours Since Leaving Pecos, ( h )</th>
<th>Distance from Dallas, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

3. Use your table and the graph to write an equation that expresses Travis’ distance \( d \) from Dallas as a function of the hours \( h \) since he left Pecos.
Travis knows that his first graph represents a model for an ideal travel scenario. In reality, he assumes that his average speed will be lower because he will need to stop to refuel. He also decides that he must account for concerns about road construction and traffic.

4. Experience tells Travis that his average speed will decrease to 45 mi/h. Travis’ ideal travel scenario is shown on the grid below. On these same axes, draw the graph of his distance from Dallas, based upon his assumption that he will maintain a 45 mi/h average speed throughout his trip.

5. Write an equation for the line you drew in Item 4.

6. Travis can average anywhere from 45 mi/h to 60 mi/h, as shown on the two previous graphs.

   a. On the Item 4 graph, sketch the vertical line \( h = 3 \). Highlight the segment of that line that gives all his possible distances from Dallas three hours after Travis leaves Pecos.

   b. How far might Travis be from Dallas after three hours of travel? Write your answer as an inequality. Show your work or explain how you determined your response.
Those distances you found are the possible distances after three hours of travel. The line segment can be described as the ordered pairs \((h, d)\) such that \(h = 3\) and \(d\) is the inequality you wrote in Item 6(b).

7. Travis can average anywhere from 45 mi/h to 60 mi/h, as shown on the two previous graphs.

a. On the Item 4 graph, sketch the vertical line \(h = 5\). Highlight the segment of that line that gives all his possible distances from Dallas five hours after Travis leaves Pecos.

b. How far might Travis be from Dallas after five hours of travel? Write your answer as an inequality.

c. Describe what the inequality and the line segment tell Travis about his trip.

8. Use the Item 4 graph.

a. Draw the line segment for \(h = 7.5\).

b. How far might Travis be from Dallas after 7.5 hours of travel? Write your answer as an inequality.

c. Describe what the inequality and the line segment tell Travis about his trip.

9. Use the Item 4 graph.

a. Draw the line segment for \(h = 9\).

b. How far might Travis be from Dallas after nine hours of travel? Write your answer as an inequality.

c. Describe what the inequality and the line segment tell Travis about his trip.
10. There is a region that could be filled by similar vertical line segments for all values of $h$ that Travis could be on his trip. Shade this region on the Item 4 graph.

11. Suppose that the speed limit on all parts of Interstate 20 has been changed to 70 mi/h. Travis finds that he can now average between 50 mi/h and 70 mi/h on the trip between Pecos and Dallas.

a. Write an equation that expresses Travis’ distance $d$ from Dallas as a function of the hours $h$ since he left Pecos if his average speed is 50 mi/h.

b. Write an equation that expresses Travis’ distance $d$ from Dallas as a function of the hours $h$ since he left Pecos if his average speed is 70 mi/h.

c. On the grid on the next page, graph the two equations that you found in Items (a) and (b).
d. Shade the region of the graph below for the ordered pairs \((h, d)\) such that \(h\) represents all the possible times and \(d\) represents all the possible distances from Dallas after \(h\) hours of travel.

![Graph showing shaded region]

e. Write an inequality that shows Travis’s possible distances from Dallas after 4 hours of travel.

**CHECK YOUR UNDERSTANDING**

Graph the equations for Items 1 and 2 in the first quadrant of the same graph.

1. \(y = -25x + 250\)
2. \(y = -15x + 250\)
3. Draw a vertical line segment along the line \(x = 5\) that connects the lines you drew in Items 1 and 2. Write an inequality that states the possible \(y\)-values that lie between the two lines.

4. Tucson is 100 miles from Phoenix. A driver leaves Tucson averaging 62 mi/h, and is traveling to Phoenix. Write an equation for finding the distance \(d\) from Phoenix, given the time \(t\) in hours since the driver left.

5. **Mathematical Reflection** What did you learn about inequalities and their relationship to linear equations?
Axl and Aneeza pay for and share memory storage at a remote Internet site. Their plan allows them upload 250 terabytes (TB) or less of data each month. Let \( x \) represent the number of terabytes Axl uploads in a month, and \( y \) represent the number of terabytes Aneeza uploads in a month.

1. Look at the four months below. Determine if Axl and Aneeza stayed within their monthly plan.

<table>
<thead>
<tr>
<th>Axl's Uploads (( x ))</th>
<th>Aneeza's Uploads (( y ))</th>
<th>Within Plan? (Yes/No)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 TB</td>
<td>120 TB</td>
<td>Yes</td>
</tr>
<tr>
<td>135 TB</td>
<td>100 TB</td>
<td>Yes</td>
</tr>
<tr>
<td>100 TB</td>
<td>150 TB</td>
<td>No</td>
</tr>
<tr>
<td>162 TB</td>
<td>128 TB</td>
<td>No</td>
</tr>
</tbody>
</table>

2. Write an inequality that models the plan’s restriction on uploading.

3. If Axl uploads 100 terabytes, write an inequality that shows how many terabytes Aneeza can upload.

4. If Aneeza uploads 75 terabytes, write an inequality that shows how many terabytes Axl can upload.
The graph below represents the possible values for the number of terabytes that Axl and Aneeza can upload. Any ordered \((x, y)\) pair represents the number of terabytes that Axl and Aneeza could upload. For example, the coordinates \((100, 75)\) mean that Axl uploaded 100 TB and Aneeza uploaded 75 TB of data.

5. Graph each ordered pair on the graph above. Determine if it will allow the siblings to remain within their plan, and explain your answers.

   a. \((200, 50)\)
   
   b. \((150, 150)\)
   
   c. \((225, 25)\)
   
   d. \((20, 120)\)
   
   e. \((120, 200)\)

The graph of Axl and Aneeza’s storage plan above is an example of a graph of a linear inequality in two variables. All the points in the shaded region are solutions of the linear inequality.
Inequalities in Two Variables

Shared Minutes

6. Follow the steps below to graph the linear inequality \( y \leq 2x + 3 \).

   a. Graph the corresponding linear equation \( y = 2x + 3 \). The line you graphed is the boundary line.

   ![Graph of the boundary line]

   b. Test a point in one of the half-planes to see if it is a solution of the inequality.

   c. If the point you tested is a solution, shade the half-plane in which it lies. If it is not, shade the other half-plane.

   In Item 6, the solution includes the boundary line. The solution is a closed half-plane. In a < or > inequality, the solution does not include the points on the boundary line, so the boundary line is dashed, and the solution is an open half-plane.

7. Graph \( y > -\frac{1}{2}x + 4 \).

   ![Graph of the inequality]

   Math Tip
   
   The origin (0,0) is usually an easy point to test if it is not on the boundary line.
8. Axl and Aneeza changed to a 350-terabyte plan. How would the graph in Item 4 change?

9. On the grid below, the $x$-axis represents the number of terabytes Axl can upload, and the $y$-axis represents the number of terabytes Aneeza can upload. Graph the inequality that would represent Axl and Aneeza’s new plan from Item 8.
10. Which ordered pairs from Item 5 will work with the new data storage plan described in Items 8 and 9?

   a. (200, 50)

   b. (150, 150)

   c. (225, 25)

   d. (20, 120)

   e. (120, 200)

11. On the grid below, the x-axis represents the number of terabytes Axl can upload, and the y-axis represents the number of terabytes Aneeza can upload. Suppose Aneeza decides not to upload for a month. Write an inequality that represents the amount of data that Axl can upload during that month. Graph the inequality on the grid below.
12. The x-axis represents the number of terabytes Axl can upload, and the y-axis represents the number of terabytes Aneeza can upload. Suppose Axl decides not to upload for a month. Write an inequality that represents the amount of data that Aneeza can upload during that month. Graph the inequality on the grid below.

CHECK YOUR UNDERSTANDING

1. Graph the linear inequality $x < -3$.
2. Graph the linear inequality $y > 2$.
3. Explain how to graph the linear inequality $y < -2x - 3$.
4. Graph the linear inequality $y \geq \frac{1}{3}x - 3$.
5. Explain how to decide if a point is a solution to a linear inequality.
6. **MATHEMATICAL REFLECTION** Use vocabulary from this unit to describe the difference when graphing $x < 3$ on a number line versus graphing it in two variables.
Graphing Inequalities and Piecewise Functions

EARNINGS ON A GRAPH

1. Steve works at a restaurant. He earns $8.50 per hour. Write an equation that indicates the amount of money \( m \) in dollars that Steve can earn as a function of the hours \( h \) that he worked.

2. The cost of any food that Steve buys while working is deducted from his earnings. Write an inequality that represents the possible amounts of money he can earn after buying food.

3. Graph the inequality you wrote in Item 2.

4. Determine if each point is a solution of the inequality you wrote in Item 2. Interpret the meaning of each point that is a solution.
   a. (16, 100)
   b. (16, 136)
   c. (16, 200)

Bob has been working at the restaurant longer than Steve. He earns $9.00/h. During some weeks he works more than 40 h/wk. The hours he works beyond 40 are considered overtime. For overtime pay Bob earns double time or $18.00/h.

5. a. How much would Bob earn if he worked for 10 h in one week?
   b. Write the function you used to solve Part A.
   c. Define the domain.

6. a. How much would Bob earn if he worked for 45 h in one week?
   b. Write the function you used to solve Part A.
   c. Define the domain.

7. a. On graph paper, graph the functions you wrote in Items 5 and 6.
   b. What term is used to describe the functions? Explain why that term is used.
   c. Clarify the scale used in the graph.
## Graphing Inequalities and Piecewise Functions

### EARNINGS ON A GRAPH

<table>
<thead>
<tr>
<th>Math Knowledge #4a, b, c; 5c, 6c, 7b</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
<td>The student:</td>
<td></td>
</tr>
<tr>
<td>• Makes a correct determination about all three points. (4a, b, c)</td>
<td>• Makes a correct determination about only two of the three points.</td>
<td>• Makes a correct determination about only one of the three points.</td>
<td></td>
</tr>
<tr>
<td>• Gives the correct domain for the functions written. (5c, 6c)</td>
<td>• Gives the correct domain for one of the functions written.</td>
<td>• Gives an incorrect domain for both of the functions written.</td>
<td></td>
</tr>
<tr>
<td>• Gives the correct term that describes the functions. (7b)</td>
<td></td>
<td>• Gives an incorrect term to describe the functions.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #5a, 6a</th>
<th>The student gives correct values for Bob's earnings. (5a, 6a)</th>
<th>The student gives a correct value for Bob's earnings in only one of 5a or 6a.</th>
<th>The student gives incorrect values for Bob's earnings.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Representations #1, 2, 3, 5b, 6b, 7a</th>
<th>The student:</th>
<th>The student:</th>
<th>The student:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
<td>The student:</td>
<td></td>
</tr>
<tr>
<td>• Writes a correct equation. (1)</td>
<td>• Provides an equation (1), an inequality (2), and functions (5b, 6b), but only three of the items are correct.</td>
<td>• Provides at least two of the equation (1), the inequality (2), and the functions (5b, 6b), but only one of the items is correct.</td>
<td></td>
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<tr>
<td>• Writes a correct inequality. (2)</td>
<td>• Draws graphs for the inequality given (3) and the functions given (7a), but only one is correct.</td>
<td>• Draws at least one graph for the inequality given (3) or the functions given (7a), but the graph may be incomplete or incorrect.</td>
<td></td>
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<tr>
<td>• Draws a correct graph, based on the inequality given. (3)</td>
<td></td>
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<tr>
<td>• Writes a correct function for Bob's earnings. (5b, 6b)</td>
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<tr>
<td>• Draws correct graphs, based on the functions given. (7a)</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Communication #4a, b, c; 7b</th>
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<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
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</tr>
<tr>
<td>• Gives a correct interpretation of the meaning of all three points. (4a, b, c)</td>
<td>• Gives a correct interpretation of the meaning of only two points.</td>
<td>• Gives a correct interpretation of the meaning of only one point.</td>
<td></td>
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<tr>
<td>• Gives a correct explanation for the term used to describe the functions. (7b)</td>
<td>• Gives an incomplete explanation for the term used to describe the functions.</td>
<td>• Gives an incorrect explanation for the term used to describe the functions.</td>
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</tbody>
</table>
Parallel and Perpendicular Lines
The Slope of Things

SUGGESTED LEARNING STRATEGIES: Activate Prior Knowledge, Predict and Confirm, Create Representations

Mathematics problems often involve parallel and perpendicular lines. Since these are such common lines, it is important that we remember some information about their slopes.

1. Consider lines $l_1$, $l_2$, $l_3$, and $l_4$ on the graph above. Determine the slope of each line.

2. In the graph above, $l_1$ is parallel to $l_2$ and $l_3$ is parallel to $l_4$. Write a conjecture about the slopes of parallel lines.

3. Determine the slope of a line that is parallel to the line whose equation is $y = -3x + 4$.

4. Write the equation of a line that is parallel to the line $y = \frac{3}{4}x - 1$ and has a $y$-intercept 5.


5. Graph each line described below on the same set of axes in the My Notes section. Which lines appear to be perpendicular?

- \( l_5 \) has slope \(-\frac{4}{3}\) and contains the point (0, 2)
- \( l_6 \) has slope \(-\frac{3}{4}\) and contains the point (0,0)
- \( l_7 \) has slope \(\frac{3}{4}\) and contains the point \((-2, -1)\)

6. Write a conjecture about the slopes of perpendicular lines.

7. Use your prediction from Item 6 to write the equations of two lines that are perpendicular. On the grid in the My Notes section, graph both lines and confirm that they are perpendicular.

TRY THESE A

Determine whether the lines containing the given slopes will be parallel, perpendicular or neither.

a. \( m_1 = 5, m_2 = \frac{1}{5} \)

b. \( m_1 = -6, m_2 = \frac{1}{6} \)

c. \( m_1 = \frac{6}{8}, m_2 = \frac{3}{4} \)

Determine the slope of a line that is parallel and the slope of a line that is perpendicular to the line with the given slope.

d. \( m_1 = -\frac{1}{2} \)

e. \( m_1 = 3 \)
8. Determine the slope of any line that is perpendicular to the line $y = -\frac{3}{2}x + 2$. Explain how you know they are perpendicular.

9. Write the equation of a line that is perpendicular to the line $x - 4y = 8$ and contains the point $(-1, 2)$.
CHECK YOUR UNDERSTANDING

1. Determine whether the lines containing the given slopes will be parallel, perpendicular, or neither.
   a. \(m_1 = -4, m_2 = \frac{1}{4}\)
   b. \(m_1 = -3, m_2 = 3\)
   c. \(m_1 = \frac{10}{12}, m_2 = -1\frac{1}{5}\)
   d. \(m_1 = \frac{1}{2}, m_2 = \frac{1}{2}\)

2. Given the equation of line \(l_1\) is \(y = \frac{1}{3}x - 2\).
   a. Write the equation of a line parallel to \(l_1\).
   b. Write the equation of a line perpendicular to \(l_1\).

3. Write the equation of a line that is parallel to the line \(3x + 4y = 4\) and contains the point \((8, 1)\).

4. Write the equation of a line that is perpendicular to the line \(y = 5x + 1\) and contains the point \((-10, 2)\).

5. **MATHEMATICAL REFLECTION**
   A line \(a\) passes through points with coordinates \((-3, 5)\) and \((0, 0)\), and a line \(b\) passes through points with coordinates \((3, 5)\) and \((0, 0)\). Explain how you would determine whether lines \(a\) and \(b\) are parallel, perpendicular, or neither.
Travis Smith and his brother, Roy, are co-owners of a trucking company. The company needs to transport two truckloads of fruit grown in Pecos, Texas, to a Dallas, Texas, distributing plant. If the fruit does not get to Dallas quickly, it will spoil. The farmers offer Travis a bonus if he can get both truckloads to Dallas within 24 hours.

Due to road construction, Travis knows it will take 10 h to drive from Pecos to Dallas. The return trip to Pecos will take only 7.5 h. He estimates it will take 1.5 h to load the fruit onto the truck and 1 h to unload it.

1. Why is it impossible for Travis to earn the bonus by himself?

2. Travis wants to earn the bonus so he asks his brother, Roy, if he will help. With Roy’s assistance, can the brothers meet the deadline and earn the bonus? Explain why or why not.

To meet the deadline and earn the bonus, Travis will leave Pecos first, and meet Roy somewhere along the interstate to give him a key to the storage area in Pecos.

3. From Pecos to Dallas, Travis averages 45 mph. If Dallas is 450 mi from Pecos, write an equation that expresses Travis’ distance $d$ in miles from Dallas as a function of the hours $h$ since he left Pecos.
4. Graph the equation you wrote in Item 3.

5. Roy leaves Dallas one-half hour before Travis leaves Pecos. In terms of the hours $h$ since Travis left Pecos, write an expression that represents the time since Roy left Dallas.

6. Roy travels 60 mph from Dallas to Pecos. Write an equation that expresses Roy’s distance $d$ from Dallas as a function of the hours $h$ since Travis left Pecos.

7. Graph the equation from Item 6 on the grid in Item 4.

8. What is the intersection point of the two lines? Describe the information these coordinates provide.
The two equations you wrote in Items 3 and 6 form a system of linear equations.

To find the solution of a system of linear equations, you must find all the ordered pairs that make both equations true. One method is to graph each equation and find the intersection point.

TRY THESE A

Use the graphing method to solve each system of linear equations.

a. \( y = 2x - 10 \)
   \( y = -3x + 5 \)

b. \( 2x + y = 7 \)
   \( x - 4y = 8 \)
TRY THESE A (continued)

Use the graphing method to solve each system of linear equations.

c. $y = 3x - 5$
   $y = -2x + 4$

![Graph showing the lines $y = 3x - 5$ and $y = -2x + 4$ intersecting at a point.]

d. $3x + y = 1$
   $6x + 2y = 10$

![Graph showing the lines $3x + y = 1$ and $6x + 2y = 10$ intersecting at a point.]

e. What made finding the solutions to Try These (c) and (d) challenging?
9. How can you determine if the intersection point you identified from the graph in Item 8 is the correct solution of the system of linear equations?

On another trip, Travis is traveling from Pecos to Dallas, and Roy is driving from Dallas to Pecos. They agree to meet for lunch along the way. Each driver averages 60 mph but Roy leaves 1.5 hours before his brother. To determine when and where they will meet, you will solve this system of linear equations.

\[ d = 450 - 60h \]
\[ d = 60h + 90 \]

10. Which equation represents Travis’ distance from Dallas? How do you know?

11. Which equation represents Roy’s distance from Dallas? How do you know?
A numeric method for solving a system of equations involves first making a table of values. Then, look for an ordered pair that is common to both equations.

12. Complete each table.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d = 450 − 60h</td>
<td>d = 60h + 90</td>
</tr>
<tr>
<td>h</td>
<td>d</td>
</tr>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
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<tr>
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<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

13. What ordered pair do the two equations have in common?

TRY THESE B

Solve each system numerically by making a table and looking for a common ordered pair.

a. \( y = 2x + 6 \)
   \( y = -3x + 16 \)

b. \( x + y = 8 \)
   \( 3x + 2y = 14 \)

c. \( y = 100 − 2x \)
   \( y = 20 + 6x \)

d. \( 2x + y = 6 \)
   \( 2x + 3y = 8 \)

e. What challenges did you encounter when solving these systems of linear equations numerically?
On another trip, Roy leaves from Pecos one hour before his brother and averages 55 mph. Travis leaves from Dallas during rush hour so he only averages 45 mph.

This system of linear equations represents each brother’s distance \( d \) from Dallas, \( h \) hours after Roy leaves Pecos.

Roy: \( d = 450 - 55h \)
Travis: \( d = 45(h - 1) \)

Solve the system algebraically by finding when Travis’ distance from Dallas is the same as Roy’s distance.

Travis’ distance = Roy’s distance

\[ 45(h - 1) = 450 - 55h \]

14. Solve the equation for \( h \) and show your work.

15. What does the answer to Item 14 represent?

16. Explain how you would use your answer to Item 14 to find where Roy and Travis drive by each other on the highway.
TRY THESE C
Solve each system algebraically.

a. \( y = 3x - 8 \) \( y = 7x + 12 \)
b. \( 6x + y = 15 \) \( 2x + 2y = 10 \)
c. \( x + 2y = 5 \) \( x - 3y = 10 \)

17. Compare and contrast the advantages and disadvantages of using the graphing method, the numerical method, and the algebraic method of solving a system of equations.
18. You found where the brothers met on the highway by solving systems of linear equations using three different methods.

   a. Write a system of linear equations for the situation described below, and then solve it by graphing, making a table or solving an equation.

      On a return trip from Dallas to Pecos, Travis averages 55 mph. Roy realizes after 30 minutes that Travis forgot his cell phone, so he starts driving after him. If Roy averages 65 mph, where and when along the highway will he catch up to his brother?

   b. Verify your solution to Item(a) by using a different method than you used to solve the problem.
CHECK YOUR UNDERSTANDING

1. Solve by graphing.
   a. \( y = -2x + 5 \)
   b. \( 3x - y = 5 \)
   \[ y = \frac{1}{8}x - \frac{7}{2} \]
   \[ 4x - 2y = 4 \]

2. Solve by making a table.
   a. \( y = 10 - 5x \)
   b. \( 4x - y = -7 \)
   \[ y = 2x - 11 \]
   \[ 3x - 2y = -4 \]

3. Solve algebraically.
   a. \( y = 50 + 3x \)
   b. \( x + 3y = 9 \)
   \[ y = 100 - 2x \]
   \[ -3x + 2y = 8 \]

4. Tom leaves for Los Angeles averaging 65 mph. Michelle leaves for Los Angeles one hour later than Tom from the same location. She travels the same route averaging 70 mph. When will she pass Tom?

5. Juan bought a house for $200,000 and each year its value increases by $10,000. Tia bought a house for $350,000 and its value is decreasing annually by $5000. When will the two homes be worth the same amount of money?

6. How do you decide on the best method for solving a system of linear equations?

7. MATHEMATICAL REFLECTION What have you learned about solving systems of linear equations as a result of this activity?
ACTIVITY

My Notes

SUGGESTED LEARNING STRATEGIES: Shared Reading, Discussion Group, Create Representations

Systems of equations are useful for solving a variety of problems. Chemists form solutions by mixing liquids. A saline solution can be formed by dissolving salt in water or mixing together other saline solutions.

EXAMPLE 1

Noah is given two beakers of saline solution in chemistry class. One contains a 3% saline solution and the other an 8% saline solution. How much of each type of solution will Noah need to mix to create 150 mL of a 5% solution?

Step 1: To solve this problem, write and solve a system of linear equations.

Let \(x\) = number of mL of 3% solution.
Let \(y\) = number of mL of 8% solution.

Step 2: Write one equation based on the amounts of liquid being mixed. Write another equation on the amount of saline in the final solution.

\[x + y = 150\] The amount of the mixture is 150 mL.
\[0.03x + 0.08y = 0.05(150)\] The amount of saline in the mixture is 5%.

Step 3: To solve this system of equations by elimination, decide to eliminate the \(x\) variable.

\[-3(x + y) = -3(150)\] Multiply the first equation by \(-3\).
\[100(0.03x + 0.08y) = 100(7.5)\] Multiply the second equation by 100 to remove decimals.

\[-3x - 3y = -450\] Add the two equations to eliminate \(x\).
\[3x + 8y = 750\] Solve for \(y\).

\[5y = 300\]
\[y = 60\]

Step 4: Find the value of the eliminated variable \(x\) by using one of the original equations.

\[x + y = 150\]
\[x + 60 = 150\] Substitute 60 for \(y\).
\[x = 90\] Subtract 60 from both sides.

Step 5: Check your answers by substituting into the original second equation.

\[0.03x + 0.08y = 0.05(150)\]
\[0.03(90) + 0.08(60) \approx 0.05(150)\] Substitute 90 for \(x\) and 60 for \(y\).
\[2.7 + 4.8 \approx 7.5\]
\[7.5 = 7.5\] check

Solution: Noah needs 90 mL of 3% solution mixed with 60 mL of 8% solution to make 150 mL of the 5% solution.
TRY THESE A

a. Mary has $25,000 to invest. She decides to invest part of that amount at 3% and part at 5% interest for one year. The amount of interest she earns for both investments is $1100. How much was invested at each rate?

b. Solve the system using the elimination method:  
   \[7x + 5y = -1\]
   \[4x - y = -16\]

c. Sylvia wants to mix 100 pounds of Breakfast Blend coffee that will sell for $25 per pound. She is using two types of coffee to create the mixture. Kona coffee sells for $51 per pound and Columbian coffee sells for $11 per pound. How many pounds of each type of coffee should she use?

EXAMPLE 2

Solve the system using the linear combination method:  
\[4x - 5y = 30\]
\[3x + 4y = 7\]

Step 1: To solve this system of equations by linear combination, decide to eliminate the \(y\) variable.

Original system: Multiply the first equation by 4. Multiply the second equation by 5.

\[4x - 5y = 30\] \[\rightarrow 4(4x - 5y) = 4(30)\] \[\rightarrow 16x - 20y = 120\]
\[3x + 4y = 7\] \[\rightarrow 5(3x + 4y) = 5(7)\] \[\rightarrow 15x + 20y = 35\]

\[31x = 155\]
\[\frac{31x}{31} = \frac{155}{31}\]
\[x = 5\]

Step 2: Find \(y\) by substituting the value of \(x\) into one of the original equations.

\[4x - 5y = 30\]
\[\rightarrow 4(5) - 5y = 30\]
\[\rightarrow 20 - 5y = 30\]
\[\rightarrow -5y = 10\]
\[\rightarrow y = -2\]

Step 3: Check \((5, -2)\) in the second equation \(3x + 4y = 7\).

\[3x + 4y = 7\]
\[3(5) + 4(-2) = 7\]
\[15 - 8 = 7\]
\[7 = 7\] check

Solution: The solution is \((5, -2)\).
TRY THESE B

a. Solve the system of equations:
   \[3x - 2y = -21\]
   \[2x + 5y = 5\]

b. Solve the system of equations:
   \[7x + 5y = 9\]
   \[4x - 3y = 11\]

c. Phuong has $2.00 in nickels and dimes in his bank. The number of dimes is five more than twice as many nickels. How many of each type of coin are in his bank?

Another method for solving systems of equations involves substitution. This method is similar to the algebraic one you learned in Activity 3.4.

EXAMPLE 3

For the Valentine’s Day Dance, tickets for couples cost $12 and tickets for individuals cost $8. Suppose 250 students attended the dance and $1580 was collected from ticket sales. How many of each type of ticket was sold?

Step 1: Let \(x = \text{number of couples and } y = \text{number of single people.}\)

Step 2: Write one equation to represent the number of people attending. Write another equation to represent the money collected.

   \[2x + y = 250\] The number of attendees is 250.
   \[12x + 8y = 1580\] The total ticket sales is $1580.

Step 3: Use substitution to solve this system.

   \[2x + y = 250\] Solve the first equation for \(y.\)
   \[y = 250 - 2x\]

   \[12x + 8(250 - 2x) = 1580\] Substitute for \(y\) in the second equation.
   \[12x + 2000 - 16x = 1580\] Solve for \(x.\)
   \[-4x = -420\]
   \[x = 105\]

Step 4: Substitute the value of \(x\) into one of the original equations to find \(y.\)

   \[2x + y = 250\]
   \[2(105) + y = 250\] Substitute 105 for \(x.\)
   \[210 + y = 250\]
   \[y = 40\]

Solution: For the dance, 105 couples’ tickets and 40 singles’ tickets were sold.
TRY THESE C

a. Solve the system $x + 2y = 8$ and $3x - 4y = 4$ by substitution.

b. Solve the system of equations:
   \[5x - 2y = 0\]
   \[3x + y = -1\]

c. Patty and Toby live 345 miles apart. They decide to drive to meet one another. Patty leaves at noon traveling at an average rate of 45 mph and Toby leaves at 3:00 pm traveling at an average speed of 60 mph. At what time will they meet?

When a system of two linear equations in two variables is solved, three possible relationships can occur.

- Two distinct lines that intersect with one ordered pair as the solution.
- Two distinct lines that do not intersect because the lines are parallel and have no solution.
- Two lines that are coincident produce the same solution set—an infinite set of ordered pairs that satisfy both equations.

Systems of linear equations are classified by the relationships of their lines. Systems that produce two distinct lines when graphed are said to be independent. Systems that have no solution are said to be inconsistent.

The three systems in the chart represent each of the possible relationships described above.

<table>
<thead>
<tr>
<th>Relationship of Lines</th>
<th>Sketch</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Intersecting Lines</td>
<td></td>
<td>Independent and Consistent</td>
</tr>
<tr>
<td>Two Parallel Lines</td>
<td></td>
<td>Independent and Inconsistent</td>
</tr>
<tr>
<td>Two Coincident Lines</td>
<td></td>
<td>Dependent and Consistent</td>
</tr>
</tbody>
</table>
1. For each system below, complete the table with the information requested.

### The Nature of Solutions to a System of Two Linear Equations

<table>
<thead>
<tr>
<th>Equations in Standard Form:</th>
<th>Graph each system:</th>
<th>Write the Number of Solutions:</th>
<th>Write the Relationship of the Lines:</th>
<th>Solve Algebraically:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + y = 2$</td>
<td>![Graph 1]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6x + 3y = 6$</td>
<td>![Graph 2]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2x + y = 2$</td>
<td>![Graph 3]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x + y = 3$</td>
<td>![Graph 4]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x + 2y = -4$</td>
<td>![Graph 5]</td>
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</tbody>
</table>

Write the Equations in Slope-Intercept Form:

Compare the Slopes and $y$-intercepts:

Classify the System:
TRY THESE D
For each system below:
   i. Tell how many solutions the system has.
   ii. Describe the graph.
   iii. Classify the system.

   a. \(2x - 2y = 6\)
      \(y - x = -3\)

   b. \(y = 1.5x + 5\)
      \(3x - 2y = 10\)

   c. \(y = \frac{2}{3}x + 1\)
      \(4x - 6y = -6\)

   d. \(3x + 4y = 1\)
      \(2x - 5y = 16\)

Sometimes a situation has more than two pieces of information. For these more complex problems, you may need to solve equations that contain three variables. The solution will be the ordered triple \((x, y, z)\).

EXAMPLE 4
Solve the system by elimination.
\[
\begin{align*}
  x + 2y - z &= -3 \\
  -x - 3y + 2z &= 7 \\
  -2x + y + z &= -2
\end{align*}
\]

First, eliminate the same variable from two pairs of equations. Next use the resulting system of two-variable equations to find the values of the two variables. Then substitute those values into an original equation to find the value of the third variable.

Step 1: Use the first and second equations to eliminate \(x\). Add the equations.
\[
\begin{align*}
  x + 2y - z &= -3 \\
  -x - 3y + 2z &= 7 \\
  \hline
  -y + z &= 4
\end{align*}
\]

Step 2: Use the first and third equations to eliminate \(x\) again. Multiply the first equation by 2 so that the \(x\)-terms add to zero.
\[
\begin{align*}
  2(x + 2y - z) &= 2(-3) \quad \rightarrow \quad 2x + 4y - 2z = -6 \\
  -2x + y + z &= -2 \quad \rightarrow \quad -2x + y + z = -2
\end{align*}
\]

\[
5y - z = -8
\]

(continued on next page)
EXAMPLE 4 (continued)

Step 3: Use the two equations you found to write a new system with two variables. Add the equations to find the value of \( y \), because the \( z \)-terms add to zero.

\[
\begin{align*}
-y + z &= 4 \\
5y - z &= -8 \\
4y &= -4 \\
y &= -1
\end{align*}
\]

Step 4: Substitute the \( y \)-value into one of these equations to find \( z \).

\[
\begin{align*}
-y + z &= 4 \\
-(\text{-}1) + z &= 4 \\
1 + z &= 4 \\
z &= 3
\end{align*}
\]

Step 5: Substitute the \( y \)- and \( z \)-values into one of the original equations to find \( x \).

\[
\begin{align*}
x + 2y - z &= -3 \\
x + 2(\text{-}1) - 3 &= -3 \\
x - 2 - 3 &= -3 \\
x - 5 &= -3 \\
x &= 2
\end{align*}
\]

Solution: The solution of the system is \((2, -1, 3)\).

TRY THESE E

a. Check the solution in Example 4 by substituting \((2, -1, 3)\) into all three of the original equations.

b. Solve the system by elimination.

\[
\begin{align*}
2x + 2y - z &= -10 \\
x - y + z &= 4 \\
2x + y + z &= 7
\end{align*}
\]

Tickets for the Spring Festival cost $5 each. Children under 5 years pay $3 and senior citizens pay $4. A total of $870 was collected from ticket sales, and there were four times as many regular tickets as senior tickets sold. There were 200 people who attended the festival. The system below represents the relationships in the problem.

\[
\begin{align*}
x + y + z &= 200 \\
3x + 5y + 4z &= 870 \\
y &= 4z
\end{align*}
\]

c. What does \( x \) represent in the problem? What does \( y \) represent? What does \( z \) represent?
TRY THESE E (continued)

d. You can use substitution to solve the system above. Substitute \(4z\) for \(y\) in the first two equations. What new system in two variables do you get?

e. Solve that two-variable system. What values of \(x\) and \(z\) did you find?

f. Substitute your \(x\)- and \(z\)-values into an original equation. What value of \(y\) did you find?

g. How many of each type of ticket was sold?

CHECK YOUR UNDERSTANDING

1. Solve \(y = -\frac{1}{2}x + 5\) and \(3x - y = 2\) by substitution.

2. Solve \(5x + 6y = 8\) and \(2x - 3y = 5\) using the elimination method.

3. Solve the system \(3x - 4y = 8\) and \(y = \frac{3}{4}x - 2\) using any method. Classify the solutions.

4. Approximate the point of intersection for the system of linear equations graphed below. Verify that the selected point is a solution for the system.

5. Jillian wants to create a 20% solution of ethanol. She has 300 mL of a 4% solution and pure 100% ethanol. How much pure ethanol should she mix with the 4% solution and how much of the 20% solution will be produced after the mixing is completed?

6. Find the solution of the system \(y = -\frac{2}{5}x + 1\) and \(2x + 5y = 3\) by any method. Classify the solutions.

7. Solve the system.

\[
\begin{align*}
x - y + z &= -1 \\
2x + y + z &= 5 \\
x - 2y - 3z &= -13
\end{align*}
\]

8. MATHEMATICAL REFLECTION: Consider the different methods that have been studied in Tale of Two Truckers and in this activity to write about when it is advantageous to use one method of solution instead of another. Be sure to consider and comment on each of the methods:

- Substitution
- Using Tables
- Graphing
- Elimination
1. Graph each inequality on the number lines and grids provided.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph all $x$</th>
<th>Graph all $(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 2$</td>
<td></td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>$x \geq -3$</td>
<td></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>$x &lt; 2 \text{ and } x \geq -3$</td>
<td><img src="image3.png" alt="Graph" /></td>
<td></td>
</tr>
</tbody>
</table>

2. Compare and contrast the graphs you made in the third row of the table. In your explanation, compare the graphs to those in the first two rows and use the following words: dimension, half-line, half-plane, open, closed, and intersection.
3. On a coordinate grid, graph the solutions common to the inequalities $y \leq 4$ and $y > 1$.

Solving a system of linear inequalities means finding all solutions that are common to all inequalities in the system.

4. For the system of inequalities below, complete the chart by deciding if each ordered pair is a solution. Explain your answers.

$$x + y > 2$$
$$2x - y \geq -5$$

<table>
<thead>
<tr>
<th>Ordered Pair</th>
<th>Is it a solution?</th>
<th>Why or why not?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-2, -3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(3, 2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(3, -1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(0, 5)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Find two more solutions of the system in Item 4.
Since a system of inequalities has infinitely many solutions, you can represent all solutions using a graph. To solve a system of inequalities, graph each inequality on the same coordinate grid by shading a half-plane. The region that is the intersection of the two shaded half-planes represents all solutions of the system.

**EXAMPLE**

Solve the system of inequalities.

\[ x + y \geq 1 \]
\[ x - 3y > 3 \]

**Step 1:** First, graph \( x + y \geq 1 \).

**Step 2:** Next, graph \( x - 3y > 3 \).

**Step 3:** Determine the solution.

**Solution:** The solution is the doubly shaded region shown in Step 3.

To graph an inequality, it may be helpful to solve the inequality for \( y \) first.

When finding the solution to a system of inequalities, it is helpful to shade each inequality with a different pattern or a different color.
6. Graph \( \frac{x + y}{2} > 2 \) on the coordinate grid below.

7. The system in Item 6 is the same as the system in Item 4. Plot the points listed in Items 4 and 5 on the graph above. Where do the points that are solutions lie? Where do points that are not solutions lie?

8. Find the solutions to the systems of inequalities by graphing.
   a. \( y \geq x - 1 \)
   \( y \leq -\frac{1}{2}x + 2 \)
Systems of Linear Inequalities

Which Region Is It?

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Discussion

b. \(2x + 3y > 6\)
   \(x - 2y < 4\)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{x} & -5 & -4 & -3 & -2 \\hline
\text{y} & -5 & -4 & -3 & -2 \\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{x} & 1 & 2 & 3 & 4 \\hline
\text{y} & 1 & 2 & 3 & 4 \\hline
\end{array}
\]

c. Describe how you found the solutions for these systems.

9. Find the solutions to the systems of inequalities by graphing.

a. \(y > \frac{1}{2}x + 2\)
   \(x - 2y > 8\)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{x} & -5 & -4 & -3 & -2 \\hline
\text{y} & -5 & -4 & -3 & -2 \\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{x} & 1 & 2 & 3 & 4 \\hline
\text{y} & 1 & 2 & 3 & 4 \\hline
\end{array}
\]
ACTIVITY 3.7
Systems of Linear Inequalities
Which Region Is It?

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Discussion, Guess and Check

My Notes

b. \(3x + y < 3\)
   \(y + 2 \geq -3x\)

10. Compare and contrast the systems in Items 9(a) and 9(b).

CHECK YOUR UNDERSTANDING

1. Use guess and check to find an ordered pair that is a solution to the system of inequalities.
   \[3x + y \geq 6\]
   \[x + 3y < 3\]
2. Solve the system in Item 1 by graphing. Confirm that the ordered pair you wrote in Item 1 is a solution. Explain.
3. Solve the system of linear inequalities.
   \[y > 2x - 3\]
   \[y < 4x - 3\]
4. **MATHEMATICAL REFLECTION** Why is graphing the preferred method of representing a solution to a system of linear inequalities?
Systems of Equations and Inequalities

ALL SYSTEMS GO

1. Rajesh and his brother Mohib are each mailing a birthday gift to a friend. Rajesh’s package weighs three more pounds than twice the weight of Mohib’s package. If each package weighed 2 pounds more, the combined weight of the packages would be 19 pounds. Write a system of equations that represents this information and tell what the variables represent. Use it to find the weight of each package.

2. Solve the systems of equations and show your work.
   a. \(2x + 3y = -7\)
   \(3x - 5y = 37\)
   b. \(y = 2x - 5\)
   \(-6x + 3y = 2\)

3. The solution set of a system of inequalities is shown in the graph below as a shaded region. The equations of the boundaries are \(x + 4y = 8\) and \(y = x - 1\). Write the system of inequalities.
## Systems of Equations and Inequalities

**ALL SYSTEMS GO**

<table>
<thead>
<tr>
<th></th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Knowledge</strong></td>
<td>The student:</td>
<td>The student:</td>
<td>The student attempts to solve both systems, but the solutions are incorrect.</td>
</tr>
<tr>
<td></td>
<td>• Defines the variables for the system of equations (1)</td>
<td>• Defines only one of the variables.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Solves both systems correctly. (2a, b)</td>
<td>• Solves only one of the systems correctly.</td>
<td></td>
</tr>
<tr>
<td><strong>Problem Solving</strong></td>
<td>The student finds the correct weight of both packages, based on the system given. (1)</td>
<td>The student finds the correct weight of one of the packages.</td>
<td>The student gives incorrect weights for both of the packages.</td>
</tr>
<tr>
<td><strong>Representations</strong></td>
<td>The student:</td>
<td>The student:</td>
<td>The student attempts to write the systems, but both are incorrect or incomplete.</td>
</tr>
<tr>
<td></td>
<td>• Writes a correct system of equations. (1)</td>
<td>• Writes only one correct equation or inequality for each system, OR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Writes a correct system of inequalities. (3)</td>
<td>• Writes both systems but only one system is complete and correct.</td>
<td></td>
</tr>
<tr>
<td><strong>Communication</strong></td>
<td>The student shows the correct and complete work for solving both systems. (2a, 2b)</td>
<td>The student shows work for solving both systems, but the work shown for only one of the solutions is correct and complete.</td>
<td>The student attempts to show the work for solving both systems, but the work is incorrect or incomplete.</td>
</tr>
</tbody>
</table>
ACTIVITY 3.1

Use the tables for Items 1–5.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>6</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>7</td>
<td>-6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
<td>-7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>9</td>
<td>-8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>10</td>
<td>-9</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>11</td>
<td>-10</td>
</tr>
</tbody>
</table>

1. Find the rate of change for each table.
2. At what $x$-value does the rate of change switch patterns?
3. Find a function that represents the first table of data, and write the domain for the function.
4. Find a function that represents the second table of data, and write the domain for the function.
5. Write the piecewise defined function that represents the data in the tables.

Use the piecewise defined function for Items 6–7.

$$f(x) = \begin{cases} 
|x|, & \text{if } x < 2 \\
-x + 4, & \text{if } x \geq 2
\end{cases}$$

6. Sketch a graph of the function.
7. Does the function in Item 6 have any gaps?

ACTIVITY 3.2

8. Fresno is 150 miles from Bakersfield. A driver leaves Fresno and averages 62 miles per hour, and is traveling to Bakersfield. Find the distance $d$ from Fresno, given the time $t$, in hours since the driver left.
9. A driver traveling from Fresno to Bakersfield averaged 57 miles per hour. Write an equation for the distance $d$ from Fresno, given the time $t$, in hours since the driver left.

ACTIVITY 3.3

12. Which ordered pairs are a solution of the inequality $5y - 3x \leq 7$?
   a. (0, 0)   b. (3, 5)
   c. (-2, -5)   d. (1, 2.5)
   e. (5, -3)
13. Write an inequality for the half-plane. Is it open or closed?

14. Write an inequality for the half-plane. Is it open or closed?
15. Sketch a graph of the inequality \( y \geq -\frac{2}{3}x + 2 \).

16. Sketch a graph of the inequality \( 3y > 7x - 15 \).

**ACTIVITY 3.4**

17. Determine whether the lines containing the given slopes will be parallel, perpendicular or neither.
   a. \( m_1 = 10, m_2 = \frac{1}{10} \)
   b. \( m_1 = -\frac{6}{10}, m_2 = -\frac{3}{5} \)
   c. \( m_1 = -\frac{3}{11}, m_2 = 3\frac{2}{3} \)
   d. \( m_1 = \frac{1}{2}, m_2 = -2 \)

18. Given the equation of line \( l_1 \) is \( y = 6x \).
   a. Write the equation of a line parallel to \( l_1 \)
   b. Write the equation of a line perpendicular to \( l_1 \).

19. Write the equation of a line that is parallel to the line \( 2x - y = 5 \) and contains the point \((3, -1)\).

20. Write the equation of a line that is perpendicular to the line \( y = \frac{1}{4}x \) and contains the point \((-2, 3)\).

21. Use the slopes to determine whether the lines in the graph below are parallel, perpendicular or neither. Explain your answer.

**ACTIVITY 3.5**

22. What is the \( x \)-coordinate of the solution to the system \( x + y = 5 \) \( x - 2y = -1 \)?
   a. \( x = 1 \)    b. \( x = 2 \)    c. \( x = 3 \)    d. \( x = 4 \)

23. Which ordered pair is a solution to the system shown at the right? \( y = \frac{2}{3}x + 3 \) \( y = -3x + 14 \)
   a. \((-3, 1)\)    b. \((5, -1)\)    c. \((3, 5)\)    d. \((5, 3)\)

24. Which system’s solution is represented by the graphs shown below?
   a. \( y = 2x + 3 \)    b. \( y = 2x - 6 \)
   c. \( y = \frac{1}{5}x - 6 \)    d. \( y = \frac{1}{5}x + 3 \)
   c. \( y = 5x - 6 \)    d. \( y = 5x + 3 \)
   c. \( y = \frac{1}{2}x + 3 \)    d. \( y = \frac{1}{2}x - 6 \)
25. A bushel of apples currently costs $10 a bushel and the price is increasing by $0.50 per week. The price of bushel of pears currently costs $15 a bushel and the price is decreasing by $0.25 per week. Which system of linear equations could be used to determine when the two fruits will cost the same amount per bushel?

a. \( y = 0.5x + 10 \)  
   \( y = -0.25x + 15 \)

b. \( y = 0.25x + 15 \)  
   \( y = -0.5x + 10 \)

c. \( y = 10x + 0.5 \)  
   \( y = 15x - 0.25 \)

d. \( y = 5x + 3 \)  
   \( y = -\frac{1}{2}x - 6 \)

26. Solve the system of linear equations using the method of your choice.

a. \( y = 3x - 4 \)  
   \( y = \frac{2}{5}x + 9 \)

b. \( x + y = 7 \)  
   \( x - 3y = -1 \)

27. Solve \( 3x + 4y = 17 \) and \( 5x - 4y = 7 \) using the elimination method, and classify the system.

28. Solve \( y = \frac{2}{3}x - 8 \) and \( 2x + y = 8 \) by substitution.

29. Solve the system \( -x + y = 1 \) and \( y = -x + 5 \) using any method.

30. Approximate the point of intersection for the system of linear equations graphed below. Verify that the point of intersection is a solution for this system.

31. Peter placed an order with an online nursery for 6 apple trees and 5 azaleas and the order came to $147. The next order for 3 apple trees and 4 azaleas came to $96. What was the unit cost for each apple tree and for each azalea?

32. Solve the system \( 3x + 4y = 8 \) and \( y = -\frac{3}{4}x + 2 \) and classify the system.

33. Solve the system.

\[
\begin{align*}
3x + y - 2z &= -1 \\
-3x + 3y + z &= 9 \\
-3x + 2y + z &= 7 
\end{align*}
\]

34. Determine which of the following ordered pairs are solutions to the given systems of inequalities.

\[
\{(5, 3), (-2, 1), (1, 2), (2, -3), (3, 5), (-2, 3), (2, 0)\}
\]

a. \( y < -x + 3 \)
   \( y \geq x - 2 \)
   \( y > -\frac{1}{2}x + 1 \)

b. \( 2x - y \leq 0 \)

35. Graph each of the systems of inequalities in Item 1. Graph the points that you chose as solutions from Item 1 on the same coordinate grid to verify that they are solutions.

36. Find four ordered pairs that are solutions to the following system of equations. Explain how you chose your points.

\[
\begin{align*}
4x + 3y &\geq -12 \\
2x - y &< 4 
\end{align*}
\]

37. Solve the following system of inequalities.

a. \( 2x + 3y \geq 15 \)
   \( 5x - y \geq 3 \)

b. \( x - 4y \geq 4 \)
   \( 4y - x > 8 \)
Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

**Essential Questions**

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - Why would you use multiple representations of linear equations and inequalities?
   - How are systems of linear equations and inequalities useful in interpreting real world situations?

**Academic Vocabulary**

2. Look at the following academic vocabulary words:
   - substitution method
   - elimination method
   - linear inequality
   - piecewise defined function
   - system of linear equations
   - system of linear inequalities

Choose three words and explain your understanding of each word and why each is important in your study of math.

**Self-Evaluation**

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. If \(|x - 3| > 3\), which of the following could be a value of \(x\)?

A. \(-1\)
B. 0
C. 2
D. 6

2. The manager of the local fair wants to know the number of children who attended the fair last night. The fair charges $8 for each adult ticket and $4 for each child ticket. Last night, 200 tickets were sold for a total of $1304. How many children attended the fair last night?

3. An artist sells earrings from a booth at the mall. Rent for the booth is $250. The artist makes $6 from each pair of earrings sold and needs to make a profit of $500.00. The inequality below represents the profit in dollars, \(P\), and \(n\) is the number of pairs of earrings sold.

\[P < 6n - 250\]

What is the minimum number of pairs of earrings that the artist must sell to earn a profit over $500?
4. The senior class at a school needs to raise $500.00 to cover the cost for the prom. The president of the senior class plans to have a talent show to raise money. The president decides to charge $5.00 per student, and adults will pay $8.00. The number of tickets available for the talent show is 82.

Part A: Write a system of equations in terms of students, $c$, and adults, $d$, that can be used to find the number of students and adults who attended the show.

Part B: How many adults attended the talent show?
How do you know?